Example 4

A population is normally distributed with mean 12 and standard deviation 3. Find the sample size such that $P(\bar{x} > 12.5) = 0.05$, where $\bar{x}$ is the sample mean.

$n = ?$

$\sigma = 3$

$\mu = 12$

$\bar{X} = 12.5$

$z = ?$

$\Rightarrow$

$P(\bar{X} > 12.5) = 0.05$

$P(\bar{X} < 12.5) = 0.95$ $\Rightarrow$ $z = 1.65$ FROM TABLES

$$z = \frac{\bar{X} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$$1.65 = \frac{12.5 - 12}{\left(\frac{3}{\sqrt{n}}\right)}$$

divide by $\frac{3}{\sqrt{n}}$ is same as multiply by $\frac{\sqrt{n}}{3}$.

$$1.65 = \frac{0.5\sqrt{n}}{3}$$

$$\sqrt{n} = \frac{1.65(3)}{0.5} = 9.9$$

$$n = (9.9)^2 \approx 98$$