Example 2

In a geometric sequence, $T_3 = 32$ and $T_6 = 4$.

Find a and r and hence write down the first six terms of the sequence.

$T_n = ar^{n-1}$

$T_3 = 32 \Rightarrow ar^{3-1} = 32$

$ar^2 = 32$ (1)

$T_6 = 4 \Rightarrow ar^{6-1} = 4$

$ar^5 = 4$ (2)

(2)/(1): $\frac{ar^5}{ar^2} = \frac{4}{32}$

$r^3 = \frac{1}{8}$

$r = \sqrt[3]{\frac{1}{8}}$

$r = \frac{1}{2}$

Sub $r = \frac{1}{2}$ into (1): $a\left(\frac{1}{2}\right)^2 = 32 \Rightarrow a\left(\frac{1}{4}\right) = 32$

$a = 32(4)$

$a = 128$

Sequence: 128, 64, 32, 16, 8, 4, 2 ...

Example 3

$3, x, x + 6, \ldots$ are the first three terms of a geometric sequence of positive terms.

Find

(i) the value of x

(ii) the tenth term of the sequence.

Geometic Sequence

$a = 3$

T_1 3, T_2 x, T_3 $x+6$, ...

$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\Rightarrow \frac{x}{3} = \frac{x+6}{x}$

multiply by LCD $= 3x$

$3x\left(\frac{x}{3}\right) = 3x\left(\frac{x+6}{x}\right)$

$x^2 = 3x + 18$

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x = 6$ ✓, $x = -3$ ✗ reject not positive

$r = 2$

Sequence: 3, 6, 12

$T_n = ar^{n-1}$

$T_{10} = (3)(2)^{10-1} = 3(2)^9 = 1536$

3. Given $T_2 = 12$ and $T_5 = 324$, find a and r and hence write down the first five terms of the sequence.

$T_n = ar^{n-1}$

$T_2 = 12 \Rightarrow 12 = ar^1 \Rightarrow r = \frac{12}{a}$

$T_5 = 324 \Rightarrow 324 = ar^{5-1}$

$324 = ar^4$

$\Rightarrow 324 = a\left(\frac{12}{a}\right)^4$

$324 = \frac{20736a}{a^{\not 4}{}^3}$

$\times a^3, \div 324$

$a^3 = 20736/324 = 64$

$a = \sqrt[3]{64} = 4 = a$

$r = \frac{12}{a} = \frac{12}{4} \Rightarrow r = 3$

Sequence: 4, 12, 36, 108, 324

5. Write down the first five terms of the geometric sequence that has a second term 4 and a fifth term $-\frac{1}{16}$.

$T_n = ar^{n-1}$

$T_2 = 4 \Rightarrow ar^{2-1} = 4$

$ar = 4$ ①

$T_5 = -\frac{1}{16} \Rightarrow ar^{5-1} = -\frac{1}{16}$

$ar^4 = -\frac{1}{16}$ ②

② ÷ ①

$\frac{ar^{\not 4}{}^3}{a\not r} = \frac{(-1/16)}{4}$

$r^3 = -1/64 \Rightarrow r = \sqrt[3]{(-1/64)}$

$r = -1/4$

Sub $r = -\frac{1}{4}$ into ①

$a(-1/4) = 4 \Rightarrow a = 4/(-1/4)$

$\Rightarrow a = -16$

Sequence: $-16, 4, -1, \frac{1}{4}, -\frac{1}{16}, \ldots$

7. The three numbers $n-2, n$ and $n+3$ are the first three terms of a geometric sequence. Find the value of n and hence write down the first four terms of the sequence.

$T_1 \quad T_2 \quad T_3$

$n-2 \quad n \quad n+3$

$r = \frac{T_2}{T_1} = \frac{T_3}{T_2} \Rightarrow \frac{n}{n-2} = \frac{n+3}{n}$

multiply by LCD $= n(n-2)$

$n(n-2)\frac{n}{n-2} = n(n-2)\frac{(n+3)}{n}$

$n^2 = n^2+3n-2n-6$

$n^2 = n^2+n-6$

$n-6=0 \Rightarrow \boxed{n=6}$

$r = \frac{T_2}{T_1}$

$r = \frac{6}{4} = \frac{3}{2}$ $\boxed{r=\frac{3}{2}}$

Sequence: $4, 6, 9, \frac{27}{2}, \ldots$