Coordinate Geometry: The Circle

Chapter 4

Section 4.5 Lines and circles: Common chord

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE HIGHER LEVEL

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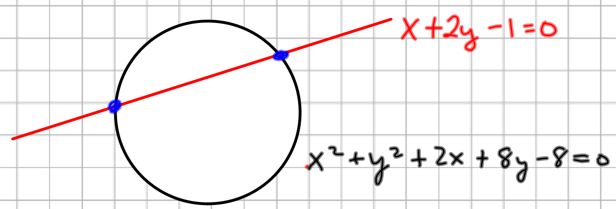
1. Points of intersection of a line and a circle

Example 1

Find the points of intersection of the line $x + 2y - 1 = 0$ and the circle $x^2 + y^2 + 2x + 8y - 8 = 0$.

Solve Simultaneous equations to find points of intersection



Step ① rewrite linear

$x = -2y + 1$ ①

Step ② sub into circle & solve

$\Rightarrow (-2y+1)^2 + y^2 + 2(-2y+1) + 8y - 8 = 0$

$4y^2 - 4y + 1 + y^2 - 4y + 2 + 8y - 8 = 0$

$5y^2 + 0y - 5 = 0$

Step ③ sub answers back into equation ①

$y^2 = 1 \Rightarrow y = \pm 1$

$x = -2(+1) + 1 = -1$

$x = -2(-1) + 1 = 3$

pts. $(3, -1)$ and $(-1, 1)$

2. Where a circle intersects the axes

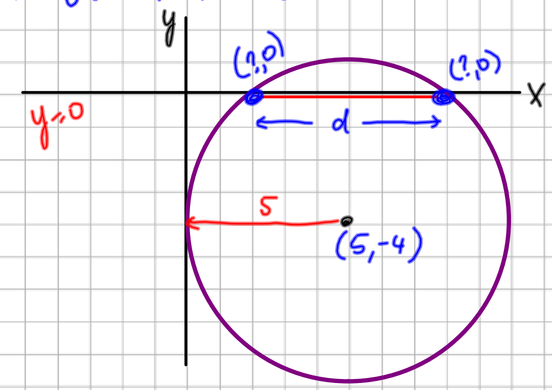
Example 3

Find the length of the intercept the circle
$x^2 + y^2 - 10x + 8y + 16 = 0$ makes on the x-axis.

$c(5,-4)$

$R = \sqrt{g^2+f^2-c}$

$R = \sqrt{5^2+4^2-16} = 5$



Sub $y=0$ into circle

$x^2+0^2-10x+8(0)+16=0$

$x^2-10x+16=0$

$(x-2)(x-8)=0$

$x=2\ ,\ x=8$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(8-2)^2+(0-0)^2} = \sqrt{6^2} = 6$

The points of intersection of two circles may be found by following these steps:
(i) find the equation of the common chord
(ii) find the points of intersection of the chord and any one of the circles.

3. The common chord or common tangent of two circles

Example 4

Find the equation of the common chord of the two circles
$x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$.

$S_1 - S_2 =$ Common chord

$$\begin{aligned}
S_1:&\ \cancel{x^2}+\cancel{y^2}-6x+2y+4=0\\
-S_2:&\ -\cancel{x^2}-\cancel{y^2}-2x+4y+6=0\\
\hline
&\ -8x+6y+10=0\\
&\ -4x+3y+5=0
\end{aligned}$$

Common chord: $4x-3y-5=0$