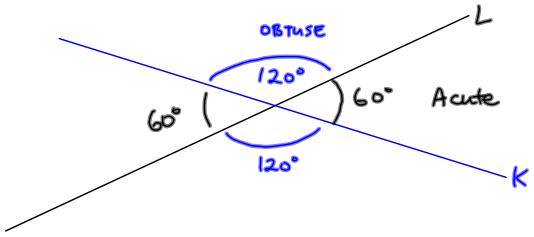
Coordinate Geometry: The Line

chapter 1

Section 1.7 The angle between two lines



PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

Example 1

Find the acute angle between the lines $y = 2x + 5$ and $3x + y = 7$.

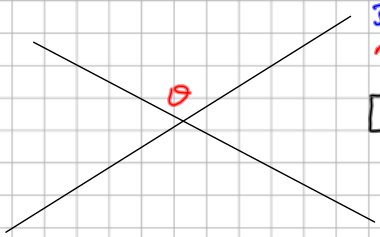
$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

use $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

or better again

$$\Rightarrow \theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Sub m_1 and m_2 values into formula



$3x + y = 7$

$m = \frac{-a}{b}$

$m_1 = -3$

$y = 2x + 5$

$y = mx + c$

$m_2 = 2$

$$\theta = \tan^{-1}\left|\frac{-3-2}{1+(-3)(2)}\right|$$

$$= \tan^{-1}\left|\frac{-5}{1-6}\right| = \tan^{-1}\left|\frac{-5}{-5}\right|$$

$$= \tan^{-1}|1| = 45^\circ$$

Example 2

Find the equations of the two lines through the point $(2, 3)$ which make angles of $45°$ with the line $x - 2y = 1$.

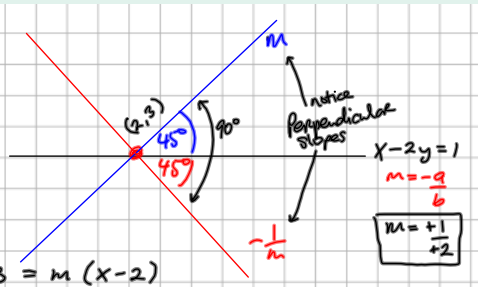
$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

equation of a line: $y - y_1 = m(x - x_1)$

SUB IN pt. $(2,3)$ $\Rightarrow$ $y - 3 = m(x-2)$

$y - 3 = mx - 2m$

$y = mx - 2m + 3$ ①



$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

SUB IN: $\theta = 45°$

SLOPES: $m = \frac{1}{2}$ and $m = m$

$\tan 45° = \left|\frac{m - \frac{1}{2}}{1 + m(\frac{1}{2})}\right|$ multiply RHS by $(\frac{2}{2})$

$1 = \left|\frac{2m - 1}{2 + m}\right| \Rightarrow |2+m| = |2m-1|$

Solve modulus equation $\Rightarrow (2+m)^2 = (2m-1)^2 \Rightarrow 4 + 4m + m^2 = 4m^2 - 4m + 1$

$\Rightarrow 3m^2 - 8m - 3 = 0 \Rightarrow (3m+1)(m-3) = 0$

*get ⊥ slopes as expected $\Rightarrow 3m + 1 = 0$ i.e. $m = -1/3$ * or $m - 3 = 0$ i.e. $m = 3$ *

SUB m values into ①

$y = -\frac{1}{3}x - 2(-\frac{1}{3}) + 3 \Rightarrow y = -\frac{1}{3}x + \frac{11}{3}$

or $y = 3x - 2(3) + 3 \Rightarrow y = 3x - 3$

1. Find the value of the tangent of the acute angle between each of the following pairs of lines:

(ii) $2x + 3y - 1 = 0$ and $x - 2y + 3 = 0$ and hence the angle

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$$

Line 1: $2x + 3y - 1 = 0$

$m = -\frac{a}{b} \Rightarrow m_1 = \frac{-2}{3}$

Line 2: $x - 2y + 3 = 0$

$m = -\frac{a}{b} \Rightarrow m_2 = \frac{+1}{+2}$

SUB INTO FORMULA $\tan\theta = \left|\frac{(-\frac{2}{3}) - (\frac{1}{2})}{1 + (-\frac{2}{3})(\frac{1}{2})}\right| = \frac{7}{4}$

$\theta = ?$ $\theta = \tan^{-1}(\frac{7}{4}) \approx 60.8°$