p.96 EDCo SAMPLE A PAPER 1 Q9

a) A spherical football is pumped with air at a rate of 25 cm³ per second. Calculate the rate at which the radius of the football is increasing when the radius is 10cm. Give your answer in terms of π.

$\frac{dR}{dt} = ?$ WANT

$\frac{dV}{dt} = 25 \text{ cm}^3/\text{s}$ GIVEN

$V = \frac{4\pi R^3}{3}$

$\frac{dV}{dR} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2) = 4\pi R^2$ GET

$\boxed{\frac{dR}{dt} = \overset{?}{\frac{dR}{dV}} \cdot \frac{dV}{dt}}$

$\frac{dR}{dV} = \frac{1}{\left(\frac{dV}{dR}\right)} = \frac{1}{4\pi R^2}$

$\frac{dR}{dt} = \frac{1}{4\pi R^2} \cdot 25 = \frac{25}{4\pi R^2}$

$\frac{dR}{dt}_{(R=10)} = \frac{25}{4\pi(10)^2} = \frac{1}{16\pi}$ cm/s

b) Differentiate $\sin^2(3x-1)$ w.r.t x

$f(x) = [\sin(3x-1)]^2$

Chain Rule

$f'(x) = 2[\sin(3x-1)]^1 \cdot (\cos(3x-1)) \cdot (3)$

$= 6\sin(3x-1)\cos(3x-1)$

c) $f(x) = \frac{2}{1-2x}$ $x \in \mathbb{R}$ $x \neq \frac{1}{2}$

(i) Show $f(x)$ is always increasing and has no points of inflection.

If always increasing
$\Rightarrow$ Slope is always positive

i.e.. $f'(x) > 0$

$f(x) = 2\left(\frac{1}{1-2x}\right) = 2(1-2x)^{-1}$

Slope= $f'(x) = -2(1-2x)^{-2}.(-2)$
$= 4(1-2x)^{-2}$

$= \frac{4}{(1-2x)^2} > 0$ $\Rightarrow$ always increasing

If no pt. of inflection
$f''(x) \neq 0$

$f''(x) = -8(1-2x)^{-3}.(-2)$
$= \frac{+16}{(1-2x)^3} \neq 0$

(ii) $y = 1x + c$ is a tangent to $f(x)$
find 2 possible values of c, $c \in \mathbb{R}$.

$m=1$ When is $f'(x)=1$ $\Rightarrow f'(x) = \frac{4}{(1-2x)^2} = 1$

$4 = (1-2x)^2 \Rightarrow 4 = 1 - 4x + 4x^2 \Rightarrow 4x^2 - 4x - 3 = 0$
$\Rightarrow (2x+1)(2x-3) = 0 \Rightarrow x = -1/2 , x = 3/2$

y-values?

$f(x) = \frac{2}{1-2x}$ $f\left(-\frac{1}{2}\right) = \frac{2}{1-2\left(-\frac{1}{2}\right)} = 1$ pt $\left(-\frac{1}{2}, 1\right)$

$f\left(3/2\right) = \frac{2}{1-2\left(3/2\right)} = -1$ pt $\left(\frac{3}{2}, -1\right)$

$\boxed{y = x + c}$

use $\left(-\frac{1}{2}, 1\right) \Rightarrow 1 = -\frac{1}{2} + c \Rightarrow c = 3/2$

use $\left(3/2, -1\right) \Rightarrow -1 = 3/2 + c \Rightarrow c = -5/2$

d) $f(x) = \dfrac{2x}{x-1}$ $\begin{matrix} u \\ v \end{matrix}$ $x \in \mathbb{R}$ $x \neq 1$

(i) A and B are two different points on the curve. The tangents at A and B are parallel. If $A(-1,1)$ then $B = ?$

What slope at A? ie if $x=-1$ $f'(-1) = ?$

QUOTIENT $f'(x) = \dfrac{(x-1)(2) - (2x)(1)}{(x-1)^2}$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$= \dfrac{\cancel{2x} - 2 - \cancel{2x}}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

Slope at A ? $f'(-1) = \dfrac{-2}{(-1-1)^2} = \dfrac{-2}{4} = -\dfrac{1}{2}$

$\Rightarrow$ Slope at B is also $-\frac{1}{2}$ if $f'(x) = -\frac{1}{2}$

$f'(x) = \dfrac{-2}{(x-1)^2} = -\dfrac{1}{2}$ $\Rightarrow$ Solve $-4 = -1(x-1)^2$

$\Rightarrow 4 = x^2 - 2x + 1 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = 3, x = -1$ $f(3) = \dfrac{2(3)}{3-1} = \dfrac{6}{2} = 3$ $B(3,3)$

(ii) Write the equations of the asymptotes to $f(x)$. Hence show that the mid-point of $[A,B]$ is the point of intersection of the asymptotes.

$f(x) = \dfrac{2x}{x-1}$

Horizontal asymptote : $y = 2$

Vertical asymptote : $x = 1$

pt. of intersection $(1,2)$

midpt $A(-1,1)$ $B(3,3) = ?$

midpt_{AB} $(1,2)$