## Probability of $k^{th}$ success on $n^{th}$ Bernoulli trial

In example 1 on the previous page, we worked out the probability of getting 3 sixes when a dice is thrown 5 times. If the same dice is thrown continuously until a six appears for the fourth time, how do we find the probability that the 4th six appears on the tenth throw?

For a 4th six to appear on the 10th throw,

(i) we need to get 3 sixes on the first nine throws, and then

(ii) get a six on the 10th throw.

Three sixes on the first nine throws is given by

$$\binom{9}{3}p^3q^{9-3} \quad \text{i.e.} \quad \binom{9}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^6$$

$$84 \times \frac{5^6}{6^9} = 0.13$$

$P(\text{six on the 10th throw}) = \frac{1}{6}$

Thus $P(\text{4th six on the 10th throw}) = 0.13 \times \frac{1}{6} = 0.0217$

$$P(r \text{ successes}) = \binom{n}{r}p^rq^{n-r}$$

## Example 3

A card is drawn at random from a normal deck of playing cards and then replaced. The process is repeated until the third diamond appears.
Find the probability that this happens when the tenth card is drawn.

P(3rd diamond on 10th draw) = ?

P(2 diamonds in 9 draws) ?
n = 9
r = 2
p = 1/4
q = 3/4

$P(2 \text{ out of } 9) = \binom{9}{2} \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^7$

$= 0.3003 \approx 0.3$

P(diamond on 10th draw) = 1/4

P(3rd diamond on 10th draw) = $0.3 \times \frac{1}{4}$

$= 0.075$

**3.** Find the probability that, in five throws of a fair dice, a 3 will occur
(i) on no occasion (ii) once only (iii) twice.

$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$

$p = \frac{1}{6}$

$p = 5/6$

(i) $P(n=5, r=0) = \binom{5}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 \approx 0.401$

(ii) $P(n=5, r=1) = \binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 \approx 0.401$

(iii) $P(n=5, r=2) = \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 \approx 0.161$

**6.** Jean either walks to school or goes by bus. The probability that she walks to school on a summer morning is 0.7. For a school week of five days during the summer, find the probability that
(i) she walks to school only once
(ii) she walks to school exactly three times.

$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$

$p = 0.7$

$q = 0.3$

(i) $P(r=1, n=5) = \binom{5}{1}(0.7)^1(0.3)^4$
$= 0.028$

(ii) $P(r=3, n=5) = \binom{5}{3}(0.7)^3(0.3)^2$
$= 0.31$

**16.** A fair dice is thrown repeatedly.

(i) Find the probability of getting two fives in the first ten throws.

(ii) Hence find the probability of getting the third five on the eleventh throw.

Bernoulli Trial

$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$

$n = 10$, $r = 2$, $p = 1/6$, $q = 5/6$

(i) $P(r=2, n=10) = \binom{10}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 = 0.2907$

(ii) $P(\text{3rd success on 11th trial}) = P(r=2, n=10) \times p$

$= (0.2907)\left(\frac{1}{6}\right) = 0.0485$

**18.** When a spinner is spun, the probability that it lands on red is 0.3. The spinner is spun until four reds are got.

Find the probability that the fourth red is got on the tenth spin.

Bernoulli Trial

$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$

$P(\text{4th red when } n=10) = P(r=3, n=9) \times p$

$p = 0.3$

$q = 0.7$

$= \binom{9}{3}(0.3)^3(0.7)^6 \times 0.3$

$= 0.08$