Exercise 7.5

1. Prove that (i) $a^2 + 2ab + b^2 \geqslant 0$ (ii) $a^2 + 2ab + 2b^2 \geqslant 0$ for all $a, b \in R$.

ii

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(\text{Something})^2 \geq 0$

Show $a^2 + 2ab + 2b^2 \geq 0$.

$a^2 + 2ab + 2b^2$

$= a^2 + 2ab + b^2 + b^2$

$= (a+b)^2 + b^2 \geq 0$

QED

4. If $\boxed{a > 0}$ and $\boxed{b > 0}$ show that

(i) $a + \frac{1}{a} \geqslant 2$ (ii) $\frac{1}{a} + \frac{1}{b} \geqslant \frac{2}{a+b}$.

If

$\frac{1}{a} + \frac{1}{b} \geq \frac{2}{a+b}$

then

$\frac{b+a}{ab} \geq \frac{2}{a+b}$

$\frac{(b+a)\cancel{(ab)}(a+b)}{\cancel{ab}} \geq \frac{2(ab)\cancel{(a+b)}}{\cancel{(a+b)}}$

$(a+b)^2 \geq 2ab$

$a^2 + 2ab + b^2 \geq 2ab$

$a^2 + b^2 \geq 0$ true

QED