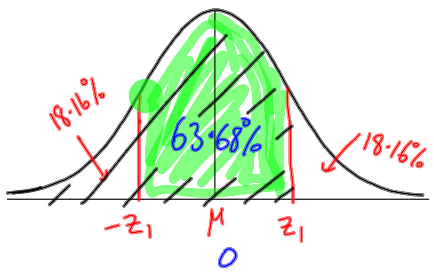
Find the value of z_1 in each of the following if z is a random variable with normal distribution.

19. $P(-z_1 \leqslant z \leqslant z_1) = 0.6368$ **20.** $P(-z_1 \leqslant z \leqslant z_1) = 0.8438$

Symmetry!



$100\% - 63.68\% = 36.32\%$

$\frac{36.32\%}{2} = 18.16\%$

$P(z \leq z_1) = 63.68\% + 18.16\% = 81.84\%$

Look up 0.8184 in tables to get z_1

z_1	0·00	0·01	0·02
0·0	0·5000	5040	5080
0·1	0·5398	5438	5478
0·2	0·5793	5832	5871
0·3	0·6179	6217	6255
0·4	0·6554	6591	6628
0·5	0·6915	6950	6985
0·6	0·7257	7291	7324
0·7	0·7580	7611	7642
0·8	0·7881	7910	7939
0·9	0·8159	8186	8212
1·0	0·8413	8438	8461

$\Rightarrow z_1 = 0.91$

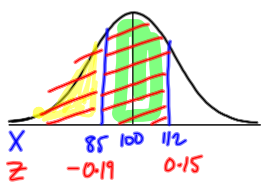
$\Rightarrow -z_1 = -0.91$

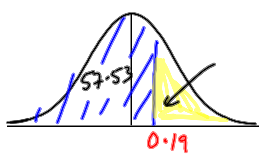
In each of the following, x is a random variable with normal distribution with a given mean (μ) and standard deviation (σ). Find the required probability in each case.

25. $\mu = 100, \sigma = 80$, find (i) $P(85 \leqslant x \leqslant 112)$ (ii) $P(105 \leqslant x \leqslant 115)$.

$\boxed{z = \frac{X - \mu}{\sigma}}$

$\mu = 100$
$\sigma = 80$





$z_1 = \frac{85-100}{80} = -0.1875 \approx -0.19$

$z_2 = \frac{112-100}{80} = 0.15$

$P(z \leq 0.15) = 55.96\%$

$P(z \leq 0.19) = 57.53\%$

$P(z \leq -0.19) = 100\% - 57.53\% = 42.47\%$

$P(-0.19 \leq z \leq 0.15) = 55.96\% - 42.47\%$
$= 13.49\%$

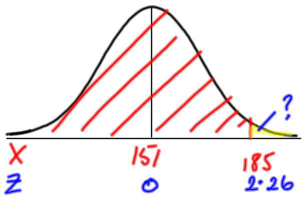
32. The mean length of 500 laurel leaves from a certain bush is 151 mm and the standard deviation is 15 mm. Assuming that the lengths are normally distributed, find how many leaves measure

(i) greater than 185 mm

(ii) between 120 mm and 155 mm.

$\mu = 151$ mm

$\sigma = 15$ mm



(i) $P(X \geq 185\text{mm})$?

$$Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{185-151}{15} = \frac{34}{15} = 2{\cdot}26$$

$P(Z \leq 2{\cdot}26) = 98{\cdot}81\%$

$P(Z \geq 2{\cdot}26) = 100\% - 98{\cdot}81\% = 1{\cdot}19\%$

How many of 500 leaves greater than 185mm $\Rightarrow$ $1{\cdot}19\%$ of $500 = 5{\cdot}95 \approx \boxed{6}$