Probability 1

chapter 1

Section 1.4 Experimental probability – Relative frequency

PROJECT MATHS
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Experiment

John suspects that a coin is biased. In an experiment, he tossed the coin 200 times and recorded the number of heads after 10, 50, 100, 150 and 200 tosses.

The results are shown in the table on the right:

Number of tosses	Number of heads	Heads ÷ tosses
10	7	0.7
50	28	0.56
100	53	0.53
150	78	0.52
200	103	0.515

As the number of tosses increase, the number of heads divided by the number of tosses gets closer to 0.5, i.e., $\frac{1}{2}$.

This value is called **relative frequency** and it gives an **estimate of the probability** that the event will happen.

Thus an estimate of the probability that an event will occur, by carrying out a survey or experiment, is given by

$$\text{Relative frequency} = \frac{\text{Number of successful trials}}{\text{Total number of trials}}$$

In general, as the number of trials or experiments increases, the value of the relative frequency gets closer to the true or theoretical probability.

Example 1

Dara collected data on the colours of cars passing the school gate.
His results are shown on the table below.

Colour	White	Red	Black	Blue	Green	Other	Total
Frequency	24	32	14	16	10	4	100

(i) How many cars did Dara survey? 100

(ii) What was the relative frequency of blue cars? $\frac{16}{100} = \frac{4}{25}$

(iii) What was the relative frequency of red cars? $\frac{32}{100} = \frac{8}{25} = 0.32$
Give your answer as a decimal.

(iv) Write down an estimate of the probability that the next car passing the school gate will be green. $\frac{10}{100} = \frac{1}{10}$

(v) How can the estimate for the probability of green cars be made more reliable?

Count more cars

Expected frequency

A bag contains 3 red discs and 2 blue discs.
A disc is chosen at random from the bag and replaced.
The probability of getting a blue disc is $\frac{2}{5}$.
This means that, on average, you expect 2 blue discs in every 5 chosen or 20 blue discs in every 50 chosen.

To find the expected number of blue discs when you choose a disc 100 times,

(i) Work out the probability that the event happens once.

(ii) Multiply this probability by the number of times the experiment is carried out.
Thus the expected number of blue discs is

$$\frac{2}{5} \times \frac{100}{1} = 40.$$

Expected frequency is probability × number of trials.

9. Paula records the number of 6s she gets when she rolls a dice 10, 100 and 1000 times. The table below shows her results.

Number of rolls	10	100	1000
Number of 6s	1	15	165

Use this information to work out the best estimate for getting a 6 on Paula's dice. Give a reason for your answer.

If unbiased $P(\text{six}) = \frac{1}{6} = 0.1\dot{6} \approx 1.67$

Most acurate relative frequency use 1000 trials

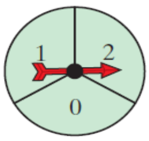
Relative frequency $(\text{six}) = \frac{165}{1000} = 1.65$

It seems very close to the theoretical value $\Rightarrow$ it's unbiased.

$\Rightarrow P(\text{six}) = \frac{1}{6}$

10. Four friends are using a spinner for a game and they wonder if it is perfectly fair. They each spin the spinner many times and record the results.

Name	**Number of spins**	**Results**		
		0	**1**	**2**
Alan	30	12	12	6
Keith	100	31	49	20
Bill	300	99	133	68
Ann	150	45	73	32
Total	580	187	267	126
Relative Freq.		$\frac{187}{580} = 0.3\dot{2}$	$\frac{267}{580} = 0.46$	$\frac{126}{580} = 0.2\dot{1}$



Expected Probability for each $= \frac{1}{3} = 0.\dot{3}\dot{3}$

Expected Value $= (\frac{1}{3})(580) \approx 191$

(i) Whose results are most likely to give the best estimate of the probability of getting each number? Bill, he spun more times.

(ii) Make a table by adding together all the results.
Use the table to decide whether you think the spinner is ~~biased~~ or unbiased. ✓

(iii) Use the results to work out the probability of the spinner getting a '2'. $P(2) = 0.2\dot{1}$

(iv) If the spinner is spun 1000 times, use the table in (ii) to write down the number of zeros you could expect.

1000 trials

Expected Value $=$ Probability $\times$ # trials

$= \left(\frac{187}{580}\right) 1000 \approx 322$