Coordinate Geometry: The Circle

chapter 4

Section 4.2 The equation of a circle with centre (h, k) and radius r

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE HIGHER LEVEL

The equation of the circle with centre (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

Example 1

Find the equation of the circle with centre $(3, -1)$ and radius 4.

$(h,k) = (3,-1)$

equation : (h,k) form

$(x-3)^2 + (y-(-1))^2 = 4^2$

$(x-3)^2 + (y+1)^2 = 16$ ✓ STOP

Expand (h,k) form to General form

$(a+b)^2 = a^2 + 2ab + b^2$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 16$

$x^2 + y^2 - 6x + 2y - 6 = 0$

GENERAL FORM :

$x^2 + y^2 + 2gx + 2fy + c = 0$

$R = \sqrt{g^2 + f^2 - c}$

$R = \sqrt{3^2 + 1^2 + 6} = \sqrt{16} = 4$

Example 2

Find the centre and radius of the circle

$(x - 3)^2 + (y + 4)^2 = 36$

$(x-h)^2 + (y-k)^2 = R^2$

Centre: (h, k)

Radius, R

Centre $(3, -4)$

$R = \sqrt{36} \Rightarrow R = 6$

The general equation of a circle

If the equation of a circle is in the form

$x^2 + y^2 + 2gx + 2fy + c = 0$

(i) the centre $= (-g, -f)$

(ii) the radius $= \sqrt{g^2 + f^2 - c}$, provided $g^2 + f^2 - c > 0$

Example 3

Find the centre and the radius of the circle $x^2 + y^2 - 2x + 4y - 8 = 0$.

$2g = -2 \Rightarrow g = -1$

$2f = 4 \Rightarrow f = 2$

$c = -8$

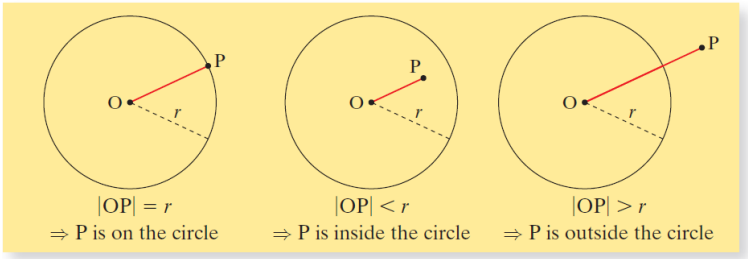
Centre: $(-g, -f) \Rightarrow$ Centre $(1, -2)$

(– HALF x, y co-efficients)

Radius: $R = \sqrt{g^2 + f^2 - c}$

$R = \sqrt{1^2 + 2^2 + 8} = \sqrt{13}$

Points and circles



Example 4

Investigate if the point P(4, 1) is inside the circle $x^2 + y^2 - 6x + 4y + 4 = 0$.

centre : $(-g, -f)$
Radius : $\sqrt{g^2+f^2-c}$

centre $(3, -2)$

$R = \sqrt{3^2+2^2-4} = \sqrt{9} = 3$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

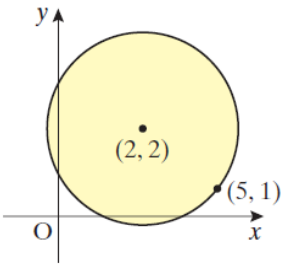
$d = \sqrt{(4-3)^2+(1+2)^2}$

$= \sqrt{1^2+3^2}$

$= \sqrt{10}$

⇒ OUTSIDE

2. The given circle has centre (2, 2). If the circle contains the point (5, 1), find its equation.



$(x-h)^2 + (y-k)^2 = R^2$

$(h, k) = (2, 2)$

⇒ $(5-2)^2 + (1-2)^2 = R^2$

$3^2 + (-1)^2 = R^2$

$10 = R^2$

equation :

$(x-2)^2 + (y-2)^2 = 10$

4. Write down the centre and radius of each of the following circles:

(i) $(x-3)^2+(y-2)^2=16$ (ii) $(x+2)^2+(y-6)^2=8$

(iii) $(x-3)^2+y^2=5$ (iv) $x^2+(y+2)^2=10$

$(x-h)^2+(y-k)^2=R^2$

i	Centre = ? Radius = ?	C (3, 2) R = 4
ii	Centre = ? Radius = ?	C (−2, 6) $R=2\sqrt{2}$
iii	Centre = ? Radius = ?	C (3, 0) $R=\sqrt{5}$
iv	Centre = ? Radius = ?	C (0, −2) $R=\sqrt{10}$

7. Find the centre and radius of each of the following circles:

(i) $x^2+y^2-4x+8y-5=0$ (ii) $x^2+y^2-2x-6y-15=0$

(i)	Centre $(-g,-f)$ $R=\sqrt{g^2+f^2-c}$	Centre $(2,-4)$ $R=\sqrt{2^2+4^2+5}=\sqrt{4+16+5}=\sqrt{25}=5$
(ii)	Centre $(-g,-f)$ $R=\sqrt{g^2+f^2-c}$	Centre $(1,3)$ $R=\sqrt{1^2+3^2+15}=\sqrt{1+9+15}=\sqrt{25}=5$

If the equation of a circle is in the form

$x^2+y^2+2gx+2fy+c=0$

(i) the centre $=(-g,-f)$

(ii) the radius $=\sqrt{g^2+f^2-c}$, provided $g^2+f^2-c>0$