1. Sequence notation

e.g. For a sequence, $u_n = 2\times 3^{n-1}$. Find the first three terms of this sequence and find the term of the sequence which has a value of 12288.

$n=1$
$n=2$
$n=3$

$u_1 = 2\times 3^{1-1} = 2\times 3^0 = 2$
$u_2 = 2\times 3^{2-1} = 2\times 3^1 = 6$
$u_3 = 2\times 3^{3-1} = 2\times 3^2 = 18$

$u_n = 12{,}288$

$2\times 3^{n-1} = 12{,}288$

$3^{n-1} = 6{,}144$

$b^n = a$
$\Leftrightarrow\ n = \log_b a$
$\Rightarrow$

$n-1 = \log_3 6144$

$\simeq 7.94$

$n = 7.94+1$

$n = 8.94$

e.g. calculate $\lim\limits_{n\to\infty} \dfrac{5(4^n)-2(3^n)}{3(4^n)+7(2^n)}$

Use calculator.

$T_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$a = ?$

$d = ?$ $T_{10} = 10$

$S_{10} = -35$

3. Arithmetic sequences and series

e.g. The tenth term of an arithmetic sequence is 10 and the sum of the first ten terms is -35. Find the first term and the common difference of the sequence.

$a + (10-1)d = 10$

$a + 9d = 10$ ①

$\frac{10}{2}[2a + (10-1)d] = -35$

$5[2a + 9d] = -35$

$2a + 9d = -7$ ②

② − ①

$$\begin{array}{r} 2a + \cancel{9d} = -7 \\ -a - \cancel{9d} = -10 \\ \hline a = -17 \end{array}$$

Sub $a = -17$ into ①

$-17 + 9d = 10$

$9d = 27$ $\Rightarrow$ $d = 3$

Geometric sequence or series

$T_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$, where $|r| < 1$

$S_n = \frac{a(1-r^n)}{1-r}$

4. Geometric sequences and series

e.g. The sum of the first six terms of a geometric series is nine times the sum of the first three terms. Find the common ratio.

$S_6 = 9S_3$

$\frac{a(1-r^6)}{\cancel{1-r}} = \frac{9(a(1-r^3)}{\cancel{1-r}}$

$\cancel{a}(1-r^6) = 9\cancel{a}(1-r^3)$

$1 - r^6 = 9 - 9r^3$

$9r^3 - r^6 - 8 = 0$

$r^6 - 9r^3 + 8 = 0$

let $r^3 = x$

$x^2 - 9x + 8 = 0$

$(x - 8)(x - 1) = 0$

$x = 8$, $x = 1$

$r^3 = 8 \Rightarrow r = \sqrt[3]{8} = 2$

$r^3 = 1 \Rightarrow r = \sqrt[3]{1} = 1$

5. Infinite geometric series

e.g. find the common ratio of a geometric series for which the sum to infinity is six times the first term.

$T_n = ar^{n-1}$

$S_\infty = \frac{a}{1-r}$

$r^0 = 1$

$S_\infty = 6T_1$

$\frac{\cancel{a}\,1}{1-r} = 6(\cancel{a}r^{1-1})$

$\frac{1}{1-r} = 6(1)$

$1 = 6 - 6r$

$-5 = -6r$

$r = \frac{-5}{-6}$

$r = \frac{5}{6}$

6. Applications of infinite geometric series

e.g. by forming an infinite geometric series, write $1\cdot\dot{4}\dot{7} = 1\cdot474747......$ in the form $\frac{p}{q}$, where $p, q \in \mathbb{N}$.

Use calculator

$1\cdot\dot{4}\dot{7} = \frac{146}{99}$

$S_\infty = \frac{a}{1-r}$

Sequence $= 1 + 0\cdot47 + 0\cdot0047 + 0\cdot000047...$

$1 + \frac{47}{100} + \frac{47}{10000} + \frac{47}{1000000} \ldots$

$a = 1$ $r = 47/100$

$S_\infty = \frac{1}{1-\frac{47}{100}} = \frac{146}{99}$