19. An examination paper consists of 12 questions, 5 in Section A and the remainder in Section B. A candidate must attempt 5 questions, at least 2 of which must be from each section. In how many different ways may the candidate select the 5 questions?

"Selecting"

Sec. A	Sec B
5 Qs	7 Qs

Attempt: 5
at least 2 from ea. Section.

Choices: ① 2 from A and 3 from B
or ② 3 from A and 2 from B.

Option ① $5C2$ and $7C3$
or option ② or $5C3$ and $7C2$

Total $(5C2)(7C3) + (5C3)(7C2)$
$= 560$

21. Find the value of $n \in N$ in each of the following:

(i) $\binom{n}{2} = 10$ (ii) $\binom{n}{2} = 45$ (iii) $\binom{n+1}{2} = 28$

$$\boxed{\binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$n = n$
$r = 2$

$\binom{n}{2} = 10 \Rightarrow \frac{n!}{2!(n-2)!} = 10$

$n! = 10(2!(n-2)!)$
$n! = 20 \; (n-2)!$

rewrite $n!$ in terms of $(n-2)!$

$(n)(n-1)\cancel{(n-2)!} = 20\,\cancel{(n-2)!}$

Solve quadratic
$n^2 - n = 20$
$n^2 - n - 20 = 0$
$(n-5)(n+4) = 0$
$n = 5$ ✓, $n = -4$ ✗

Check: with calculator
$\binom{5}{2} = 10$ ✓

Combinations

> The number of combinations of r objects, chosen from a set of n different objects, is denoted by $\binom{n}{r}$ where
>
> $$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$

21. Find the value of $n \in N$ in each of the following:

(i) $\binom{n}{2} = 10$ (ii) $\binom{n}{2} = 45$ (iii) $\binom{n+1}{2} = 28$

in Tables

$$\boxed{\binom{n}{r} = \frac{n!}{r!(n-r)!}}$$

$n = n$
$r = 2$

$$\binom{n}{2} = 45 \Rightarrow \frac{n!}{2!\,(n-2)!} = 45$$

$$\Rightarrow n! = 45[2!\,(n-2)!]$$

rewrite $n!$ in terms of $(n-2)!$

$$(n)(n-1)\cancel{(n-2)!} = 90\,\cancel{(n-2)!}$$

Solve quadratic

$$n^2 - n = 90$$
$$n^2 - n - 90 = 0$$
$$(n-10)(n+9) = 0$$
$$n = 10 \checkmark, \quad n = -9 \times$$

Check: with calculator

$$\binom{10}{2} = 45 \checkmark$$

Combinations

The number of combinations of r objects, chosen from a set of n different objects, is denoted by $\binom{n}{r}$ where

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$