chapter 7 **Algebra 3**

Section 7.7 Exponential equations

PROJECT MATHS **Text & Tests 6**

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Example 1

Solve these equations. (i) $\frac{1}{8^x} = 16^{\frac{1}{3}}$ (ii) $27^{x-3} = 3 \times 9^{x-2}$

BASE BASE

Write both sides with same base

Realise:
$8 = 2^3$
$16 = 2^4$

$\frac{1}{a^n} = a^{-n}$

$\frac{1}{8^x} = 16^{\frac{1}{3}}$

$8^{-x} = 16^{\frac{1}{3}}$

$(2^3)^{-x} = (2^4)^{\frac{1}{3}}$

$2^{-3x} = 2^{4/3}$

$\Rightarrow -3x = 4/3$

$\times 3$ $\quad -9x = 4$

$\div -9$ $\quad x = -4/9$

$x^n = y \Leftrightarrow n = \log_x y$

$\log_{\square} \square$

$\frac{1}{8^x} = 16^{\frac{1}{3}} \Rightarrow \boxed{8}^{\boxed{-x}} = \boxed{16^{\frac{1}{3}}}$ (POWER, BASE)

$-x = \log_{\boxed{8}} \boxed{16^{\frac{1}{3}}} = 4/9$

$\Rightarrow x = -4/9$

Example 2

If $y = 3^x$, express 3^{2x} in terms of y.

Hence solve the equation $3^{2x} - 4.3^x + 3 = 0$.

$y = 3^x$

$3^{2x} = (3^x)^2 = y^2$

$3^{2x} - 4.3^x + 3 = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3, \quad y = 1$

But $y = 3^x$

$3^1 = 3^x \Rightarrow x = 1$

OR $1 = 3^x \Rightarrow x = 0$

$a^0 = 1$

$3^0 = 1$

Exercise 7.7

1. Find the value of x in each of these equations:

(i) $2^x = 32$ (ii) $16^x = 64$ (iii) $25^x = 125$ (iv) $3^x = \frac{1}{27}$

Power = $\log_{\text{BASE}}$ No.

$x = \log_2 32 = 5$

$x = \log_{16} 64 = 3/2$

$x = \log_{25} 125 = 3/2$

$x = \log_3 \frac{1}{27} = -3$

2. Solve each of these index (exponential) equations.

(i) $9^x = \frac{1}{27}$ (ii) $4^x = \frac{1}{32}$ (iii) $4^{x-1} = 2^{x+1}$ (iv) $\frac{1}{9^x} = 27$

(i) $\frac{1}{a} = a^{-1}$

- Write with same Base
  $9 = 3^2$
  $27 = 3^3$
- Solve equation

$9^x = \frac{1}{27}$

$9^x = 27^{-1}$

$(3^2)^x = (3^3)^{-1}$

$3^{\boxed{2x}} = 3^{\boxed{-3}}$

$\Rightarrow 2x = -3 \quad \Rightarrow x = -3/2$

(ii) $\frac{1}{a} = a^{-1}$

- Write with same base
  $4 = 2^2$
  $32 = 2^5$
- Solve equation

$4^x = \frac{1}{32}$

$4^x = 32^{-1}$

$(2^2)^x = (2^5)^{-1}$

$2^{\boxed{2x}} = 2^{\boxed{-5}}$

$\Rightarrow 2x = -5 \quad \Rightarrow x = -5/2$

7. By letting $3^x = y$, solve the equation $3^{2x} - 12(3^x) + 27 = 0$.

$\boxed{3^x = y}$

$3^{2x} = y^2$

$x = ?$

But $y = 3^x$

$y^2 - 12y + 27 = 0$

$(y-3)(y-9) = 0$

$y = 3, \quad y = 9$

$3^x = 3^1 \quad \Rightarrow x = 1$

$3^x = 9 \quad \Rightarrow x = 2$

$(3^2 = 9)$