Probability 1

chapter 1

Section 1.7 Conditional probability

(not independent)

The first event effects the probability of the 2nd event.

Dependent

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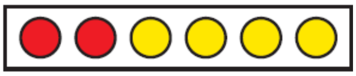
A box contains 2 red counters and 4 yellow counters, as shown.



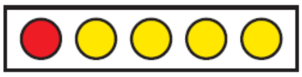
One counter is picked at random.

$P(\text{red}) = \frac{2}{6}$ and $P(\text{yellow}) = \frac{4}{6}$

Suppose the counter is **not** put back in the box.

The contents of the box will be different, depending on whether the counter taken out was red or yellow.

If it was red, the box would now contain 1 red and 4 yellow counters, as shown.



If another counter is now taken out at random, the probability that it is red is **dependent** on the colour of the first counter.

This is called **conditional probability**.

Returning to the second box above, $P(\text{red})$ is now $\frac{1}{5}$.

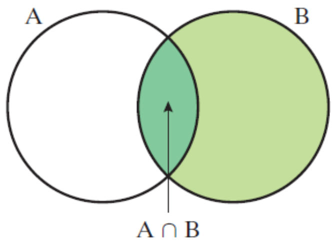
This probability is calculated on the assumption that a red was got on the first draw.

The box and counters discussed above is an example of a situation where the probability of the second event depends on the outcome of the first event.

If A and B are two events, the **conditional probability** that A occurs, **given that B has already occurred**, is written $P(A \mid B)$.

$P(A \mid B)$ is read as "the probability of A given B".

The conditional probability $(A \mid B)$ is illustrated in the given Venn diagram.



To find $P(A \mid B)$, the sample space is reduced to B only, since B has already occurred.

Thus $P(A \mid B) = \dfrac{\#(A \cap B)}{\#B} = \dfrac{P(A \cap B)}{P(B)}$.

The part of B in which A also occurs is the part denoted by $A \cap B$.

$$\text{Thus } P(A \mid B) = \frac{\#(A \cap B)}{\#B} = \frac{P(A \cap B)}{P(B)}$$

This result should be memorised as it will be used later in our study of probability.

$P(A|B)$ "Probability of A given that B has happened"

A given B

$$\boxed{P(\underset{\text{and}}{A \cap B}) = P(A|B) \times P(B)}\ ^{*}$$

This is the General multiplication Rule

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

This result can be described in words as follows:

"The probability of A given B is the probability of A and B divided by the probability of B".

The result $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ can be rearranged as follows:

$P(A \mid B) \times P(B) = P(A \cap B)$ … multiply both sides by $P(B)$.

$\therefore \quad P(A \cap B) = P(A \mid B) \times P(B)$

Also $\quad P(B \cap A) = P(B \mid A) \times P(A)$

Thus $\quad P(A \cap B) = P(A) \times P(B \mid A)$ … since $P(A \cap B) = P(B \cap A)$.

The General Multiplication law

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Example 1

The numbers 1 to 9 are written on cards and placed in a box.
A card is drawn at random from the box.
Find the probability that the number is prime, given that the number is odd.

$1, 2, 3, 4, 5, 6, 7, 8, 9$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(\text{Prime} | \text{odd}) = ?$

$1, 3, 5, 7, 9$

$P(\text{ODD}) = \frac{5}{9}$

$3, 5, 7$

$P(\text{Prime \& ODD}) = \frac{3}{9}$

$P(\text{Prime} | \text{odd}) = \frac{(3/9)}{(5/9)} = \frac{3}{5}$

Example 2

A bag contains 6 red and 4 blue discs. A disc is drawn from the bag and not replaced. A second disc is then drawn.
Find the probability that
(i) the first two discs are blue
(ii) the second disc drawn is red
(iii) one disc is red and the other disc is blue
(iv) both discs are the same colour.

(i) $P(B, B) = (4/10)(3/9) = 2/15$

(ii) $P(\text{2nd is Red}) = ?$

either RR or BR

$= P(RR) + P(BR)$

$= \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{3}{5}$

(iii) $P(\text{One Red and one Blue}) = ?$

either RB or BR

$= P(R, B) + P(BR)$

$= \left(\frac{6}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{8}{15}$

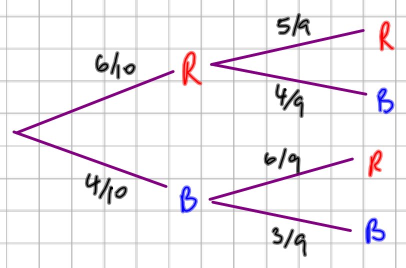
(iv) $P(\text{Same}) = P(RR)$ or $P(BB)$

$= \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) + \frac{2}{15} = \frac{7}{15}$

Example 2

A bag contains 6 red and 4 blue discs. A disc is drawn from the bag and not replaced. A second disc is then drawn.
Find the probability that
(i) the first two discs are blue
(ii) the second disc drawn is red
(iii) one disc is red and the other disc is blue
(iv) both discs are the same colour.

Take 1st Disc Take 2nd Disc P(output)



$P(RR) = \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) = \frac{5}{15}$

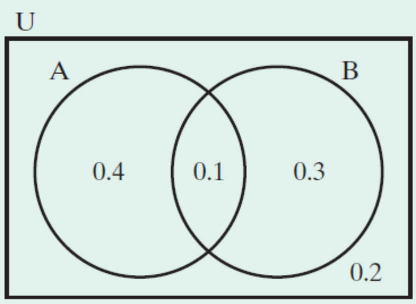
$P(RB) = \left(\frac{6}{10}\right)\left(\frac{4}{9}\right) = \frac{4}{15}$

$P(BR) = \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{4}{15}$

$P(BB) = \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) = \frac{2}{15}$

Example 3

Use the given Venn diagram to write down
(i) $P(A\,|\,B)$ (ii) $P(B\,|\,A)$

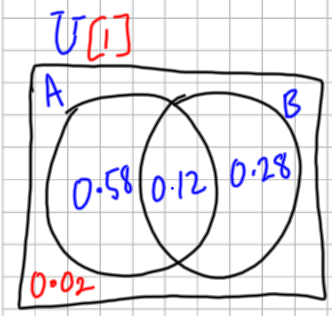


$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.4} = \frac{1}{4}$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.5} = \frac{1}{5}$

Example 4

Two events A and B are such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \mid B) = 0.3$. Determine the probability that neither A nor B occurs.



$P(A \cap B) = ?$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$0.3 = \frac{P(A \cap B)}{0.4}$$

$$0.12 = P(A \cap B)$$

GENERAL ADDITION RULE

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.7 + 0.4 - 0.12$$

$$= 0.98$$

$$P(A \cup B)' = 1 - 0.98 = 0.02$$