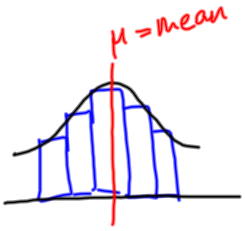
chapter 3 **Probability 2**

Section 3.6 The normal distribution



eg. • Heights of people
• I.Q.

Bell Curve
normal distribution
symmetrical

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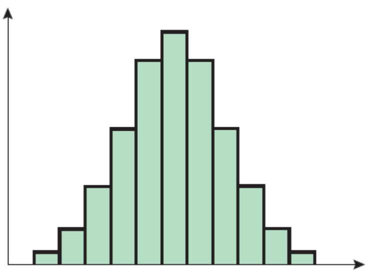
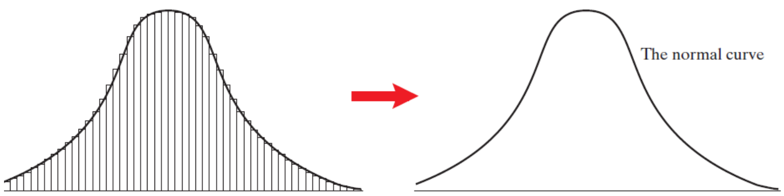
When the physical characteristics, such as height or weight, of a large number of individuals are arranged in order, from lowest to highest, in a frequency distribution, the same pattern shows up repeatedly. This pattern shows that large numbers cluster near the middle of the distribution, as illustrated by the symmetrical histogram shown below.



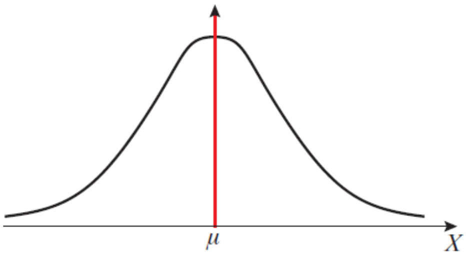
If the distribution is very large and continuous, and the class intervals become sufficiently small, the distribution forms a symmetrical bell-shaped smooth curve called **the curve of normal distribution**



The normal distribution is the most important continuous distribution in statistics.

The curve on the right shows a normal distribution with mean μ.
The red line is the axis of symmetry.

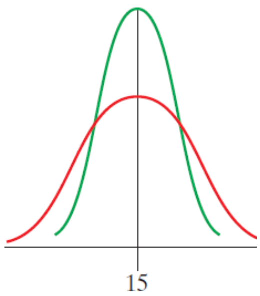
The mode, median and mean are all equal.
They lie on the axis of symmetry.



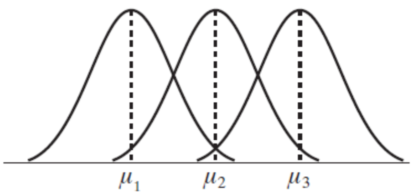
Standard deviation and the normal curve

Here are two normal curves, both with the same mean of 15.
The green curve is narrower and has the smaller spread.
It has the smaller standard deviation.

The red curve is wider and has the larger spread.
It has the larger standard deviation.



The diagram below shows three normal curves with different means but the same standard deviation.



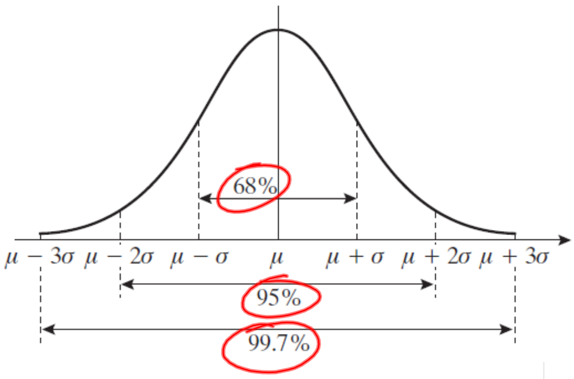
For any normal distribution:

- approximately 68% of the distribution lies within one standard deviation of the mean, i.e., 68% lies between $\bar{x} - \sigma$ and $\bar{x} + \sigma$.
- 95% lies between $\mu - 2\sigma$ and $\mu + 2\sigma$.
- 99.7% lies between $\mu - 3\sigma$ and $\mu + 3\sigma$.

This is known as the **Empirical Rule**.

The normal curve opposite illustrates the Empirical Rule.

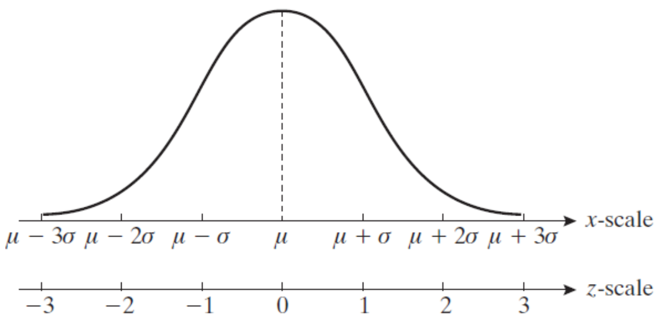
Based on the Empirical Rule, the probability that a score, selected at random, will be within one standard deviation of the mean is 68% or 0.68.



The standard normal distribution

There are many different normal distributions, all of the same bell-shape, but with different means and standard deviations. To avoid the necessity of having separate tables for each normal curve, we convert the units in a given curve to **standard units** to get the **standard normal distribution**. The standard units are often referred to as z units.

The change of scale from x-units to z-units is shown in the diagram below.



A ***z*-score** or **standard score** is the number of standard deviations that a value lies above or below the mean.

› a z-score of 1 represents a score that is 1 standard deviation **above** the mean

› a z-score of -2 represents a score that is 2 standard deviations **below** the mean

The standard normal distribution has mean 0 and standard deviation 1.

The formula which changes the given units (x-units) into z-units is given below:

Standard scores

$$z = \frac{x - \mu}{\sigma}$$

- x is the given score or variable
- μ is the given mean
- σ is the given standard deviation

mean: $\mu = 100$

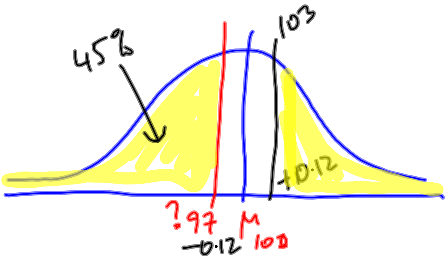
S.D.: $\sigma = 25$

Score: $X = 150$

$X = 97$

$$z = \frac{150 - 100}{25} = 2$$

$$z = \frac{97 - 100}{25} = -0.12$$



$P(z \geq +0.12) \approx 55\%$