Probability 1

chapter 1

Section 1.5 Mutually exclusive events – The addition rule

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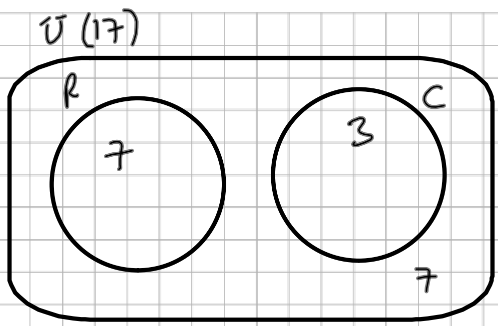


M.E.

Mutually Exclusive (can't do both) be

one or other

no intersection

$P(R) = \frac{7}{17}$

$P(C) = 3/17$

$P(R \cup C) = \frac{10}{17}$

$P(R \cup C)' = 7/17$

$P(P \cap C)' = 0$ ← M.E.

$P(R \cup C) = \frac{10}{17}$

Consider the following two events when drawing a card from a pack of 52 playing cards:

A = drawing an ace B = drawing a king.

These two events are said to be **mutually exclusive** as they cannot occur together.

If the events A and B cannot happen together, then

$$\boldsymbol{P(\mathrm{A\ or\ B}) = P(\mathrm{A}) + P(\mathrm{B})}$$

This is called the **addition law** for mutually exclusive events.

> Outcomes are mutually exclusive if they cannot happen at the same time.

So $P(\text{draw an ace } \textbf{or} \text{ king}) = P(\text{ace}) + P(\text{king})$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52} = \frac{2}{13}$$

When events are not mutually exclusive

We will now consider events which may occur at the same time.

If A is the event: selecting an ace from a pack of cards and

B is the event: selecting a heart from a pack of cards

then $P(\mathrm{A}) = \frac{4}{52}$ and $P(\mathrm{B}) = \frac{13}{52}$

In this situation, both events may occur at the same time since the *ace of hearts* is common to both.

In general, when two events A and B can occur at the same time,

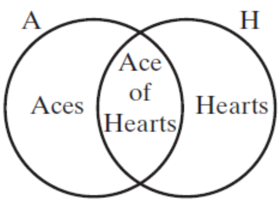
$$P(\mathrm{A\ or\ B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A\ and\ B})$$

Thus in the example given above,

$$P(\text{ace or heart}) = P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

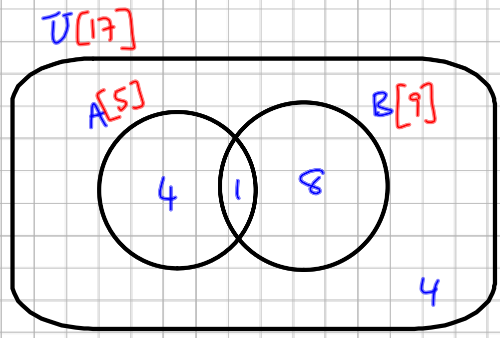


This result can be verified as there are 4 aces and 13 hearts in a pack of cards. Since one of the aces is the ace of hearts, there are 16 aces or hearts in the pack.

i.e. $P(\text{ace or a heart}) = \frac{16}{52}$, as already found.

Study
Biology
Art

not M.E.



$P(A \cup B) = \frac{13}{17}$

$P(A) = 5/17$
$P(B) = 9/17$
$P(A \cap B) = 1/17$

General Addition Rule Formula

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 5/17 + 9/17 - 1/17 = 13/17$

Example 1

A card is drawn at random from a pack of 52.
What is the probability that the card is

(i) a club	(ii) a king	(iii) a club or a king
(iv) a red card	(v) a queen	(vi) a red card or a queen

General Addition Rule:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(\text{club}) = \frac{1}{4}$

$P(\text{King}) = 1/13$

$P(\text{Club}) + P(\text{King}) - P(\text{King of clubs})$

$P(\text{Club or King}) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$
$= \frac{17}{52}$

$P(\text{Red}) = \frac{1}{2}$

$P(\text{Queen}) = 1/13$

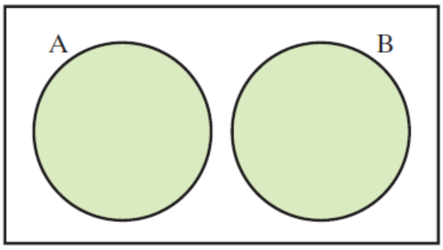
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(\text{Red}) + P(\text{Queen}) - P(\text{Red Queen})$

$P(\text{Red or Queen}) = \frac{1}{2} + \frac{1}{13} - \frac{1}{26}$
$= \frac{14}{26}$

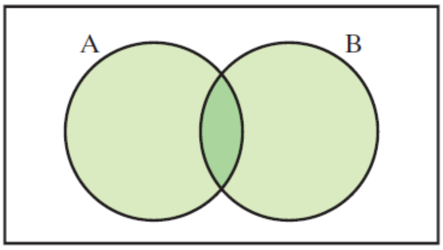
Venn diagrams for mutually exclusive events

(i) Mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B})$

(ii) Non-mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$

Example 2

A and B are two events such that $P(\text{A}) = \frac{19}{30}$, $P(\text{B}) = \frac{2}{5}$ and $P(\text{A} \cup \text{B}) = \frac{4}{5}$. Find $P(\text{A} \cap \text{B})$.

General Addition Rule

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$\frac{4}{5} = \frac{19}{30} + \frac{2}{5} - P(A \cap B)$$

$$\frac{4}{5} = \frac{31}{30} - P(A \cap B)$$

$$\frac{4}{5} - \frac{31}{30} = -P(A \cap B)$$

$$-\frac{7}{30} = -P(A \cap B)$$

$$P(A \cap B) = \frac{7}{30}$$