Prove $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ using de Moivre's theorem.

Real Imaginary

$$(\cos\theta + i\sin\theta)^3 \overset{\text{[Demoivre]}}{=} \cos 3\theta + i\sin 3\theta$$

$$(x+y)^3 = (x+y)(x+y)^2 = (x+y)(x^2+2xy+y^2)$$
$$= x^3 + 2x^2y + y^2x + x^2y + 2xy^2 + y^3$$
$$= x^3 + 3x^2y + 3y^2x + y^3$$

$$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta \cdot i\sin\theta$$
$$+ 3\cancel{i^2}\sin^2\theta\cos\theta + i^3\sin^3\theta$$

[Re = Re]

$$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$$

$$[\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta]$$

$$\cos 3\theta = \cos^3\theta - 3[1-\cos^2\theta]\cos\theta$$
$$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$$
$$= 4\cos^3\theta - 3\cos\theta$$

QED

Prove by Induction that $8^n - 3^n$ is divisible by 5, $n \in \mathbb{N}$

let $f(n) = 8^n - 3^n$

① Show it's true for some values

$f(1) = 8^1 - 3^1 = 8 - 3 = 5 = 5(1)$
$f(2) = 8^2 - 3^2 = 64 - 9 = 55 = 5(11)$

② Assume true for $n = k$

Assume $f(k) = 8^k - 3^k = 5(a)$, $a \in \mathbb{N}$

③ Prove true for $n = k+1$

$$f(k+1) = 8^{k+1} - 3^{k+1}$$
$$= 8^{k+1} - 3.8^k + 3.8^k - 3^{k+1}$$
$$= 8^k(8-3) + 3(8^k - 3^k)$$
$$= 8^k(5) + 3(5(a))$$
$\Rightarrow$ divisible by 5

④ Conclude

It's true for $n = 1, 2, k$ and $k+1$
$\Rightarrow$ its true for all values of $n \in \mathbb{N}$