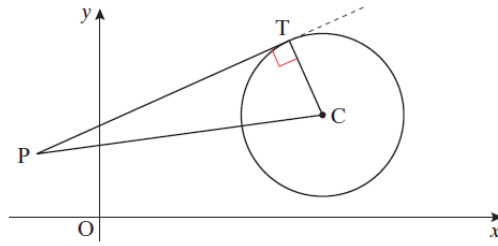


4. Length of a tangent to a circle from a given point

To find the length of the tangent [PT],
 $|CP|^2 = |PT|^2 + |CT|^2$
 $\Rightarrow |PT|^2 = |CP|^2 - |CT|^2$



Example 4

Find the length of the tangent from the point $(-5, 8)$ to the circle $x^2 + y^2 - 4x - 6y + 3 = 0$.

① get R ?
 $R = \sqrt{g^2 + f^2 - c} = \sqrt{2^2 + 3^2 - 3}$
 $R = \sqrt{10}$

② get l ? $|PC|$
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 Pythagoras:
 $a^2 = b^2 + c^2 \Rightarrow l^2 = R^2 + d^2 \Rightarrow 74 = 10 + d^2 \Rightarrow d^2 = 64 \Rightarrow d = 8$

$l = \sqrt{(2+5)^2 + (3-8)^2} = \sqrt{49+25}$
 $l = \sqrt{74}$

14. Find the equation of the line through the point $(3, 5)$ with slope m . Hence find the equations of the two tangents from the point $(3, 5)$ to the circle $x^2 + y^2 + 2x - 4y - 4 = 0$.

Line: $y - y_1 = m(x - x_1)$
 $m = m$ pt $(3, 5)$
 $C(-g, -f) = (-1, 2)$
 $r = \sqrt{g^2 + f^2 - c}$
 $r = \sqrt{1^2 + 2^2 - 4} = 3$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \Rightarrow 3 = \frac{|m(-1) - (2) + (5 - 3m)|}{\sqrt{m^2 + (-1)^2}}$

$(3\sqrt{m^2 + 1})^2 = (|-4m + 3|)^2$
 $9(m^2 + 1) = 16m^2 - 24m + 9$
 $9m^2 + 9 = 16m^2 - 24m + 9$
 $7m^2 - 24m = 0$
 $m(7m - 24) = 0 \Rightarrow m = 0, m = \frac{24}{7}$

Sub m values into ①
 $L_1: (0)x - y + 5 - 3(0) = 0 \Rightarrow y = 5$
 $L_2: (\frac{24}{7})x - y + 5 - 3(\frac{24}{7}) = 0 \Rightarrow 24x - 7y + 35 - 72 = 0$
 $\Rightarrow 24x - 7y - 37 = 0$