Probability 1

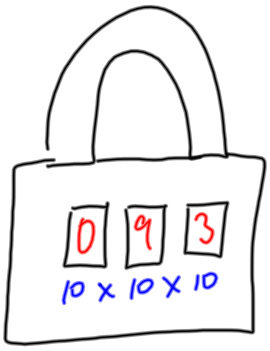
chapter 1

Section 1.1 Permutations

Arrangements

PROJECT MATHS
Text & Tests 5
LEAVING CERTIFICATE
HIGHER LEVEL
STRAND 1
PROBABILITY & STATISTICS

1



How many different possible codes?

Smallest = 0 0 0
biggest = 9 9 9

1000

Fundamental Principle of Counting

Outputs = $\boxed{10} \times \boxed{10} \times \boxed{10}$

$= 1000$

How many meals?

Outcomes $= 3 \times 4 \times 2 = 24$

How many "words" possible with 3 letters from the letters M A T H S

(i) no restrictions?

$$\boxed{5} \times \boxed{4} \times \boxed{3} = 60$$

(ii) must start with a constanant?

$$\boxed{4} \times \boxed{4} \times \boxed{3} = 48$$

Key words

the fundamental principle of counting (menu) · permutations (arrangements) · factorial ! · combinations · relative frequency · expected frequency · addition rule · mutually exclusive · exhaustive events · Venn diagrams · independent events · multiplication rule · conditional probability

Factorial !

How many 5 letter "words" possible from the letters M A T H S (use once only)

$\boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 5! = 120$

Using letters MATHS
how many 4 letter "words" possible?

$\boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} = 120$

This is a permutation – ORDER MATTERS!
MATH is not same a MAHT

Permutation

$$_nP_r = {}_5P_4 = 120$$

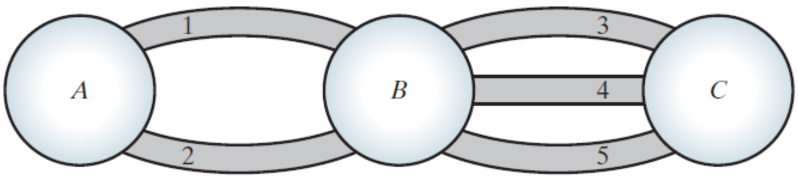
use calculator

choose from

choose how many

1. The fundamental principle of counting

The diagram below shows three towns A, B and C.
There are 2 roads from A to B and 3 roads from B to C.



The number of ways a person can travel from A to C is 6. These are listed as follows:

$(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)$.

This illustrates that there are 2 ways of going from A to B and for **each** way of going from A to B, there are 3 ways of going from B to C.
Thus there are $3 \times 2 = 6$ ways of going from A to C.

Similarly, if a car manufacturer produces 5 different models of a car and each model comes in 4 different colours, then a customer has 5×4 i.e. 20 different ways in which he can select a car.

These examples illustrate what is generally referred to as the **Fundamental Principle of Counting** which can be stated as follows:

Fundamental Principle of Counting

If one task can be accomplished in m ways **and** following this a second task can be accomplished in n ways, then the first task followed by the second task can be accomplished in $m \times n$ ways.

Note: The *Fundamental Principle of Counting* can be extended to any number of tasks. For example, if there are 3 roads from A to B, 2 roads from B to C and 4 roads from C to D, then there are

$3 \times 2 \times 4$ i.e. 24 ways of going from A to D.

Example 1

From the given number of names and surnames, determine how many different name–surname pairs are possible.

Name	Surname
Mary	Mooney
Jennifer	Byrne
Cormac	O'Brien
Kate	Lawiski
Barry	
Shane	McCarthy
6	5

Outcomes = $6 \times 5 = 30$

2. Permutations

A permutation is an arrangement of a number of objects in a certain order.

If the letters A, B and C are written in a row, one after another, there are 6 possible arrangements as follows:

$ABC \quad ACB \quad BAC \quad BCA \quad CAB \quad CBA$ ORDER MATTERS

Each arrangement is called a **permutation**; so there are 6 different permutations possible.

When calculating the number of permutations that can be made from a number of items, it is convenient to use "boxes", and working from the left, write down the number of ways in which each box can be filled.

Again, returning to the permutations of the letters A, B and C.
The first letter can be selected in 3 different ways.
Once the first place has been filled, there are only two letters left to choose from.
Thus the second place can be filled in 2 ways.
Similarly the third place can be filled in 1 way only.
Thus the letters A, B and C can be arranged in

$\boxed{3}\boxed{2}\boxed{1} = 3 \times 2 \times 1 = 6$ ways

3. Factorial notation !

To represent $3 \times 2 \times 1$, we write $3!$, pronounced "three factorial".
Similarly, $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

In general, the product of all natural numbers from any number, n, down to 1 is called $\boldsymbol{n}$ **factorial**, and written as $\boldsymbol{n!}$.

Definition

> $n!$ is the number of permutations of n different objects when all the objects are included in each arrangement.

Thus (i) 5 different objects can be arranged in $5!$ i.e. 120 ways
(ii) 8 different objects can be arranged in $8!$ i.e. 40320 ways.

Example 2

There are 6 different books, including a science book, on a shelf.

(i) In how many different ways can the 6 books be arranged on the shelf?

(ii) In how many ways can the 6 books be arranged if the science book is always on the extreme left?

lets call books: A (Science book) B C D E F

(i) arrangements $= \boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 6! = 720$

(ii) arrangements Science book on left. $= \boxed{1} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 5! = 120$

Example 3

a. In how many ways can the letters of the word SCOTLAND be arranged in a line?

b. (i) In how many of these arrangements do the two vowels come together?

(ii) How many of the arrangements begin with S and end with the two vowels?

a. arrangements of SCOTLAND $= \boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 8! = 40320$

(7 letters with vowels as one) × (2 ways of arranging the vowels)

b. (i) arrangements vowels together $= \left(\boxed{7} \times \boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}\right) \times 2$

$7!(2) = 10080$

imagine OA or AO

(ii) arrangements begin with S end with vowels $= \underset{S}{\boxed{1}} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \underbrace{\boxed{2} \times \boxed{1}}_{\text{vowels}}$

$= 5!(2) = 240$