Probability 1

Chapter 1

Section 1.5 Mutually exclusive events – The addition rule

M.E. events can not occur together.

E_1 = pick red card
E_2 = pick spade

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Text & Tests 5
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STRAND 1
PROBABILITY & STATISTICS

23

Consider the following two events when drawing a card from a pack of 52 playing cards:

A = drawing an ace B = drawing a king.

These two events are said to be **mutually exclusive** as they cannot occur together.

If the events A and B cannot happen together, then

$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

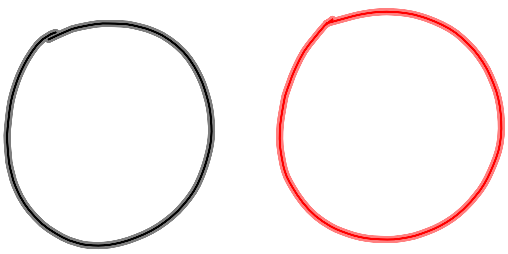
This is called the **addition law** for mutually exclusive events.

> Outcomes are mutually exclusive if they cannot happen at the same time.

So $P(\text{draw an ace } \textbf{or} \text{ king}) = P(\text{ace}) + P(\text{king})$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52} = \frac{2}{13}$$



Mutually exclusive

When events are not mutually exclusive

We will now consider events which may occur at the same time.

If A is the event: selecting an ace from a pack of cards and

B is the event: selecting a heart from a pack of cards

then $P(\mathrm{A}) = \frac{4}{52}$ and $P(\mathrm{B}) = \frac{13}{52}$

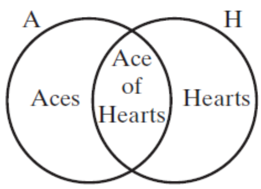
In this situation, both events may occur at the same time since the *ace of hearts* is common to both.

In general, when two events A and B can occur at the same time,

$$P(\mathrm{A\ or\ B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A\ and\ B})$$

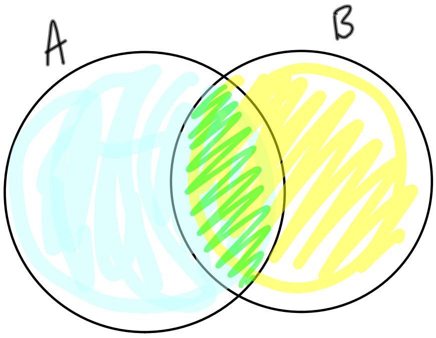
Thus in the example given above,

$$\begin{aligned} P(\text{ace or heart}) &= P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart}) \\ &= \tfrac{4}{52} + \tfrac{13}{52} - \tfrac{1}{52} \\ &= \tfrac{16}{52} \end{aligned}$$



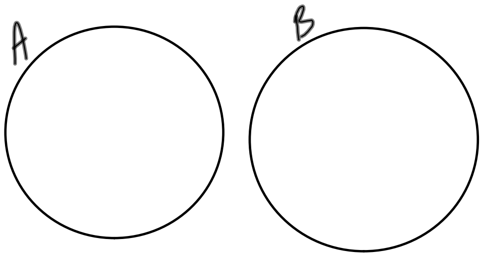
This result can be verified as there are 4 aces and 13 hearts in a pack of cards. Since one of the aces is the ace of hearts, there are 16 aces or hearts in the pack.

i.e. $P(\text{ace or a heart}) = \frac{16}{52}$, as already found.



not M.E.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



M.E.

$$P(A \cup B) = P(A) + P(B) - \overset{0}{\cancel{P(A \cap B)}}$$

Example 1

A card is drawn at random from a pack of 52.
What is the probability that the card is

(i) a club (ii) a king (iii) a club or a king
(iv) a red card (v) a queen (vi) a red card or a queen

(i) $P(\text{club}) = \frac{1}{4}$

(ii) $P(\text{king}) = \frac{1}{13}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(\text{King and Club}) = \frac{1}{52}$

(iii) $P(\text{club or King}) =$
$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$

(iv) $P(\text{Red}) = \frac{1}{2}$

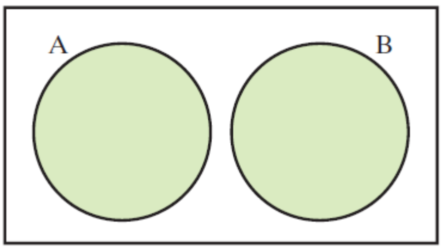
(v) $P(\text{Queen}) = \frac{1}{13}$

$P(\text{Red and Queen}) = \frac{2}{52}$

(vi) $P(\text{Red or Queen}) = \frac{1}{2} + \frac{1}{13} - \frac{2}{52}$
$= \frac{7}{13}$

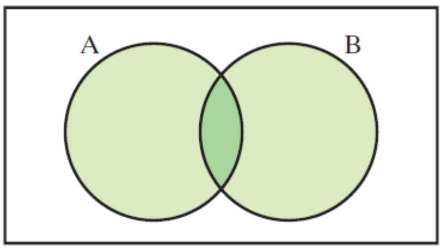
Venn diagrams for mutually exclusive events

(i) Mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B})$

(ii) Non-mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$

Example 2

A and B are two events such that $P(\text{A}) = \frac{19}{30}$, $P(\text{B}) = \frac{2}{5}$ and $P(\text{A} \cup \text{B}) = \frac{4}{5}$. Find $P(\text{A} \cap \text{B})$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{4}{5} = \frac{19}{30} + \frac{2}{5} - P(A \cap B)$$

$$\frac{4}{5} = \frac{31}{30} - P(A \cap B)$$

$$P(A \cap B) = \frac{31}{30} - \frac{4}{5} = \frac{7}{30}$$

Exhaustive events

Consider these events when throwing a dice:

A: Getting an odd number
B: Getting an even number

These two events contain all the possible outcomes when a dice is thrown.
These events are said to be **exhaustive**.
If A and B are exhaustive events, then

$$P(\text{A}) + P(\text{B}) = 1.$$

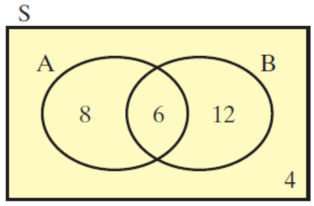
A set of events is exhaustive if the set contains all possible outcomes.

Venn diagrams

A Venn diagram is a useful way to represent data or probabilities.
Each region of a Venn diagram represents a different set of data.

The Venn diagram on the right shows two sets, A and B, in the sample space S.

The number of elements in each region is also shown.



8 is the number of elements in A but not in B. This is written as A/B.
12 is the number of elements in B but not in A. This is written as B/A.
6 is the number of elements in A and B. This is written as $A \cap B$.
4 is the number of elements in neither A nor B.
This is written as $(A \cup B)'$.

If information is presented in the form of a Venn diagram, it is easy to write down the probability of different events occurring.

the probability of either A or B is $P(A \cup B) = \dfrac{8 + 6 + 12}{8 + 6 + 12 + 4} = \dfrac{26}{30} = \dfrac{13}{15}$

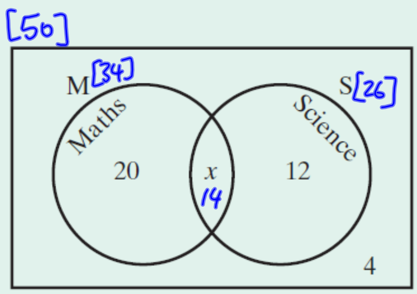
Example 3

The given Venn diagram represents the subjects taken by a group of 50 pupils.

(i) Find the value of x.

Now find the probability that a person chosen at random takes

(ii) both subjects
(iii) neither subject
(iv) Science but not Maths
(v) Maths or Science or both.



(i) $x = 50 - (20 + 12 + 4) = 50 - 36 = 14$

(ii) $P(\text{Both}) = \frac{14}{50} = \frac{7}{25}$ (28%)

(iii) $P(\text{neither}) = \frac{4}{50} = \frac{2}{25}$ (8%)

(iv) $P(S \setminus M) = \frac{12}{50} = \frac{6}{25}$ (24%)

(v) $P(M \cup S) = \frac{46}{50} = \frac{23}{25}$ (92%)

Exercise 1.5

1. A box contains discs numbered 1 to 16.
If a disc is selected at random, what is the probability that it is
(i) an odd number (ii) a multiple of 4
(iii) an odd number or a multiple of 4?

(i) $P(\text{odd}) = \frac{1}{2}$

(ii) $P(\text{multiple of } 4) = \frac{1}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = 0$
in this case
i.e.. M.E. events

(iii) $P(\text{odd or multiple of } 4) = \frac{1}{2} + \frac{1}{4}$

$= \frac{3}{4}$