Example 1
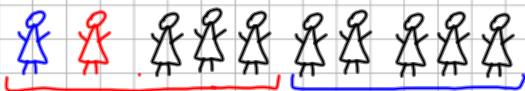
(i) In how many ways can a team of 5 players be chosen from 9 players?

(ii) In how many ways can this be done if a certain player must be selected in each team?

i

$$\binom{9}{5} = 9C5 = 126$$

ii

1   and   8 player

Choices   1 $\times$ $8C4$ $= 70$

Example 2

a In how many ways can a group of five be selected from ten people?

b How many groups can be selected if two particular people from the ten cannot be in the same group?

a

$$\binom{10}{5} = 10C5 = 252$$

b

If 2 people always together $= 8C3$
                              $= 56$

Ways apart $= 252 - 56 = 196$

Combinations from two different sets

If we have two different sets, one containing m different things and the other containing n different things, the number of combinations which can be made containing r of the first and s of the second is,

$$\binom{m}{r} \times \binom{n}{s}$$

The selections of $\binom{m}{r}$ and $\binom{n}{s}$ are **multiplied** because for each selection from $\binom{m}{r}$ we can associate every selection from $\binom{n}{s}$.

Note:

In general, when dealing with problems involving permutations, combinations or probability, the word **or** indicates that results are **added**.

Example 3

Find the number of ways in which a panel of four men and three women can be chosen from seven men and five women.

7 men   ⟹   4 men
5 women   3 women

7C4   ×   5C3   = 350

Example 4

In how many ways can a committee of six be formed from 5 teachers and 8 students if there are to be more teachers than students on each committee?

$$5T \qquad 8S$$

6 on committee

options $\quad\quad$ 5T + 1S $\quad$ } more T

(OR) 4T + 2S $\quad\quad$ than S

OR = "ADD"

$$
\begin{array}{l}
5C5 \times 8C1 \\
+\ 5C4 \times 8C2 \\
\hline
= 148
\end{array}
$$

Exercise 1.2

1. Evaluate each of the following:

(i) $\binom{6}{2}$ $\quad$ (ii) $\binom{7}{3}$ $\quad$ (iii) $\binom{10}{2}$ $\quad$ (iv) $\binom{12}{10}$ $\quad$ (v) $\binom{18}{16}$

i $\qquad$ $\binom{6}{2} = 6C2 = 15$

3

4.a In how many ways can a team consisting of 11 players be selected from a panel of 14 players?

b If the 14 players include only one goalkeeper, how many different teams can be selected if the goalkeeper is included in each team?

a $14C11 = 364$

b   TEAM
                  1 goal keeper + 13 outfield players
                       1   × $13C10$
                         $= 286$

5.a How many different selections of 5 letters can be made from the letters of the word CHEMISTRY?

b (i) How many 5-letter selections can be made if the letter C is included in each selection?

(ii) How many 5-letter selections can be made if the letter C is always included and Y is always excluded?
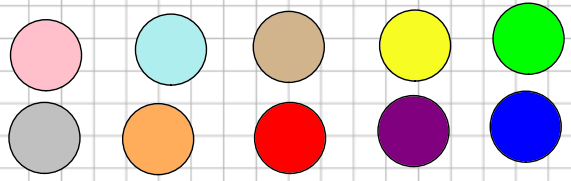
a CHEMISTRY $9C5 = 126$
   = 9 letters

b (i)   C + 4 others out of 8
                         1 × $8C4 = 70$

(ii)   C + 4 others out of 7
                         1 × $7C4 = 35$

8. In how many different ways may 5 colours be selected from 10 different colours
    including red, blue and green,
    (i)   if blue and green are always included
    (ii) if red is always excluded
    (iii) if red and blue are always included but green excluded?

Pick 5

(i) $8C3 = 56$

(ii) $9C5 = 126$

(iii) $7C3 = 35$

10. A school council consists of 10 teachers and 12 students.
    In how many ways can a group of 6 be selected if the group consists of
    (i) 3 teachers and 3 students
    (ii) 2 teachers and 4 students?

(i) $10C3 \times 12C3 = 26400$

(ii) $10C2 \times 12C4 = 22275$