

Coordinate Geometry: The Circle

chapter

4

Section 4.6 Touching circles – Chords and circles

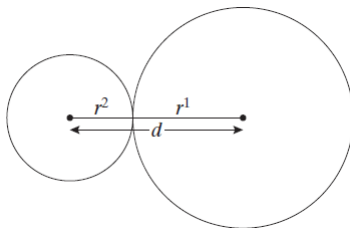
PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

135

1. Circles touching externally or internally

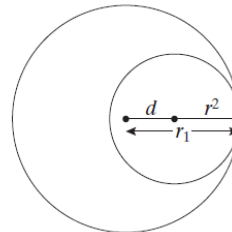
externally

$$d = r_1 + r_2$$



internally

$$d = r_1 - r_2$$



Example 1

Show that the circles $s_1: x^2 + y^2 - 6x - 4y + 11 = 0$
and $s_2: x^2 + y^2 + 4x + 6y - 19 = 0$ touch externally.

External \Rightarrow Is $r_1 + r_2 = |C_1 C_2|$?

$$C_1 (3, 2)$$

$$C_2 (-2, -3)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r_1 = \sqrt{3^2 + 2^2 - 11} = \sqrt{2}$$

$$r_2 = \sqrt{2^2 + 3^2 + 19} = 4\sqrt{2}$$

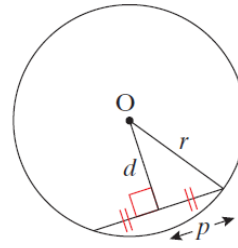
$$d = |C_1 C_2| = \sqrt{(3+2)^2 + (2+3)^2} = 5\sqrt{2}$$

$$r_1 + r_2 = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2} = |C_1 C_2|$$

\Rightarrow touch externally.

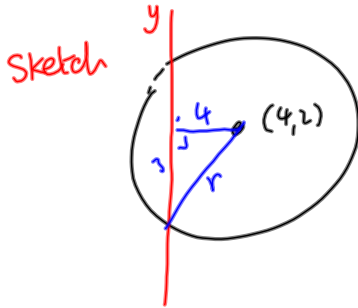
2. Chords and circles

The perpendicular from the centre of a circle to a chord bisects the chord.



Example 2

A circle k has centre $C(4, 2)$ and makes a chord 6 units in length on the y -axis. Find the equation of k .



Recognising triangle (Pythagoras)

$$\Rightarrow r = 5$$

$$C(4, 2)$$

$$\text{equation: } (x-h)^2 + (y-k)^2 = r^2$$

$$(x-4)^2 + (y-2)^2 = 25$$

Exercise 4.6

1. Find the centre and the radius of each of these circles:

$$s_1: x^2 + y^2 - 2x - 15 = 0 \quad \text{and} \quad s_2: x^2 + y^2 - 14x - 16y + 77 = 0.$$

Hence show that the circles touch externally.

Centre: $(-g, -f)$	$C_1 = (1, 0)$	$C_2 = (7, 8)$
Radius: $R = \sqrt{g^2 + f^2 - c}$	$R_1 = \sqrt{1^2 + 0^2 + 15} = \sqrt{16}$	$R_2 = \sqrt{7^2 + 8^2 - 77}$
	$R_1 = 4$	$R = \sqrt{36} = 6$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$ C_1 C_2 = \sqrt{(7-1)^2 + (8-0)^2}$	
	$= \sqrt{36 + 64} = \sqrt{100} = 10$	
$ S \quad R_1 + R_2 = C_1 C_2 $	$R_1 + R_2 = 4 + 6 = 10 = C_1 C_2 $	
	\Rightarrow they touch externally.	