Example 2

Given that 10% of apples are bad, find the probability that in a box containing 6 apples, there is

(i) no bad apple
(ii) just one bad apple
(iii) at least one bad apple.

$$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$$

Success = bad

$p = 10\% = 0.1$
$q = 90\% = 0.9$
$n = 6$

(i) $P(0 \text{ bad}) = \binom{6}{0}(0.1)^0(0.9)^6$

$r = 0$

$= (0.9)^6$

(ii) $P(1 \text{ bad}) = \binom{6}{1}(0.1)^1(0.9)^5$

$r = 1$

$= \dfrac{6 \cdot 9^5}{10^6}$

(iii) $P(\text{at least one bad}) = 1 - P(0 \text{ bad})$

$= 1 - 0.9^6$

Probability of k^{th} success on n^{th} Bernoulli trial

In example 1 on the previous page, we worked out the probability of getting 3 sixes when a dice is thrown 5 times. If the same dice is thrown continuously until a six appears for the fourth time, how do we find the probability that the 4th six appears on the tenth throw?

For a 4th six to appear on the 10th throw,

(i) we need to get 3 sixes on the first nine throws, and then
(ii) get a six on the 10th throw.

Three sixes on the first nine throws is given by

$$\binom{9}{3} p^3 q^{9-3} \quad \text{i.e.} \quad \binom{9}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^6$$

$$84 \times \frac{5^6}{6^9} = 0.13$$

$P(\text{six on the 10th throw}) = \frac{1}{6}$

Thus $P(\text{4th six on the 10th throw}) = 0.13 \times \frac{1}{6} = 0.0217$

Example 3

A card is drawn at random from a normal deck of playing cards and then replaced.
The process is repeated until the third diamond appears.
Find the probability that this happens when the tenth card is drawn.

$$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$$

$10^{th} = 3^{rd}$ diamond

$\Rightarrow$ 2 diamonds in first 9 events

$\Rightarrow P\left(\begin{smallmatrix}\text{3rd on}\\\text{10th go}\end{smallmatrix}\right) = \underbrace{P\left(\begin{smallmatrix}\text{2 successes}\\\text{out of 9}\end{smallmatrix}\right)}_{\text{Bernoulli}} \times P(\text{diamond})$

Success = diamond

$p = \frac{1}{4}$ $q = \frac{3}{4}$ $n = 9$ $r = 2$

$= \left[\binom{9}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^7\right] \times \frac{1}{4}$

$= 0.075$

2. A coin is tossed 5 times. What is the probability of getting
(i) exactly 1 head (ii) exactly 3 heads?

let "success" = head

$P(H) = \frac{1}{2} = p$

$P(T) = \frac{1}{2} = q$

$n = 5$

$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$
Bernoulli

(i) $P\left(\begin{smallmatrix}\text{1 success}\\\text{out of 5}\end{smallmatrix}\right) = \binom{5}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$

$r = 1$ $= \frac{5}{32}$

(ii) $P\left(\begin{smallmatrix}\text{3 successes}\\\text{out of 5}\end{smallmatrix}\right) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$r = 3$ $= \frac{5}{16}$

8. The probability that a first-year university student will complete four years of study is $\frac{1}{3}$. What is the probability that, of four first-year students, at least three will complete four years of study?

Success = Complete 4 years study

$p = P(\text{Complete 4 yrs}) = \frac{1}{3}$

$q = \frac{2}{3}$

$n = 4$

$P\left(\begin{smallmatrix}r \text{ successes}\\ \text{in } n \text{ trials}\end{smallmatrix}\right) = \binom{n}{r} p^r q^{n-r}$

Bernoulli

$P\left(\begin{smallmatrix}\text{that 3 OR 4 students}\\ \text{complete all 4 yrs}\end{smallmatrix}\right) = ?$

$= \underbrace{P(\text{exactly } 3)}_{r=3} + \underbrace{P(\text{exactly } 4)}_{r=4}$

$= \binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 + \binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$

$= \frac{8}{81} + \frac{1}{81} = \frac{9}{81}$

$= \frac{1}{9}$

9. If 20% of the bolts produced by a machine are defective, determine the probability that, out of 4 bolts selected at random,

(i) 2 bolts are defective

(ii) not more than 2 bolts are defective.

Success = "defective"

$p = 20\% = 0.2$

$q = 80\% = 0.8$

$n = 4$

Bernoulli

$P\left(\begin{smallmatrix}r \text{ successes}\\ \text{out of } n\end{smallmatrix}\right) = \binom{n}{r}$

(i) $\underbrace{P(2 \text{ defective})}_{r=2} = \binom{4}{2}(0.2)^2(0.8)^2$

$= \frac{96}{625}$

(ii) $P\left(\begin{smallmatrix}\text{not more than}\\ 2 \text{ defective}\end{smallmatrix}\right) = P(2 \text{ OR } 1 \text{ OR } 0 \text{ defective})$

$= \frac{96}{625} + \underbrace{\binom{4}{1}(0.2)^1(0.8)^3}_{r=1} + \underbrace{\binom{4}{0}(0.2)^0(0.8)^4}_{r=0}$

$= \frac{96}{625} + \frac{256}{625} + \frac{256}{625}$

$= \frac{608}{625}$