EDCO SAMPLE D

Q1 a

$a+bi = r\,\mathrm{CiS}\,\theta$

$r = 2$

$\theta = \frac{5}{6}\pi$

rectangular form

$u = 1\,\mathrm{CiS}\,\frac{\pi}{2}$

$w = 2\,\mathrm{CiS}\,\frac{\pi}{3}$

$u.w = ?$

$u.w = (1)(2)\,\mathrm{CiS}\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$= 2\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$

$u/w = ?$

$\frac{u}{w} = \frac{1}{2}\,\mathrm{CiS}\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$

$= \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= \frac{1}{2}\left(\sqrt{3}/2 + i\left(\frac{1}{2}\right)\right)$

$= \frac{\sqrt{3}}{4} + \frac{1}{4}i$

SAMPLE B

Q1

$\frac{3\pi}{4} = \frac{3(180^\circ)}{4} = 135^\circ$

De Moivre

$(r\,\mathrm{CiS}\,\theta)^n = r^n\,\mathrm{CiS}\,n\theta$

CUBIC EQUATION
3 SOLUTIONS

$\theta_1 = 135^\circ$
$\theta_2 = 135^\circ + 360^\circ = 495^\circ$
$\theta_3 = 495^\circ + 360^\circ = 855^\circ$

$z = 3\frac{3}{8}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$z = \frac{27}{8}\,\mathrm{CiS}\,135^\circ$

$w^3 = z \quad \Rightarrow \quad z^{1/3} = w$

$w = \left[\frac{27}{8}\,\mathrm{CiS}\,135^\circ\right]^{\frac{1}{3}}$

$w_1 = \left[\frac{27}{8}\,\mathrm{CiS}\,135^\circ\right]^{\frac{1}{3}}$

$= \frac{9}{2}\,\mathrm{CiS}\,45^\circ$

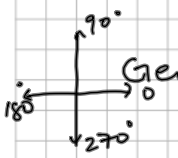
$w_2 = \left[\frac{27}{8}\,\mathrm{CiS}\,495^\circ\right]^{\frac{1}{3}}$

$= \frac{9}{2}\,\mathrm{CiS}\,165^\circ$

$w_3 = \left[\frac{27}{8}\,\mathrm{CiS}\,855^\circ\right]^{\frac{1}{3}}$

$= \frac{9}{2}\,\mathrm{CiS}\,285^\circ$

SAMPLE B
Q1



General Polar: $w =$

De Moivre

$(r\,cis\,\theta)^n = r^n\,cis\,n\theta$

$z = 3\frac{3}{8}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$w^3 = z \Rightarrow z^{\frac{1}{3}} = w$

$z = \frac{27}{8}\,cis\left(\frac{3\pi}{4} + 2n\pi\right)$

$z^{\frac{1}{3}} = \left[\frac{27}{8}\,cis\left(\frac{3\pi}{4} + 2n\pi\right)\right]^{\frac{1}{3}}$

$= \sqrt[3]{\frac{27}{8}}\,cis\left(\frac{3\pi}{12} + \frac{2n\pi}{3}\right)$

$= \frac{3}{2}\,cis\left(\frac{\pi}{4} + \frac{2}{3}n\pi\right)$

$n=0$ w_1

$\frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ$ 1st Q.

$w_1 = \frac{3}{2}\,cis\left(\frac{\pi}{4} + \frac{2(0)\pi}{3}\right)$

$= \frac{3}{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$n=1$ w_2

$\frac{11}{12}\pi = \frac{11(180^\circ)}{12} = 165^\circ$ 2nd Q

$w_2 = \frac{3}{2}\,cis\left(\frac{\pi}{4} + \frac{2(1)\pi}{3}\right) =$

$= \frac{3}{2}\left(\cos\frac{11}{12}\pi + i\sin\frac{11}{12}\pi\right)$

$n=2$ w_3

$\frac{19}{12}\pi = \frac{19(180^\circ)}{12} = 285^\circ$ 4th Q

$w_3 = \frac{3}{2}\,cis\left(\frac{\pi}{4} + \frac{2(2)\pi}{3}\right)$

$= \frac{3}{2}\left(\cos\frac{19}{12}\pi + i\sin\frac{19}{12}\pi\right)$

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5. Amortised loans, including mortgages

e.g. Olivia borrows €100000 and agrees to repay the loan by a series of 8 equal annual repayments starting in one year's time. The APR for the loan is 9%.

(i) Calculate the amount of each equal annual repayment.

(ii) Construct a schedule showing interest and principal portions of the repayments outlined in part (i).

$A = \frac{P\,i\,(1+i)^t}{(1+i)^t - 1}$

P = Loan amount = €100,000

A = repayments = ?

$i = 9\%$

$t = 8$ years

$A = \frac{100\,000\,(0{\cdot}09)(1{\cdot}09)^8}{(1{\cdot}09)^8 - 1}$

$= €18{,}067{\cdot}44$

yr.	Loan amount
0	100 000
1	109 000 − 18067·44 = 90932·56

18067·44 → Loan 9067·44

→ Interest 9000·00

Pension Fund

This is a series of present values that will each be worth €20000 when needed

$$P = \frac{F}{(1+i)^t}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

How much of a pension fund is needed if we want (APR = 4.5%) €20000 p.a. for 20yrs?

1st Payment needed now = 20000 — a

2nd needed in 1 yr. = $\frac{20000}{1.045^1}$

3rd needed in 2 yrs = $\frac{20000}{1.045^2}$ — $r = \frac{1}{1.045}$

⋮

20th needed in 19 years = $\frac{20000}{1.045^{19}}$ — $n = 20$

$$\text{Fund} = S_{20} = \frac{20000\left(1-\left(\frac{1}{1.045}\right)^{20}\right)}{1-\left(\frac{1}{1.045}\right)}$$

$$= €271{,}865.87$$

$y - y_1 = m(x - x_1)$

$y + 9 = m(x - 0)$

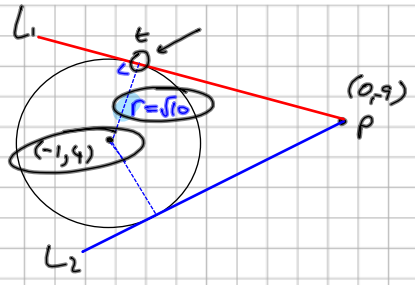
$y + 9 = mx$ (?)

$L: mx - y - 9 = 0$

$ax + by + c = 0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$\binom{159}{3}\binom{53}{}$



$$\frac{|m(-1) - 1(4) - 9|}{\sqrt{m^2 + (-1)^2}} = \sqrt{10}$$

$$\frac{|-m - 13|}{\sqrt{m^2+1}} = \sqrt{10}$$

$$|-m - 13| = \sqrt{10}\sqrt{m^2+1}$$

$$m^2 + 26m + 169 = 10(m^2+1)$$

$$m^2 + 26m + 169 = 10m^2 + 10$$

$$9m^2 - 26m - 159 = 0$$

$$(9m - 53)(m + 3) = 0$$

$m = \frac{53}{9}$, $m = -3$

Bernoulli

$P(\text{event } r \text{ times in } n \text{ trials}) = \binom{n}{r} p^r q^{n-r}$

$P(6) = \frac{2}{7}$

$P(\text{not } 6) = \frac{5}{7}$

$P(3^{rd}\ 6 \text{ on } 8^{th} \text{ throw})$?

$P(2\ 6\text{'s in } 7 \text{ throws}) \overset{*}{\text{and then}} P(6)$

$= \binom{7}{2} \left(\frac{2}{7}\right)^2 \left(\frac{5}{7}\right)^5 * \frac{2}{7}$

$= 0.091$