Example 4

a How many four-digit numbers can be formed using the digits $0, 2, 5, 7, 8$ if a digit cannot be used more than once in any number? *(not starting with 0)*

b   (i) How many of these numbers are greater than 5000? *starts with 5, 7, 8*
    (ii) How many of these numbers are odd? *ends with 5 or 7*

(a) $\boxed{4} \times \boxed{4} \times \boxed{3} \times \boxed{2} = 96$
     not 0

(b) i $\boxed{3} \times \boxed{4} \times \boxed{3} \times \boxed{2} = 72$
         5, 7, 8

     ii $\boxed{3} \times \boxed{3} \times \boxed{2} \times \boxed{2} = 36$
         not 0   *odd*

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4. Permutations of n different objects taking r of them at a time

To find the number of ways the five letters A, B, C, D, E can be arranged in a line when taking 3 at a time, we could use boxes as follows:

$\boxed{5}\ \boxed{4}\ \boxed{3} = 5 \times 4 \times 3 = 60$ ways.

The first box can be filled in 5 ways, the second in 4 ways and the third in 3 ways.

Notice that $5 \times 4 \times 3 = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \dfrac{5!}{2!} = \dfrac{5!}{(5-3)!}$

We use the notation $^5P_3$ to denote the number of permutations of 5 objects, taking them 3 at a time.

$^5P_3 = 5 \times 4 \times 3$ ... starting at 5 and going down 3 numbers

Similarly, $^8P_4 = 8 \times 7 \times 6 \times 5 \left(\text{or } \dfrac{8!}{(8-4)!}\right)$

LOG TABLES

In general, the number of arrangements of n objects, taking r at a time, is given on the right.

$^nP_r = \dfrac{n!}{(n-r)!}$

Example 5

(i) Evaluate $^{10}P_3$ (ii) Find n if $7[^nP_3] = 6[^{n+1}P_3]$

(i) $10P3 = 720$ calculator

or $\boxed{10} \times \boxed{9} \times \boxed{8} = 720$

(ii) $7(nP3) = 6(n+1 \; P3)$

$7(\boxed{n} \times \boxed{n-1} \times \boxed{n-2}) = 6(\boxed{n+1} \times \boxed{n} \times \boxed{n-1})$

$7n - 14 = 6n + 6$

$n = 20$

check is $7(20 P3) = 6(21 P3) = 47880$

Example 6

a How many different four-letter arrangements can be made from the letters of the
b word THURSDAY if a letter cannot be repeated in an arrangement?
How many of the arrangements begin with the letter D and end with a vowel?

(a) $\boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5} = 1680$

(b) $\boxed{1} \times \boxed{6} \times \boxed{5} \times \boxed{2} = 60$
       Start   Vowel
       with
       D