Coordinate Geometry: The Line

chapter 1

Section 1.5 Concurrencies of a triangle

MUST KNOW :

- CENTROID (C)
- CIRCUMCENTRE (O)
- ORTHOCENTRE (H)
- (• INCENTRE)

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE HIGHER LEVEL

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1. The Centroid (Average Point)

The given diagram shows the three medians [AE], [BF] and [CD] intersecting at the point G.

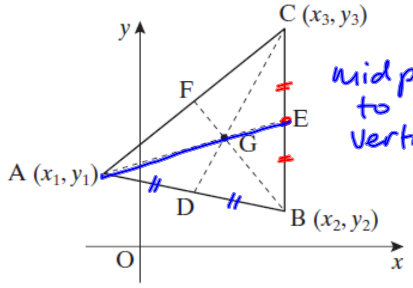
G is called the **centroid** of the triangle.

The medians of a triangle divide each other in the ratio 2 : 1.

In the given triangle,

|AG| : |GE| = 2 : 1; |BG| : |GF| = 2 : 1 and

|CG| : |GD| = 2 : 1.



mid pt to vertex

If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the coordinates of the vertices of a triangle, then the coordinates of the centroid, G, are

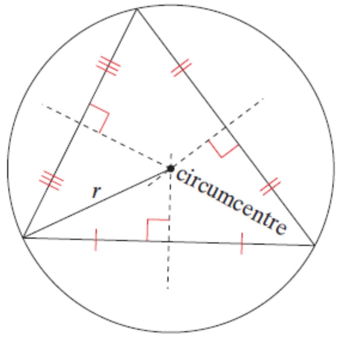
$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

2. The circumcentre

The circumcentre of a triangle is the point of intersection of the **mediators** (the perpendicular bisectors of the sides) of a triangle.

The line segment from a vertex of a triangle to the circumcentre is the radius of the circumcircle. This is shown here as *r*.

Perpendicular Bisectors meet

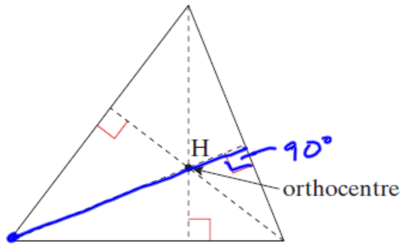


90°

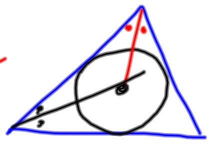
3. The orthocentre

The orthocentre of a triangle is the point of intersection of the perpendiculars from the vertices to the opposite sides.

In the given triangle, H is the orthocentre.



4. Incentre



INCENTER

Bisect angles

Example 1

The vertices of a triangle are A(−4, 3), B(0, −5) and C(3, 4).
Find the coordinates of
(i) the centroid (ii) the orthocentre of the triangle.

(i) Centroid (C)
(Average)

$$C = \left(\frac{-4+0+3}{3}, \frac{3-5+4}{3}\right)$$
$$= \left(-\frac{1}{3}, \frac{2}{3}\right)$$

(ii) Orthocentre (H)

- get slopes m_{AC} & m_{BC}
  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- get slopes of L & K $\perp$ to m_{BC} & m_{AC}
- get equations L & K
  $y - y_1 = m(x - x_1)$
- Solve Sim. eqns to get $L \cap K$