Question 5   **(25 marks)**
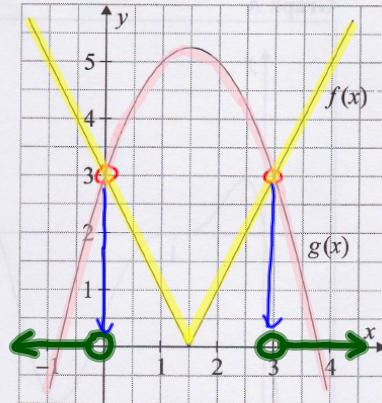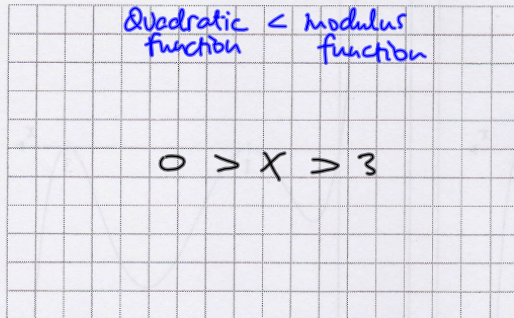
The graphs of the functions $f : x \mapsto |2x - 3|$ and $g : x \mapsto 3 + 3x - x^2$ are shown in the diagram.

(a)   Use your graph to solve the inequality

$$3 + 3x - x^2 < |2x - 3|.$$

Quadratic   <   Modulus
function   function

$$0 > X > 3$$

(b)   Use algebra to solve the inequality $3 + 3x - x^2 < |2x - 3|$.

Square both Sides

$$(3 + 3x - x^2)^2 < (2x - 3)^2$$

3 +	3x	$-x^2$
3	9	+9x	$-3x^2$
+3x	+9x	$+9x^2$	$-3x^3$
$-x^2$	$-3x^2$	$-3x^3$	$+x^4$

$$x^4 - 6x^3 + 3x^2 + 18x + 9 < 4x^2 - 12x + 9$$

$$x^4 - 6x^3 - x^2 + 30x < 0$$

FACTORISE   HCF = x   If $x(x^3 - 6x^2 - x + 30) = 0$

FACTORISE CUBIC

try x=3 in cubic ('from graph')

$$x^3 - 6x^2 - x + 30 = 0$$
$$(3)^3 - 6(3)^2 - 3 + 30 = 0$$
$$\Rightarrow (x - 3) \text{ is factor}$$

DIVIDE

$$\begin{array}{r} x^2 - 3x - 10 \\ x - 3 \overline{)\, x^3 - 6x^2 - x + 30} \\ \underline{\mp x^3 \pm 3x^2} \\ -3x^2 - x \\ \underline{\pm 3x^2 \mp 9x} \\ -10x + 30 \\ \underline{\pm 10x \mp 30} \end{array}$$

FACTORISE QUADRATIC

$$x^2 - 3x - 10$$
$$(x + 2)(x - 5)$$

4 LINEAR FACTORS

$$x^4 - 6x^3 - x^2 + 30x = 0$$
$$(x + 2)(x)(x - 3)(x - 5) = 0$$

POSSIBLE SOLUTIONS

$$x = -2 \quad, x = 0 \quad, x = 3 \quad, x = 5$$

Test Values :

$$3 + 3x - x^2 < |2x - 3|$$

x   inside 0 and 3 ? x = 1

Inside inner Values ?

$$3 + 3(1) - (1)^2 = 5$$
$$|2(1) - 3| = 1 \qquad 5 \not< 1 \Rightarrow \text{outside works}$$

Outside outer Values ?

x   outside -2 and 5 ?

$$x = -3 \qquad 3 + 3(-3) - (-3)^2 = -15$$
$$|2(-3) - 3| = 9 \qquad -15 < 9$$
$$\Rightarrow \text{outside works}$$

Conclude

$$0 > x > 3$$

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