2. Solve each of these index (exponential) equations.

(i) $9^x = \frac{1}{27}$ (ii) $4^x = \frac{1}{32}$ (iii) $4^{x-1} = 2^{x+1}$ (iv) $\frac{1}{9^x} = 27$

iii:

$4^{x-1} = 2^{x+1}$

Write in base 2 using $4 = 2^2$

$(2^2)^{x-1} = 2^{x+1}$

$2^{2x-2} = 2^{x+1}$

powers are equal

$\Rightarrow 2x - 2 = x + 1$

$x = 3$

Check:

$4^{3-1} = 4^2 = 16$

$2^{3+1} = 2^4 = 16$ ✓

10. If $y = 2^x$, write (i) 2^{2x} (ii) 2^{2x+1} and (iii) 2^{x+3} in terms of y.

Hence solve the equation $2^{2x+1} - 2^{x+3} - 2^x + 4 = 0$.

$y = 2^x$

$a^{n+m} = a^n . a^m$

(i) $2^{2x} = (2^x)^2 = y^2$

(ii) $2^{2x+1} = (2^{2x})(2^1) = 2y^2$

(iii) $2^{x+3} = (2^x)(2^3) = 8y$

Solve: $x = ?$

$2^{2x+1} - 2^{x+3} - 2^x + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y^2 - 9y + 4 = 0$

$(2y - 1)(y - 4) = 0$

$y = \frac{1}{2}$, $y = 4$

$y = 2^x$

$y = \frac{1}{2}$; $\frac{1}{2} = 2^x \Rightarrow x = -1$

$y = 4$; $4 = 2^x \Rightarrow x = 2$