Coordinate Geometry: The Circle

chapter **4**

Section 4.3 Finding the equation of a circle

PROJECT MATHS – STRAND 2
Text & Tests 4
**LEAVING CERTIFICATE**
HIGHER LEVEL

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2. **The perpendicular bisector of a chord contains the centre**
   In the given diagram, *l* is the perpendicular bisector of the chord [AB] and *m* is the perpendicular bisector of the chord [BC].

   Each of these bisectors contains the centre, O.

   The point of intersection of these two perpendicular bisectors is the centre of the circle.

   Note: This property is very useful when we require the equation of a circle containing three given points.

$R = 5.5\,cm$
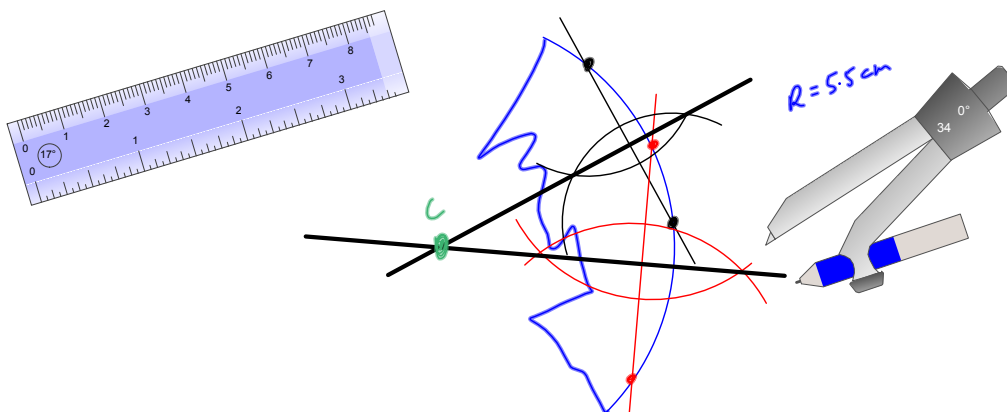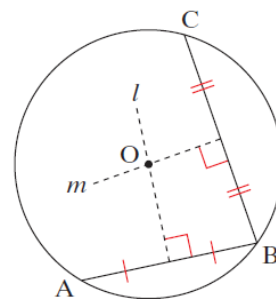
1. Equation of circle containing three given points

Example 1

Find the equation of the circle which contains the points A(2, 1), B(0, 5) and C(−1, 2).

"CHORDS METHOD"

Plan: C is L∩K

① Slopes of Chords
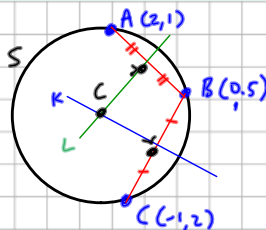$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

② Slopes of L and K
$$\frac{a}{b} \perp -\frac{b}{a}$$

③ mid-pts of Chords
average pt.

④ equations of L & K
$$y - y_1 = m(x - x_1)$$

Sketch

A (2,1)

S

K   C   B (0,5)

L

C (−1, 2)

Equation L	Equation K
$m_{AB} = \frac{5-1}{0-2} = \frac{4}{-2} = -2$	$m_{BC} = \frac{2-5}{-1-0} = \frac{-3}{-1} = 3$
$-2 \perp \frac{1}{2} = m_L$	$3 \perp -\frac{1}{3} = m_K$
midpt [AB] = (1, 3) ∈ L	midpt [BC] = $(-\frac{1}{2}, \frac{7}{2})$ ∈ K
equation L:	equation K:
$y - 3 = \frac{1}{2}(x-1)$	$y - \frac{7}{2} = -\frac{1}{3}(x + \frac{1}{2})$
(×2) $2y - 6 = x - 1$	(×6) $6y - 21 = -2x - 1$
L: $x - 2y + 5 = 0$	$2x + 6y - 20 = 0$
	K: $x + 3y - 10 = 0$

⑤ C = L∩K
Solve equations

$$\Rightarrow L: x - 2y + 5 = 0 \quad \Rightarrow \quad x - 2y = -5$$
$$K: x + 3y - 10 = 0 \quad \Rightarrow \quad x + 3y = 10$$

Ⓚ $x + 3y = 10$
Ⓛ $x + 2y = 5$
        $5y = 15 \quad \Rightarrow \quad y = 3$

Sub into L

$x - 2(3) = -5 \quad \Rightarrow x - 6 = -5 \Rightarrow x = 1$

Centre: C (1, 3)

⑥ Radius, $R = |CA|$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
A (2, 1)

$$|AC| = \sqrt{(1-2)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$$
$$R = \sqrt{5}$$

⑦ write circle, s
$$(x-h)^2 + (y-k)^2 = R^2$$

equation of circle, S

$$(x-1)^2 + (y-3)^2 = 5$$