chapter 3

Probability 2

Section 3.5 Probability involving permutations and combinations

PROJECT MATHS
Text & Tests 5
LEAVING CERTIFICATE
HIGHER LEVEL
STRAND 1
PROBABILITY & STATISTICS

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Example 2

Three cards are drawn at random, and without replacement, from a pack of 52 playing cards. Find the probability that

(i) the three cards drawn are the Jack of spades, the Queen of clubs and the King of clubs
(ii) the three cards are aces
(iii) two cards are red and the third one is a club
(iv) the three cards are of the same colour.

(i)

$\boxed{3} \times \boxed{2} \times \boxed{1} = 3! = 6$
arrangements

exact
$P(J, Q, K) = \left(\frac{1}{52}\right)\left(\frac{1}{51}\right)\left(\frac{1}{50}\right) = \frac{1}{132600}$

6 possible arrangements
$P(J, Q, K \text{ any order}) = \frac{6}{132600} = \frac{1}{22100}$

(i) $P(E) = \frac{\text{FAVOURABLE}}{\text{TOTAL}}$

$P(\underset{\text{any order}}{J, Q, K}) = \frac{\binom{3}{3}}{\binom{52}{3}} = \frac{1}{22100}$

Example 2

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(ii) the three cards are aces

(iii) two cards are red and the third one is a club

(iv) the three cards are of the same colour.

(ii) $P(3 \text{ Aces}) = \dfrac{\binom{4}{3}}{\binom{52}{3}} = \dfrac{1}{5525}$

(iii) $P\left(\begin{array}{c}\text{2 Red then}\\ \text{3rd is Club}\end{array}\right) = \dfrac{\binom{26}{2} \times \binom{13}{1}}{\binom{52}{3}} = \dfrac{13}{68}$

(iv) $P(\text{Same Colour}) = P(\text{All Black or All Red})$

$= \dfrac{\binom{26}{3} + \binom{26}{3}}{\binom{52}{3}} = \dfrac{4}{17}$

Events occurring at least once

Many questions in probability contain phrases such as "at least once", "at least one red disc", etc. Take, for example, the probability of getting at least one 4 when a pair of dice are thrown.

First get the probability that no 4 is thrown.

$$P(\text{no } 4) = P(\text{no 4 on 1st throw}) \times P(\text{no 4 on 2nd throw}) = \tfrac{5}{6} \times \tfrac{5}{6} = \tfrac{25}{36}$$

$$P(\text{at least one } 4) = 1 - P(\text{no } 4) = 1 - \tfrac{25}{36} = \tfrac{11}{36}$$

Similarly, if a coin is tossed four times, the probability of getting at least two heads is

$$P(\text{at least 2 heads}) = 1 - P(\text{no head}) - P(\text{1 head})$$

In general, if E is any event, then

$$P(E \text{ occurring at least once}) = 1 - P(E \text{ not occurring at all})$$

Exercise 3.5

1. A hand of four cards is dealt at random from a normal pack of 52 cards. Find the probability that the hand contains

(i) exactly two queens (ii) four spades

(iii) four red cards (iv) four cards of the same suit.

(i) $P(Q,Q) = \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \frac{1}{221}$

or $P(2\text{ queens}) = \dfrac{\binom{4}{2}}{\binom{52}{2}} = \frac{1}{221}$

(ii) $P(S,S,S,S) = \left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)\left(\frac{10}{49}\right) = \frac{11}{4165}$

or $P(4\text{ Spades}) = \dfrac{\binom{13}{4}}{\binom{52}{4}} = \frac{11}{4165}$

(iii) $P(R,R,R,R) = \left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{24}{50}\right)\left(\frac{23}{49}\right) = \frac{46}{833}$

4 Same Suit = $4 \times P(\text{exact suit})$

(iv) $P(4\text{ Same Suit}) = 4\left(\frac{11}{4165}\right) = \frac{44}{4165}$

see part ii

7. In an examination a candidate is required to select any seven questions from ten.

(i) In how many ways can this be done?

(ii) How many of the selections contain the first and second questions?

Now calculate the probability that the candidate selects

(iii) both the first and second questions

(iv) at least one of the first two questions.

(i) $\binom{10}{7} = 120$

(ii) If contains Q1 and Q2 we only choose 5 from remaining 8 questions

$\binom{8}{5} = 56$

(iii) $P\left(\begin{array}{c}\text{both Q1 \& Q2}\\ \text{are selected}\end{array}\right) = \dfrac{\binom{8}{5}}{\binom{10}{7}} = \frac{7}{15}$

(iv) $P\left(\begin{array}{c}\text{does at least}\\ \text{one of Q1 or Q2}\end{array}\right) = 1 - \underbrace{P(\text{neither Q1 or Q2})}_{\text{choose 7 from 8}}$

$= 1 - \dfrac{\binom{8}{7}}{\binom{10}{7}} = \frac{14}{15}$

8. A class of 16 pupils consists of 10 girls, 3 of whom are left-handed, and 6 boys, only one of whom is left-handed. Two pupils are to be chosen at random from the class to act as prefects. Calculate the probability that the chosen pupils will consist of

(i) one girl and one boy
(ii) one girl who is left-handed and one boy who is left-handed
(iii) two left-handed pupils
(iv) at least one pupil who is left-handed.

Given:

	L	R	
G	3	7	10
B	1	5	6
	4	12	16

2 Pupils choosen at Random

Total outcomes $= \binom{16}{2} = 120$

(i) $P(\text{one girl} \,\&\, 1 \text{ boy}) = \dfrac{\binom{10}{1} \times \binom{6}{1}}{120} = \dfrac{1}{2}$

(ii) $P\left(\text{Left handed girl} \,\&\, \text{left handed boy}\right) = \dfrac{\binom{3}{1} \times \binom{1}{1}}{120} = \dfrac{1}{40}$

(iii) $P(\text{both left-handed}) = \dfrac{\binom{4}{2}}{120}$

(iv) $P\left(\text{at least 1 left handed}\right) = 1 - \left(\text{no left handed i.e. all Right}\right) = 1 - \dfrac{\binom{12}{2}}{120}$