THE IRISH TIMES irishtimes.com

examTIMES

In association with The Institute of Education

How I Got My A1
Tips from an A1 student

No 3: Maths

Thursday, January 15th, 2015

Maths for the Leaving Cert

Your essential guide

THE IRISH TIMES

Paper by paper

Everything you need to know to tackle both papers at higher or ordinary level

Project Maths – a teacher's view

Page 4

DOs and DON'Ts for the exams

Pages 4-5

THE INSTITUTE OF EDUCATION FOUNDED 1969

What's Inside

Exam Times

No 3: Leaving Cert Maths

Exam Times is published by
The Irish Times
in association with the
Institute of Education

Editor: Deirdre Falvey
Production: Emma Somers
education@irishtimes.com

THE IRISH TIMES
24-28 Tara Street
irishtimes.com
Tel: 01-6758000

Advertising:
Tel: 01-6758585

Results: students at the Institute of Education, Leeson Street. PHOTOGRAPH: ALAN BETSON

Welcome

Welcome to the first Exam Times of 2015, a collaboration between the Institute of Education and *The Irish Times*. Over the coming seven weeks we will be publishing a series of study guides written by teachers from the Institute of Education, covering a variety of Leaving and Junior Certificate subjects.

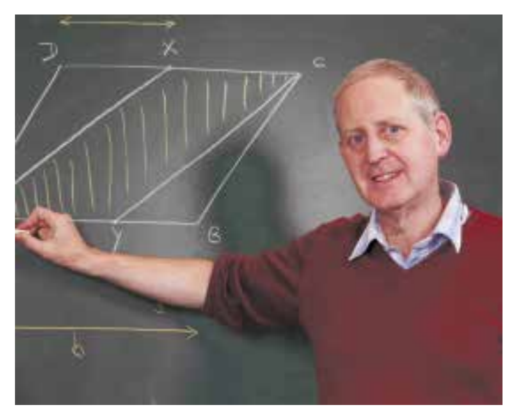
Today's supplement deals with Leaving Certificate maths, at both higher and ordinary level.

Written by Aidan Roantree, senior maths teacher at the Institute of Education, this guide looks at the final version of the new Project Maths syllabus.

This supplement gives an overview of the syllabus, with advice on exam technique and a close look at sample questions and answers.

We hope students find the advice beneficial and that this guide will help them with their exam preparation.

Peter Kearns
Director, the Institute of Education



Aidan Roantree

Senior maths teacher at the Institute of Education

Aidan has been teaching maths and applied maths at the Institute for more than 28 years. As the author of 14 books, including the most recent *Effective Maths Books 1* and *2*, he is acquainted with every detail of the new maths course.

He has also appeared on radio programmes to give advice to students prior to exams.

He is regularly called upon by branches of the Irish Mathematics Teachers Association to give talks to teachers on aspects of the maths courses.

Next Thursday
Leaving Cert languages (English, Irish, French, Spanish, German)

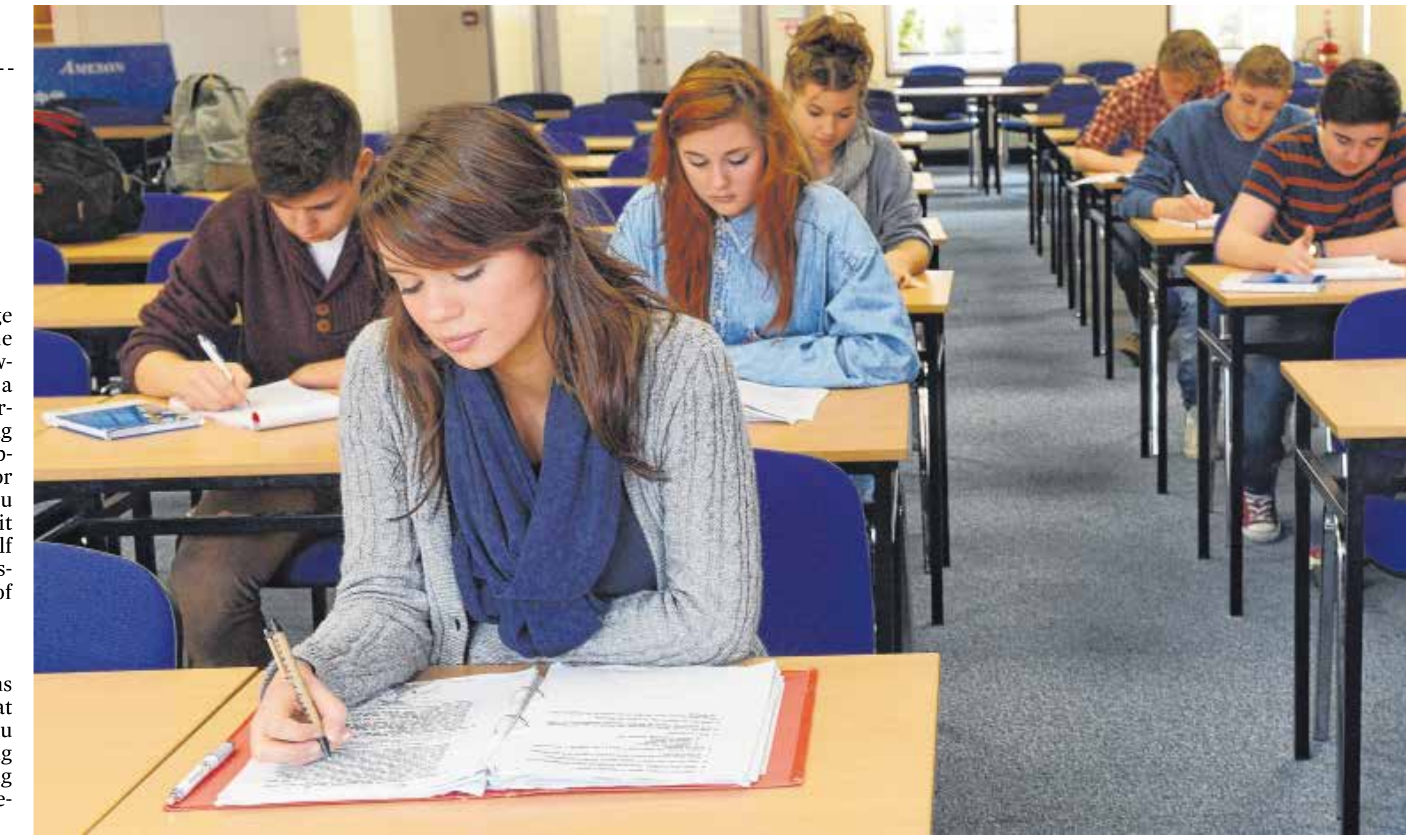
Exam Times Online
For more from the Institute of Education and the Irish Times visit irishtimes.com/examtimes

THE IRISH TIMES in association with THE INSTITUTE OF EDUCATION

Ten tips you can count on

Aidan Roantree

From revision to exam technique and timing, you can improve your maths grade with these pointers



Studying at the Institute of Education. PHOTOGRAPH: BRENDA FITZSIMONS

1 Pay huge attention to basics

Numbers and algebra are the language of maths at our level. Can you imagine writing an English essay without knowing the language? Any time you run into a problem with numbers (eg fractions, percentages) or with algebra (eg rearranging or solving equations) face up to the problem and get someone, perhaps a teacher or somebody whose mathematical ability you trust, to explain to you exactly how it works. You will not be able to do yourself justice, especially with the harder questions, if you are weak at the mechanics of maths.

2 Pay attention to detail

It is important to read instructions carefully, and to realise exactly what is being asked. For example, you should know the difference between being asked to solve a cubic equation and being asked to factorise a cubic. Maths is a precise discipline.

Whereas you will be given credit for every correct step you take, if you fail to answer what is required, you will lose marks. Paying attention to detail also applies to how you write things down. Watch the way your teacher writes equations, etc. You will notice that it is very similar to what you see in your textbook. This is the proper way to write maths, and this is what you should strive to do.

3 Master all the main techniques in each topic

In each area of maths, you have to start by learning all of the key techniques connected with that area. For example, in algebra, you have to be able to solve quadratic equations, simultaneous equations, equations with x in the index, etc.

You should bear in mind that "learning" a technique in maths should preferably not mean memorising it. Rather, if at all possible, for each technique (formula or method), you should try to understand exactly when it is used (and when it is not), its purpose, how it works, alternatives and pitfalls to avoid.

You should also practise giving precise written explanations in questions where these are required.

4 Pay proper attention to real-life questions

Section B on each paper will involve a number of questions which will be drawn from real life. Although there is no way of preparing for the exact questions you will face this June, by practising as many of these as you can find on past papers and sample papers, you will reduce the shock when exam time comes.

You should try this after you have become comfortable with all the main techniques, because the first stage in tackling a real-life problem is to decide on an approach that is likely to work. You will be able to make an informed decision only after you are aware of all the options.

5 Revise frequently

To retain information, you need to use it frequently. One of the best ways of doing this is to revise each topic on a regular basis. Each month or each term should do for most students. What you will find as you revise a topic again and again is that each occasion takes less and less time, as you remember more and more of the topic. In revision, you should attempt exam-type questions, where you can practice both the techniques and your decision-making skills.

6 Study the marking schemes

Marking schemes are available for the 2014, 2013 and 2012 papers and for the exams taken by students from the pilot schools from 2010 to 2013.

You should study the marking schemes for all of these carefully, although you should be aware that certain changes have occurred to the syllabus in the meantime. Hence a small number of these questions are no longer relevant.

You should notice the great disparity in the marks for some question parts, and the large number of marks given for attempts. This should spur you on to making as much effort as possible for the easy parts, and to write something down for each of the other parts.

For the really hard parts, resolve to write something down to give yourself every chance of getting a large portion of what will probably be a very small mark.

7 Watch your timing

Watching the time spent on each question in the exam will be crucial. Running out of time and leaving out questions at the end of an exam will have a devastating effect on your final grade.

In Section A on each paper, there will be a selection of 25-mark questions. You will have 12 and a half minutes for each of these.

In Section B, the marks for each question are unknown. You can use the rule of thumb that to find the number of minutes you can afford to spend on a question, divide the number of marks for the question by two. These times are all upper limits, and you should try to complete the questions in less time.

In particular, noting what was said above about the likely marking scheme, do not spend much time on the parts you reckon are extremely hard! They are probably going to be worth only a few marks.

8 Don't give up on a question

The majority of students will often meet questions from past papers, mocks, etc, which they don't know how to approach, or whose solutions they cannot follow. If you are looking for the best possible grade you can get, it is vital that you don't give up on these questions. Ask your friends or relations (those very good at maths!) or ask your teacher. One way or another, do your best to make sure you get an explanation you are happy with.

9 Practise all the skills

In the exams you will be required to deal with expressions and equations. Naturally, you have to become familiar with these. But there is more. In certain questions, you will have to draw graphs, use your calculator and use and adapt the formulae given in the maths tables. All of these skills need to be practised and later revised.

You will also have to draw graphs on the squared paper in the exam answer book, and so you should practise this now. Don't forget to name the axes and mark scales on the axes. For geometry, you will have to practise the constructions on the course, remembering to show all the construction lines and arcs.

10 Take on board all the advice given by your teacher

Most maths teachers are approachable, helpful and knowledgeable about your course and exams. If you have any queries or problems, you should approach your teacher. You should listen to, and take on board, any advice given by your teacher. This also applies to any comments they make about your homework or tests.

Project Maths: where are we now?

Aidan Roantree

It's hard luck for the hardworking student with the new regime but the first step is still to master the techniques on the course

This year, for the first time, all Leaving Cert maths students will sit papers based on the final version of the new Project Maths syllabus. This transition from the old syllabus to the new has been incremental, taking five years from beginning to end.

The new course, and the method of its implementation, have not been without controversy. The intense debate between committed defenders and detractors has made this change of syllabus probably the most fractious in the recent history of Irish education.

It now appears the dust has settled, and the new course and exams are here to stay. Some tinkering with the nature of exam questions may, and probably will, occur, but the word is that no further change to the syllabus will happen for the foreseeable future.

Thus, despite the accusations of the dumbing down of maths, the clamour from teachers for the course length to be reduced, the criticism by many of the exam questions (especially in the early years) and the perceived reluctance of the powers that be to engage meaningfully with teachers on their concerns about the new course, this is now a fait accompli.

So we can now begin to make a rational assessment of where we stand with this new Project Maths course.

From my point of view as a maths teacher, one of the major changes that has occurred with the new course, and exams, is in the profile of the student who will do well in the Leaving Cert Maths exams, a view largely supported by other maths teachers I have discussed this with.

The hardworking student

Under the old regime, the conscientious, hardworking student who put in long hours could expect to be duly rewarded with a high grade. Most of these students spent significant time analysing trends in past exam questions and learned to replicate successful approaches. Gifted students who didn't bother taking note of the trends and the expectations of the exam often did not perform as well as their more organised and focused classmates.

The problem was that the hardworking student very often had no real appreciation of what they were doing, treating maths like a computer game in which they want to get a high score. They never learned to interpret their maths work and place it in the real world. To be fair, they were not required to either by the syllabus or by the exams.

The effect of this was that when these students arrived at third level, they were inadequately prepared for what was required in their courses. With the exception of students who take pure maths in university (there are very few of us, but there should be more), all other students are expected to treat maths as a tool – a means to an end.

The vast majority of third-level students who use maths are expected to combine maths methods with other abilities, such as writing reports, making decisions and giving advice. For example, students of business courses are expected to be able to use maths to maximise profits or minimise costs. But they usually have to form their own mathematical model, and in particular, interpret their outcomes.

Students of the old course were ill-prepared for such requirements. These problems with the old course were identified and highlighted some years ago by key people from industry. It was largely on account of their urging that the initiative to make the fundamental change in the teaching of maths, designated Project Maths, was undertaken.

The aim of the new course is to produce students who are proficient with basic maths methods, but who are also able to appreciate the origins of maths and the applicability of maths to many areas of living, both professional and social. And so the new course stresses the understanding of concepts and the practical applications of these concepts, and gives far less importance to tackling mathematical puzzles.

Thus, for example, there is a substantial section on financial maths (at higher level), and a much expanded section on probability and statistics. Also, traditional topics such as algebra, trigonometry and calculus include far more material of a practical nature than before.

Independent thought

The new course also seeks to change the student's experience in maths class. It is intended that work in the classroom be more investigative and collaborative than on the old course. Students are encouraged to work individually and in groups to discover for themselves key concepts and methods.

These changes have many objectives: they will reinforce the student's understanding of what they learn, they will encourage independent thought and they will encourage students to learn to work in teams. All of these will benefit students when they move to the next stage: third level education and/or employment.

Of course, unless these features of the course are incorporated in the exam structure they are likely to be largely ignored in practice. The exams all contain questions of varying types: skill questions, which test a student's ability to execute mathematical procedures; concept questions, which examine a student's grasp of the maths ideas they have learned; and application questions, which require students to make decisions and develop strategies before solving the question. It is important to note that without the significant changes to the exam structure, the other aims of the new course will have been largely aspirational.

This all sounds so wonderful and obvious that it must result in students leaving secondary school with more appropriate maths skills and a better attitude to maths. However, the new regime is not without its significant problems, which we'll come to.

With the changes to syllabus, teaching methods and exam structure, the profile of the most successful students in the Leaving Cert maths exams has altered dramatically. The highest grades will go to those students who have mastered the required skills, as with the old exams, but who also show a deep understanding of the concepts and have the ability to analyse new situations and apply the full panorama of their maths knowledge to a variety of problems.

This militates against the hardworking student who has just learned to master the techniques of maths, without paying attention to understanding the concepts. It also means that students who don't practise applying their maths knowledge to new and strange problems will be at a serious disadvantage.

Ask questions

All of this should influence how you approach studying maths for the Leaving Cert in order to obtain as high a grade as you can. The first step is to master the techniques on the course. But this alone is no longer enough.

To be able to approach the concept questions, you should simultaneously try to master the different ideas and techniques on the course while all the time asking questions: what's the purpose? What's the reason for the definition? Why am I being asked to learn this?

But even more important is that you prepare for the applications questions – these will account for 50 per cent of the total marks on each paper. In light of the promise that these questions will be as unpredictable as possible, the questions you will face this year will be unlike the questions from last year and the year before. So what can be done?

You can certainly start by studying thoroughly the Section B (Applications) questions for the past few years, focusing on how the given information is changed into maths language and equations. Then you should try to source as many other sample and mock papers as you can, especially those for which you can get solutions, all the time trying to generalise what you see.

It would also be a good idea to go to examinations.ie and scrutinise in detail the marking scheme for each of the past Leaving Cert questions. You will notice that often the questions parts are given hugely disproportionate marks, with many marks going for relatively easy parts and very few marks going for the most difficult. It also seems from the marking schemes that picking up substantial partial credit marks is not that difficult. The key is to make a serious, realistic attempt at each question part.

Aidan Roantree teaching at the Institute of Education, Dublin. PHOTOGRAPH: BRENDAN DUFFY

> **The one thing you can be sure of is this year's exam papers will be unlike last year's, or any other year's. The material will be the same, but the questions will be very different**

In terms of the exams, the bottom line is that the one thing you can be sure of is this year's exam papers will be unlike last year's, or any other year's. The material will be the same, but the questions will be very different.

Yes, questions on algebra, trigonometry, differentiation, statistics, complex numbers, etc will be there. But we don't know exactly where, or exactly what they will deal with. And that's only Section A (Concepts and Skills). With Section B, anything goes.

All in all, the new course and exams seek to promote maths as a logical, structured and useful discipline, and in doing so produce students who appreciate the maths they have learned and who are better prepared to use this maths in a variety of ways in their later life.

Also, with the 25 bonus CAO points for at least a D3 grade in higher maths, we now have about 30 per cent of Leaving Cert students taking the higher-level papers. This means far more students are entering third level with a significantly greater maths background than before, which can only enhance Ireland's reputation in education.

Awkward implementation

Again, this sounds so wonderful that one would be inclined to think that it would be impossible to argue with the new course, whatever about the awkward nature of its implementation. But this is not so. Significant problems remain.

The course is undoubtedly too long. This is despite official denials, and disparaging remarks made about teachers who hold this opinion (any teacher who thinks the course is too long is not teaching it properly, or is afraid of change). The rejoinders I have heard to these comments are that any official who thinks the course is not too long is unfamiliar with the workings of a classroom, and that teachers are not averse to change – they just don't like bad change.

Even the amount of basic material to be covered is greater than on the old course, which was itself perceived as too long. Add in the necessity to get students to fully grasp the concepts, and apply their new-found expertise to a myriad of diverse problems, and it is easy to see why teachers are having to hold extra classes just to get the course covered.

Other criticisms relate to the exams. To begin with, large areas of the course may not be examined in any given year, with conversely a heavy focus on other areas. This introduces an element of luck to the exam, much more than existed with the old exams. For example, in 2014, many higher level students were disappointed there was no question on financial maths and that all the difficult material in calculus was also missing.

Also, students are concerned about question parts that are seen as having little or nothing to do with maths. These tend to be verbal questions requiring opinions based on general knowledge and common sense.

One example from the 2014 Higher Level Paper 2 was Question 7 about people at work, unemployed and not in the labour force. This question was worth 45 marks, or 7.5 per cent of the total marks. Of these marks, 25 were for giving opinions, while the remaining 20 marks were for performing elementary operations on fractions and decimals. Only 5 marks out of the 45 were awarded for anything students would have studied in school. Consequently, a first-year student with a good general knowledge could have obtained 40 out of the available 45 marks for this question.

Lost the plot

It should not be the case that students whose mathematical abilities are weak are able to overcome this deficiency and obtain a high mark in maths on the back of giving opinions in random situations. Someone needs to remember that just because something uses maths, doesn't mean that it is maths.

This does not allow a student with a good grasp of mathematical techniques to showcase their abilities.

Of course, students should be able to use their maths in a real-life setting, but questions such as the 2014 example above seem to have lost the plot. It also means that there will be fewer questions of a true mathematical nature, leading to the feeling among students that their maths result will be prone to an element of luck.

As a teacher, I have first-hand experience of this when I examine the Leaving Cert results of my students over the past few years. With the old course and the old exams, I found it possible to predict quite accurately how individual students would fare in maths in the Leaving Cert. The best students obtained As and Bs, etc, just as they had achieved in class.

With the Project Maths exams, however, I see wide discrepancies. Students who had obtained Ds throughout the years were now getting Bs, and even As, in the Leaving Cert, and vice-versa. The problem of random questions has been compounded by the awarding of high, or very high, partial credit marks, for sometimes rather poor attempts, although this problem seems to be abating a little.

All of this means that we cannot have as much faith in the mathematical abilities of students who get B, C and D grades as we could before. However, students who obtain A grades would largely be immune to this doubt.

In summary, it seems clear that the serious issue of the length of the course will not be addressed in the near future, which is a pity. However, we can only hope that some of the other problems will be dealt with, so that all stakeholders – students, teachers, parents, third level institutions and employers – can have greater confidence in the real mathematical skills of our students emanating from second level.

Exam DOs

- Do arrive in plenty of time. If you are rushing into the exam, you will be too flustered to do yourself justice, at least for the first while.
- Do have a positive frame of mind as you enter the exam. View the exam as a positive opportunity to show what you can do, and get the credit you deserve for all the work you have put in. The glass is half full, not half empty.
- Do start immediately by reading the entire paper, mapping out the questions you think you can handle, and noting those you think may be difficult. Read each question extremely carefully, making note of all that is required.
- Do start with the question you consider the easiest, then do the next easiest, and so on. Because of the layout of the answer booklet, it will be obvious which questions you have left undone, as you return later.
- Do show your calculations. You sometimes (not always) get full marks for a correct answer without work shown, but if you get the wrong answer, you may get little.
- Do plan your timing carefully. In practice, you can only plan for the total marks for a question: halve the number of marks to find the number of minutes available for a question. After that, try to play it smart by guessing which parts are likely to be awarded the most marks, and concentrate on these.
- Do underline key words. Examples include: prove, verify, show, find, solve, evaluate, graph, plot. If a question says 'hence', as distinct from 'hence, or otherwise', you must use what went before to complete what follows. Using any other method will not get the marks.
- Do check that you have answered everything required before leaving a question.
- Do make some attempt at every part of the questions you are doing. Any right step will get you at least a partial credit. Past marking schemes show this could be a huge portion of the overall marks for the question and could make a significant difference to the grade you eventually get.
- Do realise the importance of algebra and the need for accuracy when using algebra.
- Do try to be as precise and accurate as possible in any question that requires an explanation of a key concept. Vague, imprecise answers are marked down significantly.

Exam DON'Ts

- Don't buy a new calculator on the morning of the exam. You need to be fully familiar with all the common operations on your calculator.
- Don't be disorganised. If your teacher tells you that your work is hard to follow, take it that the examiner, who doesn't know you at all, will find your work even harder to follow. This may be fine if your work is correct, but if there are errors, the examiner is not obliged to try to read your mind.
- Don't spend time trying to guess answers. Even if you get lucky every now and then, the risk is too great that you are just wasting time.
- Don't daydream, or become worried about how you are doing. There will be time for both after the exam. Stay focused.
- Don't spend too much time on a difficult part: only a few marks are likely to be allocated to it.
- Don't forget to bring your calculator, pens, maths instruments, and perhaps some sweets or chocolate into the exam. By the end, you could probably do with the energy boost.
- Don't perform any difficult calculations in your head. Use your calculator.
- Don't leave early. Where are you going? If you stay you might be able to pick up a few marks that could make all the difference.
- Don't stay up late the night before (or worse still, do an all-nighter) trying to learn off things last-minute. You will be in no state to do yourself justice the following morning.
- Don't be afraid to state your opinion in questions that require written explanations. If you write down anything reasonable, you will probably score very well.

How I got my A1

Saoirse Sheehy, Maths the Institute of Education

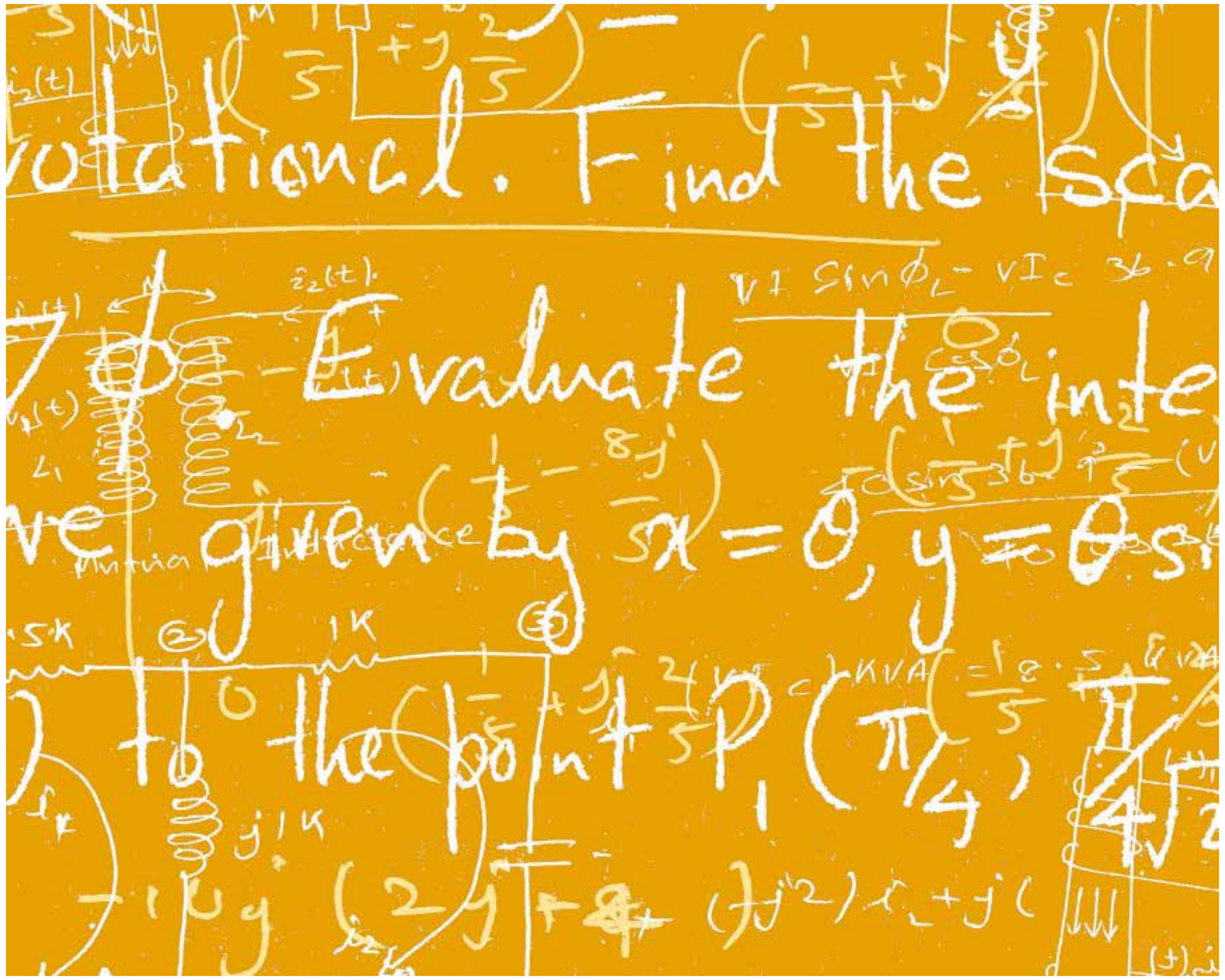
I worked hard in fifth year making sure I understood the ideas and basic principles when first taught. This made revising much simpler. In sixth year, I would advise focusing on what you are currently learning. If you can revise other topics at the same time then do so, but don't sacrifice what you're doing in class to revise on your own time. Having the ground work done took a lot of the pressure off. It is a good deal easier reminding yourself how to do a question than reteaching yourself.

Being dyslexic means that I don't find last-minute cramming very effective. Regular testing is the key. I used to get very stressed around tests. But with constant quizzes at the Institute I learned not to be. If your teacher doesn't have the time to test you then I would recommend testing yourself on topics you have just covered. Study before the test, do it in a quiet environment, time yourself and don't allow yourself to look at your notes.

Right before the actual exam is a terrifying time. The course is huge and impossible to cover in last-minute revision. I had to trust to the work I'd been doing throughout the year and pick out the parts of the course I had most difficulty with. I got help from friends at different times during the year as well. I think we all found it helpful. You get a much better understanding of the material when you go through the process of teaching it to someone, and I was able to get one-on-one tuition with a peer.

In the exam I tried everything. There were questions that I wasn't sure of what they were asking or what they wanted me to put down. For those I quickly attempted them, trying whatever relevant sums I could think of and then moved on. If I had time at the end of the exam I came back to them, but because I didn't know how the paper was going to be marked there was no sense in trying any range of methods. Time management is important in all tests but particularly here – you don't want to find out you spent 15 minutes on a question that was marked down to three marks.

Good bets and possible topics



Paper 1

This paper will consist of two sections, one on 'Concepts and Skills' and the other on 'Contexts and Applications'

This year's Paper 1 will be the second full Paper 1 on the new Project Maths course. So we don't have much by way of past papers to guide us in our preparations.

Instead, what we have is the official direction to make the papers as 'unpredictable as possible'. This will apply both to the choice of topic, or topics, within each question, and to the very nature of these questions.

Paper 1 will consist of two sections, each worth 150 marks. Section A, called 'Concepts and Skills' will contain six questions, each of value 25 marks. Section B, called 'Contexts and Applications', will contain between two and four questions. The number of marks for each question will not be known until the day of the exam.

Although we cannot be certain of the topics covered by the questions in Section A, it is a good bet that there will be at least one question each on complex numbers, algebra, sequences and series and calculus. Other possible topics are the theory of functions and their limits, proof by induction, transformations of graphs and exponential and log functions. For the most part, the questions in Section A will be of a mathematical nature. You can also expect one question to be a 'concept' question, i.e. a question requiring little or no calculation, but one which will require the student to demonstrate an understanding of some concept on the course.

By now, students have become familiar with the really unpredictable nature of the questions in Section B. However, because of the major topics covered by Paper 1, it is highly likely that there will be a major question (perhaps two) calling on students to use their knowledge of algebra and calculus together and in alternate parts.

It is also likely that another question will cover either exponential growth/decay or financial maths. To the surprise of many, there was no question on financial maths last year. This may be rectified this year.

Number Systems

It is necessary to clearly understand the ideas of factors, prime numbers, prime factors, HCF and LCM of natural numbers. These concepts are also important when dealing with rational fractions, e.g. when adding two fractions with different denominators. An understanding of these ideas can then be called on to fully understand how algebraic expressions are dealt with.

Another key idea is the distinction between rational and irrational numbers. A concept question, such as classifying some numbers, is quite possible on this distinction.

From the higher level section of the syllabus, we also have to geometrically construct $\sqrt{2}$ and $\sqrt{3}$. We also have to be able to prove that $\sqrt{2}$ is irrational. This is a difficult proof, and you should go through it thoroughly.

1. **HCF**
   e.g. a, b and c are natural numbers. If $\text{HCF}(a,b)$ stands for the highest common factor of a and b, and
   $\text{HCF}(a,b)=12$
   $\text{HCF}(a,c)=16$
   what is the least possible value of
   $\text{HCF}(b,c)$?
   Explain your reasoning.
2. **LCM**
   e.g. if the lowest common multiple of 5, 8 and k is 120, list the possible values of k.
3. **Constructing roots**
   e.g. given a line segment of length 1 unit, show how to construct a line segment of length $\sqrt{2}$, using only a straight edge and a compass.
4. **Scientific notation**
   e.g. An electronics company invests $2\cdot 5$ billion euro in research and development in 2014. It intends to increase this by 10% each year after that. Write in scientific notation the amount it intends to invest in research and development between 2014 and 2016 inclusive.

Algebra

Algebra is the most important and widely used of the mathematical tools that we use. Algebra is also the language of maths, and a weakness with basic algebraic techniques, such as being able to deal with expressions and equations, will severely hamper your ability to tackle questions in many other areas of maths.

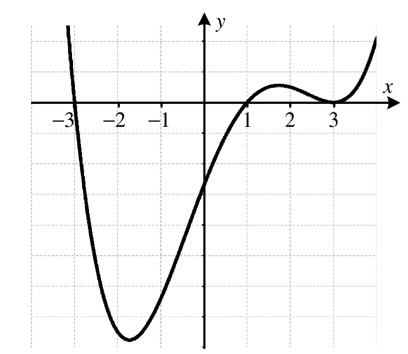
For this reason, algebra will be examined directly and indirectly in a number of questions on Paper 1.

To begin with, the theory and mechanics of algebra are likely to be examined in one or more questions in Section A. Thus we can expect to see polynomials, algebraic fractions, surds, modulus, exponential and log functions figuring prominently. You should be familiar with all the methods used to solve the types of equation on our course, including, but not limited to factors, the Factor Theorem and by means of graphs.

As stated above, algebra is likely to figure prominently in Section B, whether alone or more likely combined with other topics such as differentiation and integration. It is important that you practise recognising what aspects of algebra are required in different cases.

Below is a list of the topics that you should study in algebra.

1. **Binomial Theorem**
   e.g. use the Binomial Theorem to expand fully
   $$(3x-2y)^5$$
2. **Fractions**
   e.g. write as a single fraction in its simplest form
   $$\frac{x-1-\frac{6}{x}}{x-\frac{4}{x}}$$
3. **Surds**
   e.g. if $x=\sqrt{a}+\frac{1}{\sqrt{a}}$ and $y=\sqrt{a}-\frac{1}{\sqrt{a}}$
   where $a\in\mathbb{R}$, $a>0$, evaluate
   $$x^2-y^2$$
4. **Making and manipulating formulae**
   e.g. The hot tap of a bath can fill the bath in h minutes, when turned fully on. The cold tap can fill the bath in c minutes when turned fully on. If t is the number of minutes it takes to fill the bath when both taps are fully on, express t in terms of h and c.
5. **Linear simultaneous equations in two variables**
   e.g. solve the simultaneous equations
   $$3x+2y=9$$
   $$2x-y=-1$$
   (i) by algebra
   (ii) by drawing a graph
6. **Linear simultaneous equations in three variables**
   e.g. A father's age (in years) is three times the sum of the ages (in years) of his son and his daughter. The sum of the father's and the son's ages is nine times the daughter's age. The sum of all three ages is 40. Find the age of each person.
7. **Solving quadratic equations**
   e.g. solve the equation
   $$28=x(31+5x)$$
   (i) by factors,
   (ii) by completing the square,
   (iii) by the quadratic formula
8. **Quadratic graphs**
   e.g. Construct a graph of the function
   $$f:x\to 2x^2-6x+11$$
   by using the complete square form to find the co-ordinates of the turning point.
   Use your graph to estimate the values of x for which $f(x)=10$.
9. **Nature of quadratic roots**
   e.g. Show that the roots of the equation
   $$px^2-(p+q)x+q=0$$
   are real for all $p,q\in\mathbb{R}$. Express these roots in terms of p and q.
10. **Linear, non-linear simultaneous equations**
    e.g. solve the simultaneous equations
    $$3x-y=1$$
    $$x^2+4xy=9$$
11. **Rational equations**
    e.g. Solve the equation
    $$\frac{1}{x-2}+\frac{4}{x+1}=2$$
12. **Irrational equations**
    e.g. solve
    $$\sqrt{2x+1}-\sqrt{x-3}=2,$$
    for $x\in\mathbb{R}$
13. **Identities**
    e.g. if
    $$(x+a)^2-(x+b)^2=12x+12$$
    for all $x\in\mathbb{R}$, find the values of the constants a and b
14. **Use of the Factor Theorem to factorise cubics and solve cubic equations**
    e.g. if $x-1$ and $x-2$ are factors of
    $$f:x\to x^3+ax^2-7x+b,$$
    find the values of the constants a and b, and find the three solutions of the equation $f(x)=0$
15. **Quadratic factor of a cubic**
    e.g. x^2-px+1 is a factor of ax^3+bx+c, where $a\neq 0$. Show that $c^2=a(a-b)$.
16. **Graphing polynomial curves**
    e.g. The graph of the polynomial
    $$y=f(x)$$
    of degree 4 is shown below.



    (i) Find an expression for the polynomial $f(x)$.
    (ii) If the curve contains the point $(0,-54)$, find the equation of the curve $y=f(x)$.
17. **Modulus inequalities**
    e.g. solve $|2x+5|<3$
18. **Abstract inequalities**
    e.g. if $x,y\in\mathbb{R}$, prove that
    $$x^2+y^2\geq\frac{1}{2}(x+y)^2$$
19. **Rational inequalities**
    e.g. solve
    $$\frac{5-x}{x-2}<1,\qquad x\in\mathbb{R},\ x\neq 2$$
20. **Logs and log equations**
    e.g. solve the equation
    $$\log_5 x=1+\log_2\left(\frac{3}{2x-1}\right),$$
    for $x>\frac{1}{2}$
21. **Equations with the unknown in the index**
    e.g. solve the equation
    $$2^{2x+1}-17(2^x)+8=0$$
    for $x\in\mathbb{R}$.
22. **Exponential and log functions**
    e.g. Each month 2% of the quantity of a radioactive substance present decays. The amount of radioactive substance present at the start is P. Let $f(n)$ be the quantity of the radioactive present n months after the start.
    (i) Express $f(n)$ as an exponential function of n.
    (ii) Show that $\frac{f(n+12)}{f(n)}$ is independent of n, and find its value, correct to four decimal places. Give an interpretation of this result.
    (iii) Find the quantity of radioactive substance present 80 months after the start.

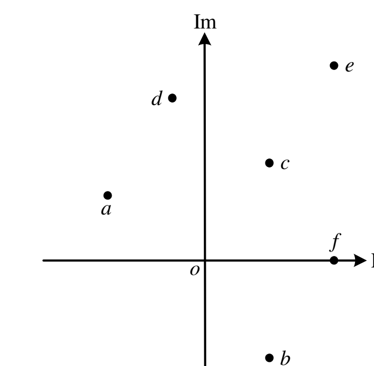
Complex Numbers

The set of complex numbers is formed when imaginary numbers and combinations of real and imaginary numbers are combined with the set of real numbers. You will need to know how to perform the standard operations on complex numbers, such as addition, subtraction, multiplication and division. You will also have to represent complex numbers on an Argand diagram, and deal with the idea of modulus.

The link between operations on complex numbers and their effects on points on an Argand diagram is extremely important. You should also realise that elements of co-ordinate geometry and trigonometry may be required to answer questions on the Argand diagram, as happened in 2014.

Other topics you need to revise are the use of algebra to solve complex equations and the polar form of a complex number. In particular, the polar form of a complex number, and the associated De Moivre's Theorem, have many applications, such as dealing with powers and roots.

1. **Equality of complex numbers**
   e.g. find the complex number $z=x+yi$ if
   $$5z+2i\bar{z}=11-4i,$$
   where $\bar{z}$ is the conjugate of z
2. **Addition, subtraction and multiplication**
   e.g. if $z=5-3i$ and $w=-2+4i$, express in the form $a+ib$:
   $$3w(2\bar{z}-z)$$
3. **Conjugate and division**
   e.g. express $\frac{30+i}{3+5i}$ in the form $a+bi$
4. **Square roots**
   e.g. find the real numbers a and b if
   $$(a+bi)^2=3-4i$$
5. **Argand diagram and modulus**
   e.g. $z=3+4i$ and $w=1-2i$. Plot $z+w$ on an Argand diagram and investigate if
   $$|z+w|=|z|+|w|$$
6. **Interpreting an Argand diagram**
   e.g. The Argand diagram below shows six points called a, b, c, d, e and f. These points represent the complex numbers,
   $$z,\ 2z,\ \bar{z},\ z+\bar{z},\ iz,\ (1+i)z$$
   but not necessarily in that order.



   (i) Identify which point represents each of the complex numbers listed above.
   (ii) If o is the origin, determine the acute angle formed by z, o and $(1+i)z$.
7. **Complex equations**
   e.g. if $2+i$ is a root of the equation
   $$z^2-(a-i)z+(8+bi)=0,$$
   find the values of $a,b\in\mathbb{R}$, and find the other root of this equation
8. **Conjugate Roots Theorem**
   e.g. if $3+2i$ is a root of the equation
   $$z^3+az^2+bz-52=0,\quad a,b\in\mathbb{R},$$
   find the values of a and b and the other roots of the equation
9. **Polar Form**
   e.g. express $-2\sqrt{3}+2i$ in the form
   $$r(\cos\alpha+i\sin\alpha)$$
10. **De Moivre: Trigonometric identities**
    e.g. use De Moivre's Theorem to express $\sin 3\theta$ as a polynomial in $\sin\theta$
11. **De Moivre: Large powers**
    e.g. express $\left(1-\sqrt{3}i\right)^8$ in the form $a+bi$
12. **De Moivre: Roots**
    e.g. express the solutions of the equation
    $$z^3=-64$$
    in the form $a+bi$. Show these solutions on an Argand diagram.

Exam Times

THE IRISH TIMES in association with THE INSTITUTE OF EDUCATION

Maths Higher Level

Proof by Induction

Proof by induction is a method of proof that is required for a number of different types of statement, many of which will allow no other method of proof. The method is complicated, and is generally one we will try to avoid unless no other method suggests itself. Obviously, if we are asked to use this method, we will have no choice.

Like any other method of proof, there is a number of steps that must be completed before we can say that the statement or theorem is proved. You should make sure that you learn these steps precisely.

On our course, we are required to be able to prove three types of statement by induction. These are outlined below.

1. **Formula for the sum of a series**
   e.g. prove by induction that
   $$1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$$
2. **Divisibility proofs**
   e.g. prove by induction that
   $5^n-4n+15$
   is divisible by 16, for all $n\in\mathbb{N}$
3. **Inequality proofs**
   e.g. prove by induction that
   $2^n>2n+1$,
   for all $n\in\mathbb{N}, n\geq 3$.

Arithmetic & Money

The topic of arithmetic is mostly a repeat of what you studied at Junior Cert level. As such, you can revise it in a reasonably short period.

However, it should still not be underestimated, as it could make a crucial difference in a practical question in Section B.

1. **Percentage error**
   e.g. The number of spectators at a match is estimated to be 6500. The accurate number is later found to be 6389. Calculate the percentage error in the estimate.
2. **Accumulation of error**
   e.g. If $p=(8\cdot 9\pm 0\cdot 25)$ km and $q=(7\cdot 3\pm 0\cdot 2)$ km, calculate the percentage error in
   (i) $p+q$, (ii) $p-q$.
3. **Tolerance and tolerance intervals**
   e.g. A ruler is scaled with marks showing every two millimetres.
   (i) What is the tolerance of this ruler?
   (ii) This ruler measures a length to be $12\cdot 8$ cm. What is the tolerance interval for the length?
4. **Order of magnitude**
   e.g. Obtain an order of magnitude estimate for the number of heart-beats in a human life. (Average number of heartbeats per minute is about 80, average lifespan is about 80 years.)
5. **Household finances**
   e.g. The present reading on the electricity meter in John's house is 63792 units. The previous reading was 62942 units.
   (i) How many units of electricity were used since the previous reading?
   (ii) What is the cost of the electricity used, if electricity costs $14\cdot 1$ cent per unit?
   (iii) There is also a daily standing charge of €$0\cdot 252$ and a PSO levy of €$7\cdot 56$. John's bill covers a period of 62 days. If VAT is charged on all the previous amounts and John's bill comes to €$162\cdot 34$, what is the rate of VAT?
6. **Income Tax**
   e.g. Barry has a gross income of €72000 and an income after tax of €53295 for a certain year. The standard rate cut-off point is €36500 and the standard and higher rates of tax are 20% and 41% respectively. Determine his tax credits for the year.

Sequences & Series

When we are presented with a series of patterns or progressions, we have to be able to identify the pattern or connection and be able to develop it to a formula, or a rule for the general term. Once we have this, we can obtain any term, or indeed find a term that satisfies a given condition.

It is also important to understand the concept of the limit of a sequence and how to calculate this limit, when it exists.

We are required to study in detail two specific types of sequence and series: arithmetic and geometric. We also need to look at practical examples of each.

Geometric sequences and series, in particular, have many real life applications. Many of these are in the area of financial maths, but there are others, e.g. recurring decimals.

1. **Sequence notation**
   e.g. For a sequence, $u_n=3\times 2^{n-1}$. Find the first three terms of this sequence and find the term of the sequence which has a value of 98304.
2. **Limits of sequences**
   e.g. calculate $\lim\limits_{n\to\infty}\frac{2(3^n)-3(2^n)}{5(3^n)+6}$
3. **Arithmetic sequences and series**
   e.g. The sum of the first twenty terms of an arithmetic is 45, and the sum of the first forty terms is 290. Find the first term and the common difference.
4. **Geometric sequences and series**
   e.g. The sum of the first seven terms of a geometric series is 7 and the sum of the next seven terms is 896. Find the common ratio.
5. **Infinite geometric series**
   e.g. express in terms of x the sum of the infinite geometric series
   $$x+\frac{x}{1+x}+\frac{x}{(1+x)^2}+\ldots$$
6. **Applications of infinite geometric series**
   e.g. by forming an infinite geometric series, write $3\cdot\dot{2}\dot{8}=3\cdot 282828......$ in the form $\frac{p}{q}$, where $p,q\in\mathbb{N}$.

Financial Maths

The topic of financial maths deals with the concept of compound interest and depreciation. Then it develops to the idea of present value and its use for tackling questions on loans, mortgages, regular savings, annuities and pensions.

After the no-show last year, much to everyone's surprise, it is highly likely that a question on financial maths will figure in Section A or, more likely, in Section B. If such a question does occur in Section B, it is probable that it will contain parts looking for an opinion or assessment based on figures obtained.

1. **Compound interest**
   e.g. €8200 is invested at 6% per annum compound interest. What is the value of the investment after seven years? How much interest was gained in this time?
2. **Depreciation**
   e.g. A new car costs €25000. It depreciates at the rate of 20% in each of its first two years. Then a newer model becomes available and the car depreciates at the rate of 30% each year after that. Find the amount of the depreciation in the fourth year.
3. **Present value**
   e.g. If the effective annual rate of interest is $4\cdot 25\%$, find
   (i) the value in ten years time of €12500 due in two years time,
   (ii) the value two years ago of €6500 due in three and a half years time,
   (iii) the value in two years time of €6000 due in sixty six months time.
4. **Interest rates other than annual**
   e.g. Donal has inherited a substantial sum of money which he wants to invest. He researches the interest rates offered by different institutions. A offers a rate of $5\cdot 5\%$ per annum, B offers a rate of $0\cdot 45\%$ per month and C offers an annual rate of 5%, compounded monthly. If Donal wants to maximise his income, which of the three institutions should he invest his money with?
5. **Amortised loans, including mortgages**
   e.g. Emily borrows €80000 and agrees to repay the loan by a series of 6 equal annual repayments starting in one year's time. The APR for the loan is 7%.
   (i) Calculate the amount of each equal annual repayment.
   (ii) Construct a schedule showing interest and principal portions of the repayments outlined in part (i).
6. **Investments, annuities and bonds**
   e.g. Dean wants to save for his retirement, and so needs to calculate the size of the fund he will need to purchase an annuity, starting on the date of his retirement and lasting for 20 years. Take the AER on the date of his retirement to be $4\cdot 5\%$.
   (i) Find the value of the fund, on the date of his retirement, required to buy an annual payment of €20000 per year (20 payments).
   (ii) Find the value of the fund, on the date of his retirement, required to buy an annual payment which starts at €20000 and increases by 3% per year (20 payments).

Students at the Institute of Education, Lower Leeson Street, Dublin 2. PHOTOGRAPH: ALAN BETSON

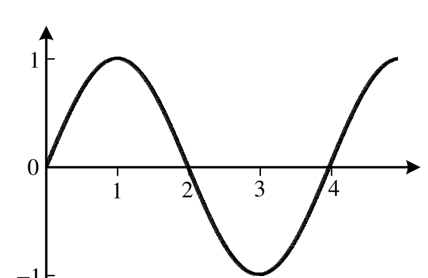
Functions

Although you will have been using the term 'function' for some time, it is only at this level that we get to properly examine what exactly it means to be a function. Understanding exactly what a function is, and being able to use function notation, is very important for a proper appreciation of many other areas of maths.

One of the key ideas in functions is that of the inverse of a function, especially when the inverse is itself a function. For the inverse to be a function, we require the original function to be bijective. This is why we study bijective functions.

As well as the theory, we need to be able to get the rule of an inverse function, and to find the rule for a composite function.

1. **Definition of a function**
   e.g. The function $g:X\to Y:x\to\frac{3}{2x-1}$ is defined on the set $X=\{1,2,3,4,5\}$.
   List the elements that Y must contain.
2. **Types of functions**
   e.g. The curve shown below represents part of a function $f:\mathbb{R}\to\mathbb{R}$. The shape of the curve continues as shown in both direction.



   (i) Use the diagram to explain if f is injective.
   (ii) State the range of the function, and use it to explain if f is surjective.
   (iii) If we restrict the domain to $[1,3]$ and the codomain to $[-1,1]$, explain why the curve now represents a bijective function.
3. **Composite functions**
   e.g. $f:x\to 2x^2-1$ and $g:x\to 4x-3$ are two functions defined for all $x\in\mathbb{R}$.
   (i) Investigate if $g\circ f(x)=f\circ g(x)$, for all $x\in\mathbb{R}$.
   (ii) Express $g^2(x)$ in terms of x.
4. **Inverse functions**
   e.g. f is the function $f:x\to\frac{3x+1}{x-4}$.
   (i) Find $f^{-1}(x)$.
   (ii) Verify that $f^{-1}\circ f(x)=x$.

Differentiation

Differentiation is one of the most important areas of maths, especially at third level. On our course, differentiation can be broken down into three areas.

1. Concepts and Proofs
   First up, we start with limits and continuity. Limits, in particular, are then used to introduce the concept of differentiation as the slope or the instantaneous rate of change, and look at the first principles proofs.
2. Methods of Differentiation
   Next, we consider the mechanics of differentiation, which are the rules and methods used to obtain derivatives as quickly and efficiently as possible.
3. Applications of Differentiation
   After learning how to obtain derivatives, the next step is to consider the many and varied applications of differentiation that are on our course. These include finding turning points, helping with curve sketching, maximising and minimising quantities, and dealing with rates of change.

1. **Limits and continuity**
   e.g. The function $f:\mathbb{R}\to\mathbb{R}$ is defined by
   $$f:x\to\begin{cases}\frac{x^2-9}{x-3}, & \text{for } x<3\\ 4, & \text{for } x=3\\ \frac{3x-3}{x-2}, & \text{for } x>3\end{cases}$$
   (i) What is $f(3)$?
   (ii) Determine if $\lim\limits_{x\to 3}f(x)$ exists.
   (iii) Determine if f is continuous at $x=3$.
2. **Theory of differentiation**
   e.g. The function f is defined for all $x\in\mathbb{R}$ by $f:x\to x^2-4x+5$.
   (i) Find, from first principles, the derivative of $y=f(x)$.
   (ii) The derivative is sometimes described as the 'slope function'. Explain what this means in the light of this derivative.
   (iii) Find the instantaneous rate of change of f at $x=3$.
   (iv) Find the average rate of change of f over the interval from $x=3$ to $x=4$.
3. **Differentiation by rule**
   e.g. find $\frac{dy}{dx}$ if
   (i) $y=\tan^{-1}\left(\frac{3}{x^2}\right)$
   (ii) $y=e^{1-2\sin x}(1+2\cos x)$
   (iii) $y=\ln\left(x\sqrt{x^2+2}\right)$
4. **Curve sketching**
   e.g. The equation of a curve is
   $y=-x^2(x-2)^2$.
   (i) Show that the curve has a local minimum at the point $(1,-1)$.
   (ii) Find the co-ordinates of the two local minimum points.
   (iii) Draw a rough sketch of the curve, for $-1\leq x\leq 3$.
5. **Maximum and minimum problems**
   e.g. A charity organisation is planning a fund-raising campaign in a major city having a population of 2 million. The proportion, *P*, of the population who will make a donation is estimated by the function
   $P=1-e^{-0\cdot 02x}$
   where x is the number of days for which the campaign is conducted. Past experience indicates that the average contribution per donor is €2. The cost of the campaign is estimated to be €10000 per day.
   (i) By calculating the total contribution minus the total costs, find an expression for $N(x)$, the net proceeds after x days.
   (ii) For how many days should the campaign be conducted to maximise the net proceeds?
   (iii) Determine the maximum net proceeds.
6. **Rates of change**
   e.g. A spherical snowball rolling down a hill is increasing in volume at the constant rate of $3\text{ cm}^3\text{s}^{-1}$.
   (i) Find the rate of change of the radius, r, in terms of r.
   (ii) Find the rate of change of the surface area of the snowball when its radius is 12 cm.

Integration

Integration is initially seen as the inverse operation to differentiation, but has many applications in its own right, a few of which we see on our course.

We start by learning the methods of integration, using the standard integrals and rules on our course. You should practise these methods, paying attention to being careful with notation.

After that, the main topic for study is the use of integration to calculate areas and find the average value of a function over an interval. Of these, calculating areas under curves, and between curves, is the more important and more difficult. That studying how to calculate complicated areas is essential was borne out by Question 8 on the 2014 Paper 1.

1. **Indefinite integrals**
   e.g. find
   (i) $\int\left(x+\frac{1}{x}\right)^2dx$
   (ii) $\int\cos 6x\,dx$
   (iii) $\int e^{4x+1}dx$
2. **Definite integrals**
   e.g. evaluate
   (i) $\int_0^2\left(5x^2+3^x\right)dx$
   (ii) $\int_0^{\frac{\pi}{4}}\sin 4x\,dx$
3. **Integration as the inverse of differentiation**
   e.g. The derivative of the function $f(x)$ is
   $f'(x)=x^2+\sin x$.
   If $f(0)=3$, express $f(x)$ in terms of x.
4. **Area by integration**
   e.g. find the area of the region bounded by the curve $y=x^2-6$ and the line $y=x$
5. **Average value of a function**
   e.g. A particle is moving in a straight line such that after t seconds its velocity, $v\text{ ms}^{-1}$, is given by
   $v=6t+12t^2$.
   Find the average velocity of the particle during the first two seconds of its motion.

Paper 1: Sample Questions

1. Algebra

Question
A bacteria colony grows at an exponential rate. $Q(t)$ represents the size, in grams, of the colony t hours after the initial measurement. $Q(t)$ can be written as follows.
$Q(t)=Ce^{bt}$,
where b and C are constants. The initial amount present is 1000 g, i.e. when $t=0$, $Q=1000$.
Also $Q(3)=1191$, to the nearest gram.

(i) Find the values of the constants b and C. Give the value of b correct to four decimal places.
(ii) Find, correct to the nearest gram, the size of the colony after eight hours.
(iii) Find the number of hours it takes the colony to double in size. Give your answer correct to the nearest hour.
(iv) If we can also write
$Q(t)=Ca^t$
find the value of the constant a, correct to two decimal places.
(v) Show that
$$\frac{Q(t+24)}{Q(t)}$$
is independent of t, and find its value, correct to two decimal places.
Give an interpretation of this result.
(vi) Use the result of part **(v)** to estimate the size of the colony after a week after the initial amount.

Contined on page 10

Exam Times

Solution

(i) $Q(0) = 1000$:

$1000 = Ce^{0}$

$C = 1000$

$Q(3) = 1191$:

$1191 = 1000e^{b(3)}$

$1\cdot191 = e^{3b}$

$3b = \log_e 1\cdot191$

$3b = 0\cdot1747932904$

$b = 0\cdot0583$

(ii) $Q(t) = 1000e^{0\cdot0583t}$

$Q(8) = 1000e^{0\cdot0583(8)}$

$Q(8) = 1594$,

correct to the nearest gram.

(iii) $Q(t) = 2000$

$1000e^{0\cdot0583t} = 2000$

$e^{0\cdot0583t} = 2$

$\log_e 2 = 0\cdot0583t$

$t = \dfrac{\ln 2}{0\cdot0583}$

$t = 11\cdot8893$

$t = 12$,

correct to the nearest hour.

(iv) $Q(t) = Ca^{t}$

$Ca^{t} = Ce^{0\cdot0583t}$

$a^{t} = (e^{0\cdot0583})^{t}$

$a = e^{0\cdot0583} = 1\cdot06$

(v) $\dfrac{Q(t+24)}{Q(t)} = \dfrac{1000e^{0\cdot0583(t+24)}}{1000e^{0\cdot0583t}}$

$= \dfrac{e^{0\cdot0583t}.e^{1\cdot3992}}{e^{0\cdot0583t}}$

$= e^{1\cdot3992}$

$= 4\cdot0519$

$= 4\cdot05$

which is independent of t.

Thus

$Q(t+24) = 4\cdot05Q(t)$

which means that as there are 24 hours in a day, the size of the colony increases by a factor of approximately 4 over the period of one day.

(vi) By part **(v)**, as the size of the colony is increasing by a factor of $4\cdot05$ each day, its size at the end of a week is

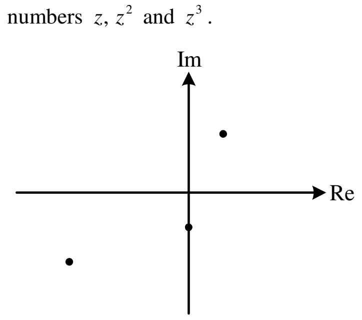
$1000(4\cdot05)^{7} = 17872494$,

i.e. nearly 18 million.

2. Complex Numbers

Question

(a) The modulus of z is less than 1. The Argand diagram below shows the complex numbers z, z^{2} and z^{3}.



(i) Copy the diagram and label each of the points shown.

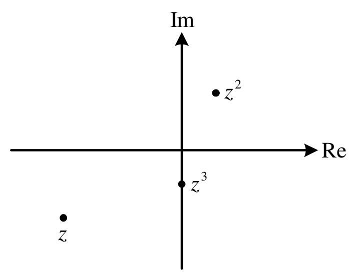
(ii) Find θ, the argument of z.

(iii) If $\left|z^{3}\right| = \frac{1}{4}\left|z\right|$, write z in the form $r(\cos\theta + i\sin\theta)$.

(b) Express $\left(\dfrac{1-3\sqrt{3}i}{2+\sqrt{3}i}\right)^{4}$ in the form $a+bi$.

Solution

(a) **(i)** The points are labelled below. (As the modulus of z is less than 1, each successive power of z is nearer to the origin.



(ii) Let $z = r\,\text{cis}\,\theta$, where from the diagram, $-180° < \theta < -90°$.

Then

$z^{3} = \left(r\,\text{cis}\,\theta\right)^{3}$

$= r^{3}\,\text{cis}\,3\theta$

From the diagram, the angle of z^{3} is $-90°$, or more generally, $-90° + n(360°)$, for $n \in \mathbb{Z}$, from trigonometry. Thus

$3\theta = -90° + n(360°)$

$\theta = -30° + n(120°)$

For $-180° < \theta < -90°$, we put $n = -1$.

Thus

$\theta = -30° - 120° = -150°$

(iii) We are given that

$r^{3} = \frac{1}{4}r$

$r^{2} = \frac{1}{4}$

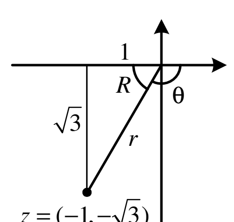
$r = \frac{1}{2}$, as $r > 0$.

(b) $\dfrac{1-3\sqrt{3}i}{2+\sqrt{3}i} = \dfrac{1-3\sqrt{3}i}{2+\sqrt{3}i} \times \dfrac{2-\sqrt{3}i}{2-\sqrt{3}i}$

$= \dfrac{2-\sqrt{3}i-6\sqrt{3}i-9}{2^{2}+(\sqrt{3})^{2}}$

$= \dfrac{-7-7\sqrt{3}i}{7}$

$= -1-\sqrt{3}i = (-1,-\sqrt{3})$



$r = \sqrt{1^{2}+(\sqrt{3})^{2}} = 2$

$\tan R = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$R = 60°$

$\theta = -120°$

$-1-\sqrt{3}i = 2\left(\cos(-120°) + i\sin(-120°)\right)$

$= 2\,\text{cis}(-120°)$

$\left(\dfrac{1-3\sqrt{3}i}{2+\sqrt{3}i}\right)^{4} = \left(2\,\text{cis}(-120°)\right)^{4}$

$= 16\,\text{cis}(-480°)$

$= 16(\cos 480° - i\sin 480°)$

$= 16\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right)$

$= -8-8\sqrt{3}i.$

3. Financial Maths

Question

Gavin is due to retire in exactly 40 years. He decides to save for a pension fund on the day he retires.

He saves €200 at the beginning of each month, starting today. He makes his last saving one month before he is due to retire. Take the effective annual rate of interest to be 8%.

(i) Find the total value of Gavin's savings on his retirement date.

Gavin plans to use his savings to buy an annuity for a guaranteed 25 years, starting on the date of his retirement. He wants the amount he receives to rise each year by 5%, to take account of inflation. Take the effective annual rate of interest after Gavin's retirement to be 6%.

(ii) Find the value of the first pension payment Gavin receives on the date of his retirement.

Solution

(i) **Method 1:** Evaluation date is date of retirement

Gavin makes $40 \times 12 = 480$ savings.

The effective monthly rate of interest is

$i = (1\cdot08)^{\frac{1}{12}} - 1 = 0\cdot00643403011$

Present Value of all his savings.

1st: accumulate for 480 months

PV $= 200(1+i)^{480}$

2nd: accumulate for 479 months

PV $= 200(1+i)^{479}$

... ...

480th: accumulate for 1 month

PV $= 200(1+i)$

Total PV of all savings

$= 200(1+i)^{480} + 200(1+i)^{479} + \ldots + 200(1+i)$

$= 200(1+i) + \ldots + 200(1+i)^{479} + 200(1+i)^{480}$

$= \dfrac{200(1+i)\left(1-(1+i)^{480}\right)}{1-(1+i)}$

... geometric series,

$a = 200(1+i)$

$r = 1+i,\ n = 480$

$= 648360\cdot77$

Thus the value of Gavin's retirement fund on his retirement date is €648360·77.

Method 2: Amortisation formula

We can start by pretending that Gavin's savings are paying off a loan of P, given one month before his first saving. We can use the amortisation formula to find P.

$A = 200,\ P = ?,$

$i = 0\cdot00643403011,\ t = 480$

Then

$200 = P\dfrac{i(1+i)^{480}}{(1+i)^{480}-1}$

$200 = P(0\cdot006744485052)$

$P = 29653\cdot8577$

Now, to find the value of the fund on Gavin's retirement date, we need to accumulate this forward by 40 years and 1 month, i.e. 481 months.

PV $= 29653\cdot8577(1+i)^{481}$

$= 648360\cdot77$,

as before.

(ii) According to his plans, Gavin will receive:

Retirement date:	
A	1st payment
1 year later:	
$A(1\cdot05)$	2nd payment
2 years later:	
$A(1\cdot05)^{2}$	3rd payment
...	...
24 years later:	
$A(1\cdot05)^{24}$	25th payment

Take the evaluation date to be the date of his retirement.

1st payment: on date

PV $= A$

2nd payment: discount 1 year

PV $= \dfrac{A(1\cdot05)}{1\cdot06} = A(0\cdot99056)$

3rd payment: discount 2 years

PV $= \dfrac{A(1\cdot05)^{2}}{(1\cdot06)^{2}} = A(0\cdot99056)^{2}$

... ...

25th payment: discount 24 years

PV $= \dfrac{A(1\cdot05)^{24}}{(1\cdot06)^{24}} = A(0\cdot99056)^{24}$

Total PV of Gavin's pension payments

$= A + A(0\cdot99056) + A(0\cdot99056)^{2} + \ldots + A(0\cdot99056)^{24}$

$= \dfrac{A(1-(0\cdot99056)^{25})}{1-0\cdot99056}$

... geometric series,

$a = A,\ i = 0\cdot99056,\ n = 25$

$= A(22\cdot36429726)$

Equation of Value:

PV of savings fund = PV of pension

$648360\cdot77 = 22\cdot36429726A$

$A = 28990\cdot89$

Thus Gavin's first pension payment will be €28990·89.

■ **Students at the Institute of Education, Lower Leeson Street, Dublin 2.**
PHOTOGRAPH: ALAN BETSON

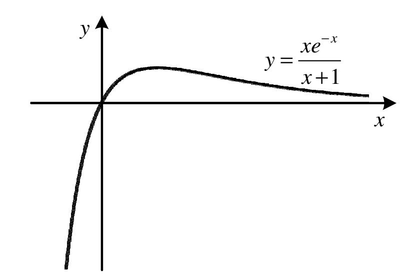
4. Differentiation

Question

(a) Find the stationary point of the curve $y = \log_e(x^{2}+1)$ and determine the nature of this point.

(b) The diagram below shows a graph of part of the curve

$y = f(x) = \dfrac{xe^{-x}}{x+1}$.



Copy this graph and indicate underneath a sketch of the slope function of this curve, i.e. sketch

$\dfrac{dy}{dx} = f'(x)$.

Solution

(a) $y = \log_e(x^{2}+1)$

$\dfrac{dy}{dx} = \dfrac{1}{x^{2}+1}.(2x) = \dfrac{2x}{x^{2}+1}$

Put $\dfrac{dy}{dx} = 0$: $\dfrac{2x}{x^{2}+1} = 0$

$2x = 0$

$x = 0$

$x = 0$: $y = \log_e 1 = 0$

Then $(0,0)$ is a stationary point.

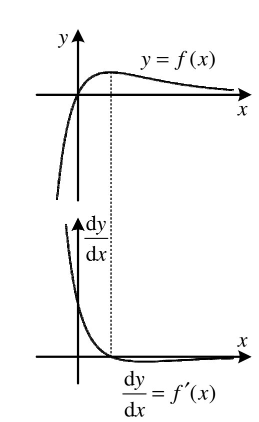
$\dfrac{d^{2}y}{dx^{2}} = \dfrac{(x^{2}+1)(2)-(2x)(2x)}{(x^{2}+1)^{2}}$

$= \dfrac{-2x^{2}+2}{(x^{2}+1)^{2}}$

$x = 0$: $\dfrac{d^{2}y}{dx^{2}} = \dfrac{2}{1} = 2 > 0$

Thus $(0,0)$ is a local minimum.

(b) The graph and its derivative are shown below.



Some changes in store

Paper 2

A couple of changes have been made to Paper 2 since last year, including the conversion of geometry to the new course

■ **Studying at the Institute of Education.** PHOTOGRAPH: ALAN BETSON

As with Paper 1, Paper 2 will be made of two sections, Section A and Section B, which will each be worth 150 marks. Paper 2 covers the material in the first two strands of the syllabus, and areas and volumes from Strand 3. Strand 1 is concerned with probability and statistics, while Strand 2 involves geometry, trigonometry and co-ordinate geometry.

There are a couple of changes to the course and the exam from last year. You should note these changes when looking at the past paper questions. First of all, the geometry section of the course has finally converted to the new course. Thus there will not be a Question 6 with a choice of two possible questions. In fact, we cannot be sure where geometry will appear on the paper from now on.

Secondly, the changes to the statistics section of the course that were originally deferred for the transition period are now on for examination. The details will be discussed in the section below on Probability and Statistics.

Section A is called 'Concepts and Skills' and will contain six questions, each worth 25 marks, covering topics from the first two strands. We have no way of knowing for sure how the questions will pan out, but we can make an educated guess. It is likely that there will be two questions on Strand 1, one on probability and one on statistics. However, it is possible that these concepts could be combined with each other, or indeed with topics from Strand 2. This happened in Question 4 in 2014.

Section B will contain between two and four questions of a practical nature. You won't know until you open the paper in the exam how many questions there are going to be, and how the marks will be split between the questions.

However conventional wisdom and a short track record do give us some pointers. On the other hand, when we bear in mind the promise that the papers will be unpredictable, you should not be too surprised if some changes do occur.

For starters, it is quite likely that there will be a major practical question based on trigonometry, with possibly some input from geometry. This has consistently been the case for the last number of years. Last year, for the first time, there was also a practical question on co-ordinate geometry. This may occur again, but on average, there is likely to be more questions on trig and geometry than on co-ordinate geometry in Section B. For these questions, it is obvious that you have to be on top of all the methods from trig and geometry.

But it should also be clear that you must learn to recognise when each method is required. This is the key to practical questions, and is generally not given the attention it deserves by students. You should practise with all the questions of this type that you can get your hands on.

Probability and statistics is likely to be equally important as trig and geometry in Section B. Last year, probability and statistics accounted for 90 of the 150 marks in Section B, although in the two previous years it was 75 marks each.

The major issue for most students with questions on probability and statistics is that there is likely to be many parts requiring definitions and opinions. You can bet that these questions will be unlike anything that has occurred over the last few years. Nevertheless, looking at the questions and the acceptable answers to these past paper questions will give you valuable guidelines as to what to expect this year.

Probability

In all likelihood, one of the questions in Section A will deal with the topic of probability. It is also likely that probability will comprise parts of one or more of the long questions in Section B. In 2014, a major question in Section B was largely on probability, although we cannot rely on this happening every year.

It is important to understand the concept of probability as the mathematical treatment of the idea of chance. You must also understand the concepts of mutually exclusive events, conditional probability and independent events. You can be asked to explain these concepts, and not just perform calculations. This occurred in 2013, and may occur again.

It is also necessary to learn all the relevant formulae, as all bar the binomial distribution formula are not present in the *Formulae and Tables*.

Other techniques, such as Venn diagrams, two-way tables and tree diagrams, which are used in the calculation of probabilities, should also be practised. Tree diagrams were asked in 2014.

The main formulae that you have to know are as follows.

* For equally likely events,

$P(E) = \dfrac{\#(E)}{\#(S)}$,

where S is the sample space and E is the event.

* For mutually exclusive events,

$P(A \cup B) = P(A) + P(B)$

* The General Addition Rule for two events which may not be mutually exclusive,

$P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

* Conditional probability,

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

* General Multiplication Rule,

$P(E \cap F) = P(E).P(F|E)$

and for independent events

$P(E \cap F) = P(E).P(F)$.

You should also realise that many probability questions can be tackled in a number of different ways.

Another section of the probability course is Bernoulli trials. These are repeated identical trials, each of which has only two possible outcomes, which we call success and failure.

To begin with, it is necessary to be able to recognise when the formulae and methods for Bernoulli trials are required. The basic formula

$P(X = r) = \dbinom{n}{r}p^{r}q^{n-r}$

is on page 33 of the *Formulae and Tables*. The other formulae can then be expressed in terms of this formula.

Finally, it is also necessary to know the formula for the expected value of a random variable, and how to interpret the result.

1. **Fundamental principle of counting**
e.g. A password for a mobile phone consists of five digits, from 0 to 9 inclusive. Eithne chooses her own password.

■ **Continued on page 12**

THE IRISH TIMES in association with THE INSTITUTE OF EDUCATION

Maths Paper 2 (Higher Level)

Continued from page 11

Students at the Institute of Education.

(i) How many passwords are possible?
(ii) If she wants her password to begin with a 4 and end with an odd number, how many passwords are now possible?

2. **Arrangements (permutations)**
e.g. All the letters of the word TRIBUNE are to be arranged in order.
(i) How many arrangements are possible?
(ii) How many arrangements have the T and the R side by side?

3. **Combinations (choices)**
e.g. Four different digits are to be chosen from the digits 1 to 9 inclusive.
(i) If there are no restrictions, how many choices are possible?
(ii) How many of these choices contain two even digits and two odd digits?
(iii) How many choices of four digits contain at least three odd digits?
(iv) How many choices of four digits contain at least one even digit?

4. **Concept of probability**
e.g. Two different numbers are chosen at random from the whole numbers from 1 to 10 inclusive.
(i) How many choices are possible?
(ii) List the choices that give a total of 5 when the two numbers are added.
(iii) What is the probability that we get a total of 5 when two different numbers are chosen from the whole numbers from 1 to 10 inclusive?

5. **Equally likely outcomes**
e.g. Four letters are chosen at random from the letters of the word LEAVING. What is the probability that we get exactly two vowels and two consonants?

6. **Probability and set theory**
e.g. E and F are two events for an experiment. $P(E)=\frac{3}{5}$ and $P(F)=\frac{2}{7}$.
(i) If $P(E\cup F)=\frac{29}{35}$, are E and F mutually exclusive? Explain.
(ii) If E and F are mutually exclusive, calculate $P(E\cup F)$.

7. **Success and Failure**
e.g. Seven people are chosen at random and asked to name their birth-months. Assuming that each month is equally likely, what is the probability that at least two of the people chosen have the same birth-month?

8. **Addition Rule**
e.g. Two dice are thrown. Let E be the event that the dice show identical numbers, and let F be the event that the numbers on the dice have a total of eight.
(i) Calculate $P(E)$.
(ii) Calculate $P(F)$.
(iii) Describe $E\cap F$ and calculate $P(E\cap F)$.
(iv) If two dice are thrown, what is the probability that the two numbers obtained are either identical or have a total of eight?

9. **Conditional probability**
e.g. In a class of all girls, 35% of the girls have brown hair, 20% have brown eyes and 12% have both brown hair and brown eyes. A girl is selected at random from the class.
(i) If she has brown hair, what is the probability that she also has brown eyes?
(ii) If she has brown eyes, what is the probability that she also has brown hair?

10. **Independent events**
e.g. A family has three children. A is the event that the family has at most one boy. B is the event that the family has children of both sexes. Assume that each child is equally likely to be a boy or a girl.
(i) Calculate $P(A)$ and $P(B)$.
(ii) By calculating $P(A\cap B)$, investigate if A and B are independent.

11. **Multiplication Rule**
e.g. In a game, cards are dealt successively, without replacement, from a standard pack of cards until a queen is obtained.
(i) What is the probability that the game ends on the fourth card?
(ii) What is the probability that at least three cards are dealt?

12. **Bernoulli trials**
e.g. A coin is biased so that a tail is three times as likely to occur as a head. This coin is tossed repeatedly. What is the probability that the seventh head appears on the tenth toss?

13. **Expected value**
e.g. A die is loaded so that the even numbers are twice as likely to occur as the odd numbers.
(i) Calculate the probability of getting a '1' and the probability of getting a '2'.
(ii) If the random variable X represents the number obtained when the die is thrown, calculate $E(X)$, the expected value of X.

Statistics

Section B is almost certain to contain at least one significant question on statistics, possibly in connection with probability or other topics. It is also likely that there will be a number of unusual parts asking for information on a number of issues that you cannot prepare for. All you can do is use common sense and rely on your knowledge of statistics.

Statistics involves the gathering of data in a scientific way, representing this data graphically and analysing the data so that conclusions can be drawn. For more advanced analysis, probability is used in conjunction with statistical methods. This is covered in the next section.

The first part of this section involves the methods of gathering data by means of surveys, controlled experiments and observational studies. Each of these terms must be clearly understood.

Also important is being aware of the different types of data, the difference between populations, sampling frames and samples, the conditions necessary for a reliable sample, the different types of sample and any ethical issues that may be involved.

For surveys, it is necessary to know how to properly design a questionnaire. This involves the variety and the nature of questions.

The next topic is the representation of the data obtained by means of pie charts, line plots, bar charts, histograms and stemplots, including back to back stemplots. At higher level, this is unlikely to be a major issue.

After that, you have to be able to calculate the measures of central tendency and variability, i.e. the mean, median, mode, range, percentiles, quartiles, IQR and standard deviation. You should also know how to interpret given graphs in terms of skewness, position of mean etc, and the standard deviation.

Finally, we come to the idea of investigating correlation, and the distinction between correlation and causality. You must be able to draw and interpret scatterplots, calculate the correlation coefficient by calculator and understand exactly what it represents. You must also be able to draw the line of best fit by eye, and find its equation.

1. **Sampling and surveys**
e.g. A polling company wants to find the reaction of the public in Ireland to a controversial television programme. It chooses five counties in Ireland at random. Then, in each of the selected counties it divides the population into a number of groups. It selects five groups at random in each of the chosen counties. It then interviews forty people at random in each of these groups.
(i) Describe the type of sampling being used by the company.
(ii) Give a reason why the company might have chosen this type of sampling.
(iii) List three possible types of bias that may be present in the data obtained.

2. **Controlled experiments and observational studies**
e.g. You have heard it stated that taking an aspirin tablet once a day reduces the risk of a heart attack. You want to conduct a study to test this theory.
(i) Explain why a survey is not the appropriate type of study to undertake.
(ii) Assuming that taking a small dose of aspirin once a day does not cause a health risk, are there any ethical issues involved in this study? Discuss.
(iii) Explain why a controlled experiment is the most reliable approach for this type of study.
(iv) Detail exactly how you would go about designing the controlled experiment.
(v) What issues could cause bias in the results, and how could you deal with them?

3. **Graphs**
e.g. Twelve sample employees were chosen from each of two large companies, A and B. On a given day, the time taken (in minutes) by each employee to arrive at their place of work, measured from the moment they left their accommodation, was recorded.

A:	10	2	17	25	12	18	14
	12	10	6	32	11		
B:	8	17	25	16	24	33	32
	7	29	36	45	22		

(i) Draw a back to back stem plot to represent this data.
(ii) If one of these companies is based in a county town and the other in a large city such as Cork or Dublin, could you guess which is which? Give a reason.

4. **Mean, median and mode**
e.g. A dataset contains the positive whole numbers 4, 7, 2, 6, x.
(i) If the median is 5, what is the value of x?
(ii) If the median is 4, what are the possible values of x?
(iii) If the median is 6, what are the possible values of x?

5. **Range, percentiles and IQR**
e.g. The ages, in years, of the guests at a wedding are recorded below.

33	22	45	44	70	34	72	75
28	27	29	31	34	44	68	25
21	28	27	29	64	25	28	49
70	26	30	29	43	23	26	28.

(i) Show this data on a stem plot.
(ii) Calculate the lower quartile and the upper quartile.
(iii) Calculate the interquartile range.

6. **Standard deviation**
e.g. calculate the mean and the standard deviation of the distribution:

Result	0	1	2	3	4
Frequency	6	10	16	12	5

7. **Analysis of graphs**
e.g. A frequency curve has a mode of $7\cdot5$ and a mean of 9.
(i) Give a rough estimate of where the median should lie.
(ii) Would you describe the frequency curve as symmetric, skewed left or skewed right? Give a reason.
(iii) Draw a frequency curve that fits the information given.

8. **Scatterplots and correlation coefficient**
e.g. A number of students volunteered for a medical test. The lung capacity, in cm^3, (X) of each student and the length of time, in seconds, (Y) for which they could hold their breath were measured. The data is given below in the form of couples (x, y).

$(400,32)$, $(380,34)$, $(450,40)$,
$(420,36)$, $(370,30)$,
$(390,40)$, $(400,38)$, $(430,42)$,
$(380,33)$, $(440,38)$.

(i) Represent this data on a scatterplot.
(ii) Describe the correlation that exists between X and Y.
(iii) Draw the line of best fit.
(iv) Use the line of best fit to predict the time holding breath of a person with a lung capacity of 410 cm^3.
(v) Find the equation of the line of best fit, by taking two points on the line.
(vi) What does the slope of the line of best fit tell us?
(vii) Use the equation of the line of best fit to estimate the time holding breath for a person with a lung capacity of 520 cm^3.

Probability and Statistics

The theory from probability can be used to help analyse the data obtained from statistical studies. This forms the last part of Strand 1.

You should watch this topic very carefully, as this section of the course has been extended from last year. The new topics are likely to be examined very soon, probably this year.

These new topics involve using z tables to form confidence intervals for the population proportion and the population mean (from a large sample), using hypothesis tests on the population mean and using p-values.

The key idea here is that of a probability distribution and the corresponding probability histogram or curve. The most important example is the standard normal probability distribution. The values of the probabilities, i.e. the areas under this curve, are given on pages 36 and 37 of the *Formulae and Tables*. You should practise calculating areas in right tails, left tails and between two given values of z.

By converting to standard units, we can also calculate probabilities for any normal variable. Associated ideas are the empirical rule and using standard units (z scores) to determine relative standing.

This theory from probability can be used to determine the reliability of statistics derived from sample data. The margin of error of a sample proportion can be used to give a confidence interval for a population proportion. From this year, we also have to be able to approach this from the point of view of z tables.

As stated above, we also now have to be able to construct a confidence interval for a population mean using z tables.

Another key area is being able to test a hypothesis, i.e. test whether or not to accept a claim made about a population proportion or population mean based on data obtained.

For the population proportion, we use the margin of error expression to make the decision. For a population mean, we use z tables to make the decision. An alternative method is to use p-values.

None of the calculations in this section is in any way challenging. Rather the difficulty lies in understanding the concepts, and being able to decide what to do in any given situation.

1. **Standard normal tables**
e.g. If Z is a standard normal variable, calculate $P(-1\cdot2\le Z\le 1\cdot75)$

2. **Other normal distributions**
e.g. The heights of 800 babies are normally distributed with mean 66 cm and standard deviation 5 cm.
(i) Calculate the probability that a baby chosen at random will have a height greater than 74 cm.
(ii) Estimate the number of babies of height greater than 72 cm.
(iii) Estimate the number of babies of height between 65 cm and 70 cm.

3. **Empirical rule**
e.g. X is a continuous random variable which is normally distributed with mean $10\cdot8$ and standard deviation $1\cdot2$.
Calculate $P(8\cdot4\le X\le 12)$, i.e. the probability that X lies between $8\cdot4$ and 12, using the empirical rule.

4. **Relative standing**
e.g. Luke sits two tests on the same day: history and geography. In history, he scores 88%, while the mean and standard deviation for his class were 68% and 12% respectively. In geography, he scores 72%, while the mean and standard deviation for his class are 62% and 5% respectively. In which exam did Luke perform better relative to his classmates?

5. **Confidence interval: population proportion**
e.g. A survey of 1200 consumers indicates that 525 intend to buy a new brand of dairy spread. Find the 95% confidence interval for the true proportion of consumers that intend to buy the new brand
(i) using the margin of error approximation,
(ii) using z tables.

6. **Confidence interval: population mean**
e.g. The standard deviation of the weight of a certain type of glass bottle produced by a factory is known to be $4\cdot5$ g. A random sample of 500 of these bottles has a mean weight of 270 g. Obtain a 95% confidence interval for the mean weight of all glass bottles of this type produced by the factory.

7. **Hypothesis testing: population proportion**
e.g. Election workers for a candidate, X, in an election have confidently predicted that she will gain 45% of the first preference votes and so top the poll. To test this claim, the workers for another candidate, Y, take a random sample of 1024 voters. This sample suggest that the support for candidate X is at 42%.
(i) State the null hypothesis and the alternative hypothesis.
(ii) On what basis should the hypothesis be rejected or not rejected?
(iii) Do you think the election workers for candidate Y have evidence to dispute the claim made by the election workers for candidate X? Give a reason.

8. **Hypothesis testing: population mean**
e.g. A car battery manufacturer claims that the average life of the batteries they produce is $4\cdot6$ years, with a standard deviation of $0\cdot8$ years. To test this claim, a random sample of 120 batteries was tested and their average life was found to be $4\cdot8$ years. Determine if there is enough evidence to conclude that the claim made by the battery manufacturer is not true, at the 5% level of significance.

Synthetic Geometry

The syllabus on synthetic geometry refers to the following (in short):
* constructions 16 to 22
* language of proof (9 terms)
* use of theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6
* proofs of theorems 11, 12 and 13.

We should also include in here the topic of enlargements, which is closely related to Theorem 13.

However, in the pre-amble to Strand 2, it states that knowledge of Junior Cert material is assumed, and by implication may be examined. Thus all other theorems, such as the isosceles triangle theorem and Pythagoras' Theorem, can occur, and have occurred, in questions at this level. So you really need to familiarise yourself with all the theorems from 1 to 21.

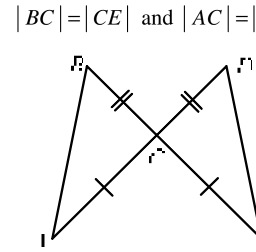
The proofs of theorems 11, 12 and 13 should be learned very well. Learn the diagram so that you can reproduce it the same every time. In each formal proof, you will need to enter information in the sections 'Given', 'To prove', 'Construction' and 'Proof'. For each of the three theorems, you must be able to write out the contents of each of these with great accuracy. These proofs have been examined frequently since the start of the new exams, and there is no reason to see this pattern changing.

You should also study the constructions 1 - 15, although most of these constructions are covered anyway when dealing with the Leaving Cert constructions.

Where exactly geometry theorems appear on the exam paper is now up in the air. It is likely that at least one question in Section A will deal with some aspect of geometry. It is also likely that a knowledge of the results from geometry will be required in one or more of the questions in Section B.

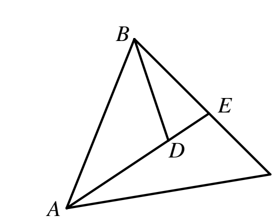
1. **Language of proof**
e.g. Explain what is meant by the phrase 'if and only if' in the statement of a theorem.
Give an example of a theorem in which this phrase can be used.

2. **Congruent triangles**
e.g. In the diagram below, $|BC|=|CE|$ and $|AC|=|CD|$.



(i) Explain why the triangles ABC and CDE are congruent.
(ii) Prove that $|AB|=|DE|$.
(iii) Prove that $|\angle BDC|=|\angle CEA|$.

3. **Other triangle theorems**
e.g. Let D be any point on the inside of the triangle ABC.
Let AD meet BC at E between B and C.



(i) Prove that $|\angle ADB|>|\angle AEB|$.
(ii) Prove that $|\angle ADB|>|\angle ACB|$.

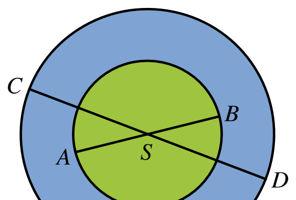
Continued on page 14

Exam Times

Maths Paper 2 (Higher Level)

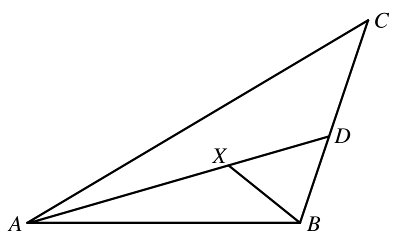
Continued from page 13

4. **Quadrilaterals and parallelograms**
e.g. A circular pond contains a circular island, both with centre S. A straight path across the island, $[AB]$, passes through S, as does a bridge $[CD]$, which goes from one edge of the pond to the other edge.



Show that $ACBD$ is a parallelogram.

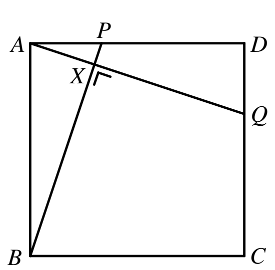
5. **Similar triangle theorems**
e.g. In the diagram, $[AD]$ bisects the angle $\angle BAC$ and $|BD| = |BX|$.



(i) Show that the triangles ACD and AXB are similar.
(ii) Deduce that
$$\frac{|AC|}{|AB|} = \frac{|CD|}{|DB|}.$$

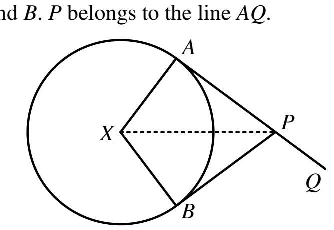
6. **Theorem of Pythagoras**
e.g. The triangle PQR is right angled at R, and X and Y are the midpoints of $[QR]$ and $[PR]$ respectively. Prove that
$$5|PQ|^2 = 4|PX|^2 + 4|QY|^2.$$

7. **Area theorems**
e.g. In the square $ABCD$, P is on $[AD]$ and Q is on $[CD]$ such that $AQ \perp BP$. AQ and BP intersect at the point X.



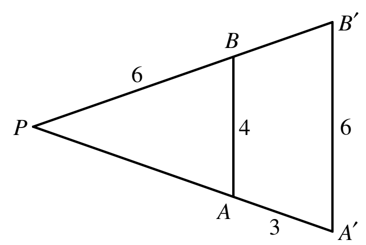
(i) Prove that the triangles ABP and AQD are congruent.
(ii) Prove that the area of ΔAXB is equal to the area of the quadrilateral $PXQD$.

8. **Circle theorems**
e.g. PA and PB are tangents to the circle with centre X and containing the points A and B. P belongs to the line AQ.



(i) Prove that the triangles APX and BPX are congruent.
(ii) Deduce that $|PA| = |PB|$ and that $|\angle PXA| = |\angle PXB|$.
(iii) Prove that $|\angle BPQ| = 2|\angle AXP|$.
(iv) Explain why A, X, B and P lie on a circle whose centre is the midpoint of $[XP]$.

9. **Enlargements**
e.g. The triangle $PA'B'$ is the image of the triangle PAB under an enlargement with centre P.
$|AA'| = 3$, $|PB| = 6$, $|AB| = 4$ and $|A'B'| = 6$.



(i) Find the scale factor of the enlargement.
(ii) Find $|PA|$.
(iii) Find $|BB'|$.
(iv) The area of $\Delta PA'B'$ is $9\sqrt{10}$. Express the area of ΔPAB in the form $c\sqrt{10}$, where $c \in \mathbb{R}$.

10. **Constructions**
e.g. (i) Construct a triangle PQR such that $|PQ| = 10$ cm, $|PR| = 7$ cm and $|\angle QPR| = 120°$.
(ii) Show how to construct its circumcentre and its circumcircle.
(iii) Describe the position of the circumcentre.

Trigonometry

The trigonometry portion of our course can be divided into six sections:
* the basic definitions of angles and angle measure, trig ratios and trig functions for all angles,
* constructing and interpreting trig graphs,
* using practical trigonometry, e.g. sine rule, cosine rule, area of a sector, Pythagoras' theorem, to solve triangles, especially in 3D,
* proving the trig identities specified on the syllabus,
* using the 24 trig identities on the course to prove unseen identities and evaluate expressions,
* solving simple trig equations, being able to write down expressions for all solutions.

It should also be remembered that ideas and results from synthetic geometry are often required when answering questions on trigonometry. This is especially true when it comes to the applications questions in Section B.

In preparation for the trig questions, it is important to become familiar with all the formulae on pages 9 to 16 in the 'Formulae and Tables'. This is not to suggest that you should learn the formulae and special values, but at least you should be familiar with what is where. You should also learn to recognise expansions, e.g. should you meet
$$2\sin A\cos A$$
you should be able to look up that this is $\sin 2A$.

Being realistic, it is likely that the one 25 mark

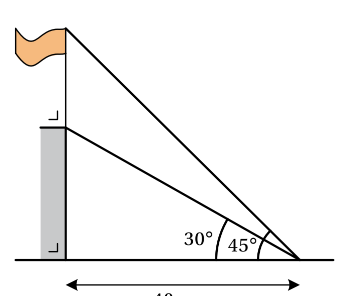
Students at the Institute of Education in Dublin. PHOTOGRAPH: BRENDAN DUFFY.

* using practical trigonometry, e.g. sine rule, cosine rule, area of a sector, Pythagoras' theorem, to solve triangles, especially in 3D,
* proving the trig identities specified on the syllabus,
* using the 24 trig identities on the course to prove unseen identities and evaluate expressions,
* solving simple trig equations, being able to write down expressions for all solutions.

It should also be remembered that ideas and results from synthetic geometry are often required when answering questions on trigonometry. This is especially true when it comes to the applications questions in Section B.

In preparation for the trig questions, it is important to become familiar with all the formulae on pages 9 to 16 in the 'Formulae and Tables'. This is not to suggest that you should learn the formulae and special values, but at least you should be familiar with what is where. You should also learn to recognise expansions, e.g. should you meet
$$2\sin A\cos A$$
you should be able to look up that this is $\sin 2A$.

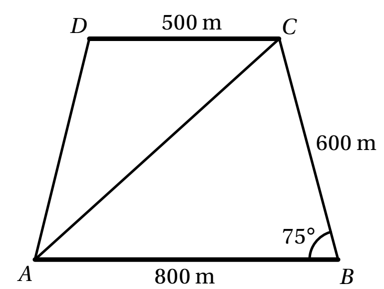
Being realistic, it is likely that the one 25 mark question on trig will probably deal with the more abstract areas of trig, e.g. trig graphs, proving trig identities. The more practical tops of the flagpole and the building are $45°$ and $30°$ respectively.



Find the height of the flagpole.

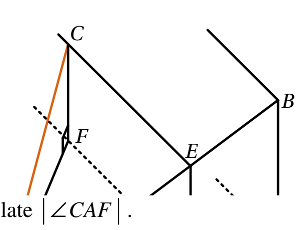
4. **Solving triangles**
e.g. In the diagram, $[AB]$ and $[DC]$ are two parallel roads, where $|AB| = 800$ m and $|DC| = 500$ m. By measurement, it is determined that $|\angle ABC| = 75°$ and that $|BC| = 600$ m.

e.g. In the diagram, $[AB]$ and $[DC]$ are two parallel roads, where $|AB| = 800$ m and $|DC| = 500$ m. By measurement, it is determined that $|\angle ABC| = 75°$ and that $|BC| = 600$ m.



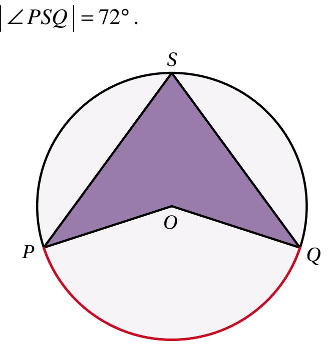
Find
(i) $|AC|$, correct to the nearest metre,
(ii) $|\angle BAC|$, in degrees to two decimal places,
(iii) $|AD|$, correct to the nearest metre.

5. **3D problems**
e.g. A hillside may be considered to be a plane inclined at an angle of $15°$ to the horizontal. $[AC]$, a straight path up the hillside makes an angle of $30°$ with $[AB]$, the line of greatest slope up the hillside. E belongs to $[AB]$ and CE is perpendicular to AB.



i.e. calculate $|\angle CAF|$.

6. **Arcs and sectors**
e.g. The diagram below shows a company logo. The circle, with centre O, is of radius 5 cm, $|PS| = |QS|$ and $|\angle PSQ| = 72°$.



Find
(i) the length of the minor arc PQ in terms of π,
(ii) the area of the coloured region in the centre of the logo.

7. **Trig proofs**
e.g. In a triangle, the sides a, b and c are opposite the angles A, B and C respectively. Prove that
$$a^2 = b^2 + c^2 - 2bc\cos A.$$

8. **Trig identities**
e.g. use pages 13 and 14 of the *Formulae and Tables* to prove that
$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}$$

9. **Trig equations**
e.g. Find the general solution of the equation
$$\cos 2x = -\frac{1}{2}$$
and use it to find all the solutions for $0° \le x \le 720°$.

Co-ordinate Geometry

Co-ordinate geometry is likely to account for one, possibly two, of the short questions in Section A. It may also make an appearance in one of the long questions in Section B, as happened in 2014.

In Section A, there are likely to be question parts which test both your understanding of key concepts such as the slope of a line, and your ability to use the formulae and the methods on the course. In Section B, you may very well have to be inventive in how you set up your co-ordinate system and in what approach you use.

With co-ordinate geometry, it is important to remember that tougher questions often involve using ideas from synthetic geometry. You should be prepared for this.

A. The Line

1. **Basic concepts**
e.g. A metal rod has length $\sqrt{61}$. One end of the rod is to welded to the point $A(1,-2)$. The other end of the rod is to be welded to a point B on the vertical line $x = 6$. By letting $B = (6,k)$, find the co-ordinates of the two possible points B.

2. **Area of a triangle**
e.g. The area of the triangle ABC where $A = (3,-1)$, $B = (5,3)$ and $C = (x,4)$, is 13 square units. Find the possible values of x.

3. **Divisor of a line segment**
e.g. If $A = (-1,5)$ and $B = (14,0)$, find the co-ordinates of the point which divides $[AB]$ in the ratio $2:3$.

4. **Angle between two lines**
e.g. find, correct to the nearest degree, the larger angle between the lines $3x + 4y = 9$ and $2x - y + 6 = 0$

5. **Perpendicular distance formula**
e.g. A ship is travelling along the line $5x - y = 18$. A lighthouse is located at the point $(2,4)$. Find, correct to two decimal places, the closest that the ship gets to the lighthouse.

$5x - y = 18$. A lighthouse is located at the point $(2,4)$. Find, correct to two decimal places, the closest that the ship gets to the lighthouse.

B. The Circle

1. **Equation:** $x^2 + y^2 = r^2$
e.g. find the equation of the circle, with centre $(0,0)$ and which has the line $3x - 2y = 13$ as a tangent

2. **Equation:** $(x-h)^2 + (y-k)^2 = r^2$
e.g. find the equation of the circle that has the line segment from $(-2,5)$ to $(6,-1)$ as a diameter

3. **Equation:** $x^2 + y^2 + 2gx + 2fy + c = 0$
e.g. find the equation of the circle which passes through the points $(0,2)$, $(1,5)$ and has its centre on the line $x + 5y - 15 = 0$.

4. ***g, f, c* method**
e.g. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has the y axis as a tangent. Show that
$$f^2 = c.$$
Find the equations of the two circles that $x^2 + y^2 + 4x - 6y - 12 = 0$ and $x^2 + y^2 - 3y - 4 = 0$ touch internally.

6. **Intersection of a line and a circle**
e.g. show that the line $2x + y - 10 = 0$ is a tangent to the circle $x^2 + y^2 - 4x - 2y = 0$ and find the coordinates of the point of contact

7. **Tangent at a point**
e.g. find the equation of the tangent to the circle $x^2 + y^2 + 4x - 8y - 5 = 0$ at the point $(2,7)$

8. **Tangents and chords**
e.g. find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ which are perpendicular to the line $2x - y - 1 = 0$.

Areas and Volumes

Areas of plane objects and volumes of solid objects comprises Section 3.4 of our syllabus and has been moved to Paper 2. The rest of Strand 3 is still on Paper 1.

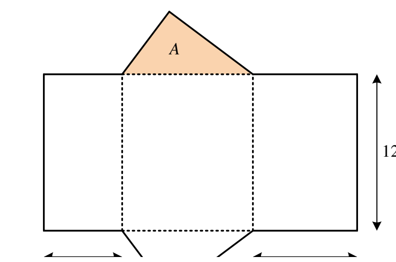
It makes sense that this topic is examined on Paper 2, as it is closely allied to geometry and trigonometry, i.e. Strand 2, which is examined on this paper.

All of the areas and volumes part of the syllabus is also on for students at Ordinary Level, and so is unlikely to be the sole subject of a question on Paper 2.

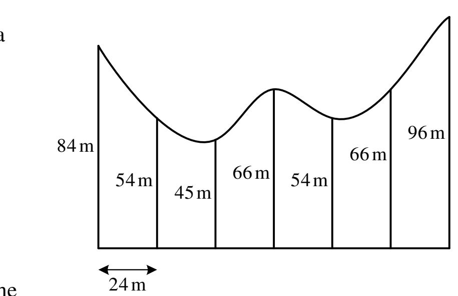
is unlikely to be the sole subject of a question on Paper 2.

However, most of the ideas in this section have been seen at Junior Cert and are assumed as we tackle practical problems in geometry and especially trigonometry. The notable exceptions are nets and the trapezoidal rule.

1. **Nets**
e.g. The diagram shows the net of a prism. The base of the prism is the triangle A.
(i) Draw a diagram of the prism that can be made from this net.
(ii) Determine the area of the base, A.
(iii) Calculate the volume of the prism.
(iv) Calculate the total surface area of the prism.



2. **Trapezoidal Rule**
e.g. A field is bounded on three sides by ditches and on the fourth side by a stream, as is shown below.



At equal intervals of 24 m perpendicular measurements of 84 m, 54 m, 45 m, 66 m, 54 m, 66 m and 96 m are made to the stream. Use the trapezoidal rule to estimate the area of the field.

3. **Finding a dimension**
e.g. A steel-works buys steel in the form of solid cylindrical rods of radius 10 cm and length 30 m. The steel rods are melted to produce solid spherical ball-bearings. No steel is wasted in the process.
(i) Find the volume of steel in one cylindrical rod, in terms of π.
(ii) The radius of a ball-bearing is 2 cm. How many such ball-bearings are made from one steel rod?
(iii) Ball-bearings of a different size are also produced. One steel rod makes 225000 of these new ball-bearings. Find the radius of the new ball-bearings.

Maths Paper 2 (Higher Level)

Sample questions

1. Probability and Statistics

Question

The average cholesterol level in a population is known to be $5 \cdot 7$. A study is conducted of the sufferers of a particular disease to see if they have a different average cholesterol level from the population. A random sample of 80 sufferers of the disease have their cholesterol level measured. They are found to have a mean of $5 \cdot 87$ and a standard deviation of $0 \cdot 78$.

(i) Obtain the test statistic for the sample mean.

(ii) Calculate the p-value for the sample statistic.

(iii) Is this result significant at the 5% level of significance? Give a reason.

(iv) Is this result significant at the 1% level of significance? Give a reason.

(v) Give a reason why the authors of the study might report the p-value rather than just saying whether the result was significant or not.

Solution

(i) $\mu = 5 \cdot 7,\ \bar{x} = 5 \cdot 87,\ n = 80,\ s = 0 \cdot 78$

$$T = \frac{5 \cdot 87 - 5 \cdot 7}{\frac{0 \cdot 78}{\sqrt{80}}} = 1 \cdot 95$$

(ii) p-value $= 2(1 - P(Z \le 1 \cdot 95))$

$= 2(1 - 0 \cdot 9744)$

$= 0 \cdot 0512$

(iii) $\alpha = 0 \cdot 05$. As the p-value is greater than α, the result is not significant at the 5% level of significance.

(iv) $\alpha = 0 \cdot 01$. As the p-value is greater than α, the result is not significant at the 5% level of significance.

(v) Reporting the p-value leaves it to the reader of the report to decide for themselves whether or not the result is significant. The reader will decide this based on the level of significance they think appropriate.

2. Geometry

Question

(a) Use

$\cos(A+B)$

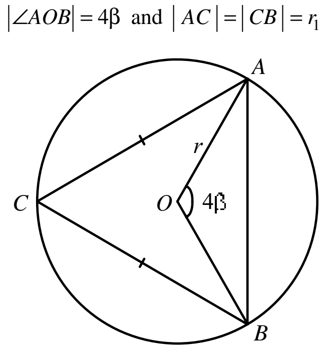
$= \cos A \cos B - \sin A \sin B$

to show that

$\cos 2A = 1 - 2\sin^2 A$.

(b) In the diagram, O is the centre of a circle of radius r.

$|\angle AOB| = 4\beta$ and $|AC| = |CB| = r_1$.



(i) Write down the area of the triangle AOB in terms of r and β.

(ii) Express r_1 in terms of r and β.

(iii) Show that

$$\frac{\text{area } \Delta AOB}{\text{area } \Delta ACB} = \frac{\cos 2\beta}{2\cos^2 \beta}.$$

Solution

(a) $\cos(A+B)$

$= \cos A \cos B - \sin A \sin B$

Putting $B = A$,

$\cos(A+A)$

$= \cos A \cos A - \sin A \sin A$

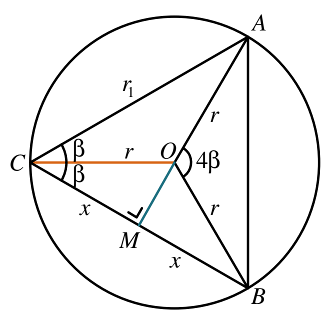
$\cos 2A = \cos^2 A - \sin^2 A$

Using $\cos^2 A = 1 - \sin^2 A$,

$\cos 2A = (1 - \sin^2 A) - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

(b)



(i) Area $\Delta AOB = \frac{1}{2} r.r \sin 4\beta$

$= \frac{1}{2} r^2 \sin 4\beta$

(ii) [$|\angle ACB| = 2\beta$, by the angle at the centre of a circle is twice the angle at the circle standing on the same arc theorem.

The triangles OAC and OCB are congruent (SSS). Thus

$|\angle ACO| = |\angle BCO| = \beta$.

Let the perpendicular from O to $[BC]$ meet this chord at M.

Then

$|CM| = |MB| = x$

by the theorem which says that a perpendicular from the centre of a circle to a chord bisects the chord.

Thus $r_1 = 2x$.]

From the triangle OCM,

$\cos \beta = \frac{x}{r}$

$x = r\cos\beta$

and $r_1 = 2r\cos\beta$.

(iii) Area $\Delta ACB = \frac{1}{2} r_1^2 \sin 2\beta$

$= \frac{1}{2}(2r\cos\beta)^2 \sin 2\beta$

$= \frac{1}{2}\left(4r^2\cos^2\beta\right)\sin 2\beta$

$= 2r^2 \cos^2\beta \sin 2\beta$

$$\frac{\text{Area } \Delta AOB}{\text{Area } \Delta ACB} = \frac{\frac{1}{2} r^2 \sin 4\beta}{2r^2\cos^2\beta \sin 2\beta}$$

$$= \frac{\frac{1}{2}(2\sin 2\beta \cos 2\beta)}{2\cos^2\beta \sin 2\beta}$$

$$= \frac{\cos 2\beta}{2\cos^2 \beta}.$$

Maths Ordinary Level

Take two for Project Maths

Paper 1

For the second time, all students at ordinary level will sit Paper 1 based entirely on the new syllabus

This will be only the second occasion when all students will tackle the completely new Paper 1 which is based on the new Project Maths syllabus. The meaning of this is that we do not have too many past paper questions to go on. But the ones we do have (and this includes the questions from the exams for the Pilot Schools) you should study very carefully.

The general feedback from students is that there is no great affection for the new style exams, with the lack of predictability and the need to answer questions looking for verbal replies. However, it has to be said that the papers for the last two years have been very fair and have examined the material that most students would have studied well in school.

On top of this, the marking schemes have been generous, with a large proportion of the marks going for the earlier and easier parts of questions. Also, significant marks were often awarded for partially correct solutions to questions.

With this in mind, it is important that you provide an answer (your best guess, if you don't know) to every question part. Remember, it will be fairly easy to get some partial credit for any correct element of your response, but if you have nothing written down, you can't get any marks.

Paper 1 will consist of two sections, each worth 150 marks. You will not have any choice in either section: you will have to attempt all questions. Section A, called 'Concepts and Skills' will have six questions, each worth 25 marks, which will be of a mathematical nature. These questions will focus on the methods you have learned in school, and on testing your understanding of the concepts you have met. The major areas of study for Paper 1 are algebra, complex numbers, arithmetic and money, sequences and series, functions and differentiation.

Section B will involve between two and four questions (you won't know until you open the paper on the day). These questions will be of a practical nature, and are intended to be new so that no student can prepare for them. One of the problems you will face with these questions is trying to identify what is the correct approach to apply. This is why you should try to understand the purpose of each of the methods you study.

Another problem you can expect is a number of question parts that ask for an explanation of a method you use, or an opinion on a method or problem. Try to be precise in answering questions of this type.

Number Systems

First of all, it is necessary to have a very clear understanding of each of the sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ and $\mathbb{R}$, and to be able to assign given numbers to these sets and their differences. For example, $\frac{5}{3}$ belongs to $\mathbb{Q}$ but not to $\mathbb{Z}$, i.e. it belongs to $\mathbb{Q} \setminus \mathbb{Z}$.

For natural numbers, you must understand factors, prime numbers, highest common factor (HCF) and lowest common multiple (LCM). You must also be able to use these ideas when working with fractions, and be able to deal with other types of numbers, such as fractions, decimals, percentages and numbers in scientific notation.

1. **HCF**
   e.g. find the highest common factor of the numbers 108, 180 and 288
2. **LCM**
   e.g. A bookcase has been designed so that it can hold an exact number of box files of width 12 cm. It can also hold an exact number of box files of width 16 cm and an exact number of box files of width 20 cm. What is the least possible width of the bookcase?
3. **Fractions and Decimals**
   e.g. place the following fractions in ascending order:
   $\frac{7}{8},\ \frac{17}{20},\ \frac{31}{36}$
   (i) by finding the lowest common denominator,
   (ii) by converting each to decimals.
4. **Scientific Notation**
   e.g. The distance from earth to the Andromeda galaxy is approximately 2 million light years, and a light year is approximately $9 \cdot 47 \times 10^{12}$ km.
   (i) Express the distance from earth to Andromeda in kilometres in scientific notation.
   (ii) In the future, a new spaceship is capable of travelling at 4×10^8 km/h. How long would it take this spaceship to reach Andromeda from earth?

Algebra

Algebra is the key to maths, being the language that is used in most of the different areas of maths. It is very important to get to grips with the techniques of algebra, ranging from evaluating expressions, rewriting expressions including fractions to solving the types of equation and inequality on our course.

Equations and inequalities can be solved by other means than just pure algebra, and we need to be aware of this. Graphs, tables and mental calculations can give very good approximate solutions, and we can be required to use these as alternatives to pure algebra.

Algebra is likely to be the subject of one, and possibly a second, question in Section A. It is also likely to occur directly in some parts of the questions in Section B.

Some of the main formulae from algebra, including the quadratic formula and the laws of indices, are given in the 'Formulae and Tables' on pages 20 and 21. However, none of the methods from algebra are contained in the tables, and you will need to become proficient will all the methods on the course.

Here is a list of the topics that we need to cover in algebra.

1. **Factors**
   e.g. factorise
   $6x^2 - 7x - 5$
2. **Evaluating expressions**
   e.g. find the value of $4x^2 y - 3x + y$ when $x = 2$ and $y = -1$
3. **Re-writing expressions**
   e.g. remove the brackets from the expression
   $2x(x-3) + (x-1)(2x+3)$
4. **Fractions**
   e.g. write the following as a single fraction in its simplest form
   $\frac{3}{2x-1} - \frac{1}{x+4}$
5. **Re-arrange formula**
   e.g. if
   $p(x+2) = 3(p-1)$
   express p in terms of x
6. **Linear equations**
   e.g. solve for x:
   $3(2x-11) = 2(x-5) - 5(6-x)$
7. **Linear equations with fractions**
   e.g. solve for x:
   $\frac{x-1}{3} + \frac{x+1}{5} = 2$
8. **Linear simultaneous equations**
   e.g. solve the simultaneous equations
   $2x + y = 2$
   $3x - 2y = -11$
9. **Quadratic equations**
   e.g. solve for $x \in \mathbb{R}$,
   $3x^2 - 5x - 2 = 0$
10. **Constructing a quadratic equation**
    e.g. construct a quadratic equation with roots 3 and -2
11. **Equations with fractions**
    e.g. solve the equation
    $\frac{2}{x+1} + \frac{3}{x+2} = 2, \quad x \ne -1,\ x \ne -2$
12. **Linear, non-linear simultaneous equations**
    e.g. solve the simultaneous equations
    $x + 2y = 5, \quad x^2 - y^2 = 8$
13. **Powers**
    e.g. write as a power of 2:
    $\sqrt{\frac{16^x}{8}}$
14. **Equations with the unknown in the index**
    e.g. solve for x:
    $4^{x+1} = \left(\frac{8}{\sqrt{2}}\right)^3$
15. **Inequalities**
    e.g. solve the inequality for $x \in \mathbb{R}$:
    $2(3x-5) \ge 3x + 2$.

Complex Numbers

Complex numbers are numbers that involve the imaginary unit $i = \sqrt{-1}$. With this definition, it is now possible to find the solutions of any quadratic equation, $az^2 + bz + c = 0$, where $z \in \mathbb{C}$, using the formula

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This is because the square roots of any negative number can be written in terms of i. For example,

$\sqrt{-16} = \sqrt{(16) \times (-1)} = \sqrt{16} \times \sqrt{-1} = 4i$.

Complex numbers are numbers of the form $a + bi$, where a and b are real numbers. We must be able to add, subtract and multiply complex numbers and simplify the answers. We must also be able to perform division, using the complex conjugate.

A key concept on our course is the representation of a complex number as a point on an Argand diagram. An associated idea is that of the modulus, which is the distance of the point representing the complex number from the origin.

1. **Addition and subtraction**
   e.g. if $z = -1 + 2i$ and $w = 4 - 3i$, express in the form $a + bi$:
   (i) $3z - w$, (ii) $z + 2w$
2. **Multiplication**
   e.g. express in the form $a + bi$:
   (i) $(2+i)(3-2i)$,
   (ii) $(1-3i)^2$
3. **Conjugate and division**
   e.g. express $\frac{11-10i}{3+2i}$ in the form $a + bi$,
   where $i^2 = -1$
4. **Argand diagram**
   e.g. represent each of the following complex numbers on an Argand diagram:
   (i) $4 - 3i$, (ii) $-2 + 5i$,
   (iii) 3, (iv) $-7i$
5. **Modulus**
   e.g. If $z_1 = -1 + 2i$ and $z_2 = 2 - 3i$,
   calculate (i) $|z_1|$, (ii) $|z_2|$,
   (iii) $|z_1 + 2z_2|$.
   Which of the three complex numbers z_1, z_2 and $z_1 + 2z_2$ is nearest the origin?
6. **Complex equations**
   e.g. if $z = 4 - 3i$, write $z^2 + 17$ in the form $a + bi,\ a, b \in \mathbb{R}$. Hence solve for real k:
   $k(z^2 + 17) = |z|(1-i)$.
7. **Quadratic equations**
   e.g. solve the following equation for $z \in \mathbb{C}$:
   $z^2 - 8z + 25 = 0$
8. **More quadratic equations**
   e.g. find the value of the real number a if $8 - 2i$ is a root of $z^2 + az + 68 = 0$, and find the other root.

Arithmetic & Finance

Arithmetic covers estimation, error and accumulation of error, the metric and imperial systems, profit, loss, margin, markup and other calculations that amount to using decimals, percentages and scientific notation. It also deals with speed as a rate of change with respect to time.

Arithmetic may not be a full question in the exam next June, but the ideas contained in this topic are likely to occur in a number of questions, possibly in Section A, but certainly in Section B.

Financial maths covers many types of financial transactions, including currency conversions, household finances, income tax, compound interest and depreciation. These topics are likely to be examined in a short question in Section A, and may enter a long question in Section B.

For compound interest, the formula we need is on page 30 of the *Formulae and Tables*,

$F = P(1+i)^t$

When using this formula it is important to have a very clear idea of what exactly is represented by each of the letters.

Also, for questions involving depreciation, the formula is also present on the same page of the *Formulae and Tables*:

$F = P(1-i)^t$.

Again it is necessary to understand the meaning of each of the letters used.

1. **Metric and imperial systems**
   e.g. An athlete runs around a racing circuit, which has a length of 440 yards.
   (i) If the athlete runs eight complete circuits, i.e. eight laps, how many kilometres has she travelled?
   (ii) How many laps of the circuit would be required for a 10000 metre race?
   (iii) If she covers three laps in 228 seconds, express her average speed in metres per second, and kilometres per hour. Give your answers correct to two decimal places.
   [1 yard $= 0 \cdot 914$ m]
2. **Profit, loss, markup and margin**
   e.g. A shop buys a TV screen for €120 and sells it for €199. Express, to the nearest percent,
   (i) the markup, i.e. the percentage profit,
   (ii) the margin.
3. **Estimation and error**
   e.g. In trying to calculate $3 \cdot 2 \times 1 \cdot 5$, Dermot accidently presses $3 \cdot 2 + 1 \cdot 5$. Calculate, correct to one decimal place, his percentage error.
4. **Tolerance**
   e.g. A pilot reckons that he has fuel enough left for 800 km flight. He knows that the reading is correct to within 40 km.
   (i) Give a tolerance interval for the flight distance remaining.
   (ii) If the true distance remaining is 832 km, calculate the percentage error.
5. **Time and speed**
   e.g. Sunita drives from Wexford to Sligo. She leaves Wexford at 9:38 and arrives in Sligo at 16:26. If the distance from Wexford to Sligo is 307 km, what was her average speed, correct to the nearest km/h?
6. **Currency conversions**
   e.g. Des has to make an extended business trip to a number of countries. For spending money, he changes €2000 into Indian rupees at an exchange rate of €1 = 64 rupees. Commission is charged at 2% for this transaction.
   (i) Find how much Des gets, correct to the nearest rupee.
   (ii) On leaving India for China, he changes some of his rupees into Chinese yuan. There is a 3% fee for this transaction. If he gets 6510 yuan and the exchange rate is €1 = 9·26 yuan, how many rupees did he change?
   (iii) On his way back through Kuwait, he changes 3400 yuan into Kuwaiti dinar. If €1 = 0·4 dinar, find how many dinar he gets, after a 2% fee.
7. **Household finances**
   e.g. A waste collection company charges €0·20 per kilogram of waste and €3·20 per 'lift', i.e. for each time the bin is emptied. Over a three month period, Michael has waste of weight 287 kg, and his bin was emptied 11 times. If VAT is charged at 13·5%, find Michael's total waste bill for the period.
8. **Income tax and net pay**
   e.g. Yvonne earns €61000 in a year. Her standard rate cut off point is €32700 and tax credits are €3520 for the year. The standard rate of tax is 20% and the higher rate is 41%. She pays PRSI at the rate of 4% on all her income. She also pays USC at the rate of 2% on the first €10036 of her income, at 4% on the next €5980, and 7% on all income above this.
   (i) Calculate Yvonne's take home pay for the year and her monthly take home pay.
   (ii) She is offered the opportunity to earn an extra €6000 a year by working overtime for three nights a week. If she takes the offer, she will have to pay extra childcare costs of €220 a month. Do you think she should take the offer? Give reasons.
9. **Compound Interest**
   e.g. Stephen has a sum of money which he wants to invest for 10 years. He investigates bonds being offered by

Continued on page 18

Maths Paper 1 (Ordinary Level)

■ Continued from page 17

different financial institutions.
Institution A offers an annual equivalent rate (AER) of $4\cdot5\%$.
Institution B offers a ten year bond giving 50% after ten years.
Institution C offers a five year bond giving 25% after five years. It also offers the option of re-investing the money at the end of the five years for another five years at the same rate.
If Stephen wants to maximise his return at the end of ten years, determine which institution he should invest his money with.

10. Depreciation
e.g. A machine originally cost a manufacturing company €120000. Its value depreciates at the rate of 20% per annum. Find
(i) the value of the machine after four years,
(ii) the depreciation amounts in each of the first four years after being bought.

Sequences & Series

Sequences and series is another topic whose position in Paper 1 is difficult to predict. The concept might appear as a whole or part question in Section A, and possibly in Section B, although this is thought unlikely.

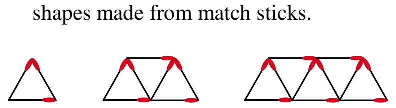
The idea of a sequence starts with recognising patterns, of symbols and numbers. Then it moves to a more algebraic approach. In practice, expressing a pattern as a formula involves using algebra.

The two main types of sequences and series on our course are arithmetic and geometric. We have to be able to recognise whether a given sequence is arithmetic, geometric or neither.

It is a bonus that most of the key formulae are now in the 'Formulae and Tables', page 22. These are:
(i) arithmetic sequences and series:
$$T_n = a+(n-1)d$$
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
(ii) geometric sequences and series:
$$T_n = ar^{n-1}$$

However, it is also vital to understand what a, d and r represent, and that S_n is given by
$$S_n = u_1+u_2+\ldots+u_n$$
for all series.

1. Patterns
e.g. The diagram below shows a pattern of shapes made from match sticks.



(i) Draw the next two shapes in the pattern. Describe by a rule how to construct the next shape.
(ii) Write down the sequence for the number of equilateral triangles present in each shape in the pattern. Describe this number sequence by a rule. Give a formula for this sequence.
(iii) Write down the sequence for the number of matches used in each shape. Describe this number sequence by a rule. Give a formula for this sequence, u_n, and find u_{86}.

2. Sequence notation
e.g. the nth term of a sequence is given by $T_n = 4n-1$.
(i) Find the first three terms of the sequence.
(ii) Show that $T_1+T_4 = T_2+T_3$.

3. Series
e.g. for the series $a+b+9+\ldots$, $S_1 = 2$ and $S_2 = 6$.
(i) Find the value of a and the value of b.
(ii) Find S_3.

4. Arithmetic sequences
e.g. the first two terms of an arithmetic sequence are 7, 10. Find
(i) a, the first term,
(ii) d, the common difference,
(iii) T_n, in terms of n,
(iv) the value of n if $T_n = 55$.

5. Arithmetic series
e.g. for the arithmetic series
$3+7+11+\ldots$
(i) Find a and d.
(ii) Express S_n in terms of n.
(iii) Hence find S_{12}.

6. Arithmetic problems
e.g. for an arithmetic series, $T_3 = 7$ and $S_5 = 35$. Find a, the first term, d, the common difference, T_n and S_n.

7. Proving a sequence is arithmetic
e.g. $T_n = 3n+5$ is a sequence.
(i) Express T_{n-1} in terms of n.
(ii) Hence prove that T_n is an arithmetic sequence.

8. Geometric sequences
e.g. The nth term of a geometric sequence is
$$T_n = 3^{n+1}.$$
(i) Find a and r.
(ii) Show that $T_3 > T_1+T_2$.

9. Geometric problems
e.g. find the values of x for which $x-2$, $x+2$, $4x+2$ are consecutive terms in a geometric sequence

10. Proving a sequence is geometric
e.g. $T_n = 4(2^n)$ is a sequence.
(i) Express T_{n-1} in terms of n.
(ii) Hence prove that T_n is a geometric sequence.

11. Investigating the nature of a sequence
e.g. The first three terms of a sequence are
5, 9, 12
Investigate if the sequence is arithmetic, geometric or neither.

Functions, Graphs & Differentiation

Functions and graphs involves using function notation, perhaps solving simple equations, drawing graphs and answering questions on graphs.

Differentiation involves finding the slope of a tangent to a graph, which can also be interpreted as the instantaneous rate of change of the graph.

Differentiation is a very powerful technique that can be used to help sketch curves, to find the maximum and minimum points on a curve and the maximum and minimum values of a speed and acceleration.

You should be aware that the standard derivative
$$\frac{d(x^n)}{dx} = nx^{n-1}$$
is given on page 25 of the *Formulae and Tables*.

1. Function notation
e.g. $f:\mathbb{R}\to\mathbb{R}: x\to 5x-3$.
(i) Find $f(-1)$.
(ii) Find the value of the real number k if $f(7) = kf(5)$.
(iii) Find x if $f(2x-1) = 72$.

2. Composite functions
e.g. $f: x\to x^2-2$ and $g: x\to 2x+1$ are two functions.
(i) Find $f(g(2))$ and $g(f(3))$.
(ii) Write $f(g(x))$ in terms of x.
(iii) Investigate if
$$g(f(x)) = f(g(x)).$$

2. Linear graphs
e.g. The temperature, C, in degrees Celsius, of a liquid in an insulated container is related to time, t, in hours, by
$C = 86-6t$.
(i) Draw the straight line graph of this relation, putting t on the horizontal axis, for $0\le t\le 8$.
(ii) Find the slope of this line, and express it as a rate of change.
(iii) Use your graph to estimate the temperature when $t = 5\cdot5$ hours.
(iv) Use your graph to estimate the time it takes for the temperature to fall from 80 degrees to 60 degrees.
(v) Describe a method of determining the answer to part (iv) by performing a quick mental calculation.

3. Quadratic graphs
e.g. Sketch a graph of
$f(x) = x^2-4x-3$, for $-1\le x\le 5$
(i) Use your graph to estimate the values of x for which $f(x)=0$.
(ii) Use algebra to check your answers.
(iii) Using the same axes and the same scales, sketch the function
$g(x) = x-6$.
(iv) Use your graph to estimate the values of $x\in\mathbb{R}$ for which
$f(x) = g(x)$.

4. Cubic graphs
e.g. sketch a graph of
$f(x) = x^3-4x^2+2x+6$,
for $-2\le x\le 4$

5. Exponential graphs
e.g. $f: x\to 2^x$
(i) Complete the following table

x	-3	-2	-1	0	1	2	3
$f(x)$							

(ii) Construct the graph $y = f(x)$ for $-3\le x\le 3$.
(iii) Use your graph to estimate the solution of the equation
$f(x) = 6$.

6. Questions on graphs
e.g. draw a graph of the function
$f(x) = x^3-2x^2-7x+4$,
in the domain $-3\le x\le 4$.
Use your graph to estimate the values of x for which
(i) $x^3-2x^2-7x = 0$,
(ii) $f(x)$ is increasing,
(iv) $x>0$ and $f(x)$ is decreasing

7. Differentiation
e.g. find $\frac{dy}{dx}$ if
$y = 2x^3+4x^2-7x+11$
Use the derivative to find the slope of the tangent to the curve at the point $(1,10)$.

8. Turning points
e.g. let $f(x) = x^3+6x^2-36x+10$.
(i) Find $f'(x)$, the derivative of $f(x)$.
(ii) Find the co-ordinates of the local maximum and local minimum points of the curve $y = f(x)$.
(iii) Hence sketch a rough graph of $y = f(x)$.

9. Rates of change
e.g. the external surface area, A cm^2, is given by
$$A = 2x^2+30x,$$
where x is one of the dimensions of the base. Find the rate of change of the surface area as x changes, when $x = 4$.

10. Velocity and acceleration
e.g. a car passes a point P as it slows down to rest. The distance, x metres, travelled by the car t seconds after passing P is given by
$$x = 18t-\frac{1}{2}t^2.$$
(i) Find the speed of the car as it passes P.
(ii) Find the time taken by the car to come to rest.
(iii) Find the distance travelled by the car in coming to rest.

P1: Sample Questions

1. Algebra

Question
The table below shows the cost of producing different weights of vegetables in a small factory.

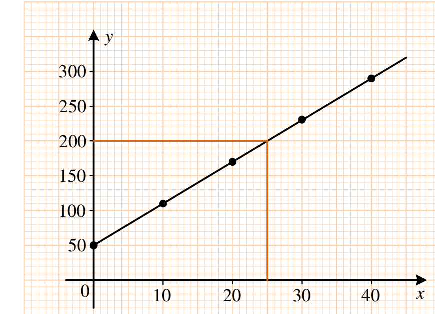
Weight (in kg)	Cost (in €)
0	50
10	110
20	170
30	230
40	290

(i) Verify that there is a linear relationship between the weight of vegetables produced and the cost.
(ii) Identify the independent and dependent variables.
(iii) If x represents the independent variable and y represents the dependent variable, what is the y-intercept?
(iv) Sketch a graph, showing the independent variable on the horizontal axis.
(v) Find the slope of the line and express it as a rate of change.
Is the line increasing or decreasing? How is this reflected in the slope?
(vi) Express the formula for the cost in words.
(vii) Describe three different ways of determining the weight of vegetables that could be produced for €200.

Solution
(i) The following table shows that the changes in the cost are constant.

Weight (kg)	Cost (in euro)	Change
0	50	
1	110	60
2	170	60
3	230	60
4	290	60

Thus there is a linear relationship between the weight of vegetables and the cost.
(ii) The independent variable is the weight and the dependent variable is the cost.
(iii) The y-intercept is 50, as this is the y value when $x = 0$.
(iv) The graph is shown below.

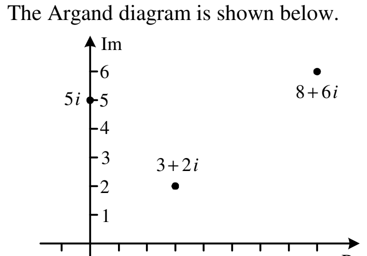


(v) Two points on the line are $(0,50)$ and $(10,110)$. Thus the slope is
$$m = \frac{110-50}{10-0} = \frac{60}{10} = 6$$
Thus the rate of change of the cost is €6 for every extra kilogram in weight.
The line is increasing, and thus the slope is positive.
(vi) The cost, in euro, can be found by multiplying the number of kilograms by 6 and adding 50.
(vii) Mental: subtract 50 from 200 and divide 150 by 6 to get 25 kg.
Graph: go across from 200 to the graph and down to 25 kg.
Algebra: the equation is $y = 50+6x$. If $y = 200$, then
$200 = 50+6x$
$150 = 6x$
$x = 25$ kg.

2. Complex Numbers

Question
(a) Represent on an Argand diagram:
$3+2i$, $5i$, $2i(3-4i)$,
where $i^2 = -1$.
(b) $z = 2+i$ and $w = 1+3i$.
(i) Write
$$\frac{z+w}{\bar{z}}$$
in the form $a+bi$. ($\bar{z}$ is the conjugate of z.)
(ii) Evaluate
$$|z+w|-|z|-|w|,$$
correct to one decimal place.
(c) (i) If $z = 4-i$ and $w = 2+3i$, find the values of the real numbers k and t if
$$kz = tw+8-9i.$$
(ii) Find the roots of the equation
$$z^2+10z+29 = 0.$$

Solution
(a) $3+2i = (3,2)$
$5i = 0+5i = (0,5)$
$2i(3-4i) = 6i-8i^2$
$= 6i-8(-1)$
$= 8+6i = (8,6)$
The Argand diagram is shown below.

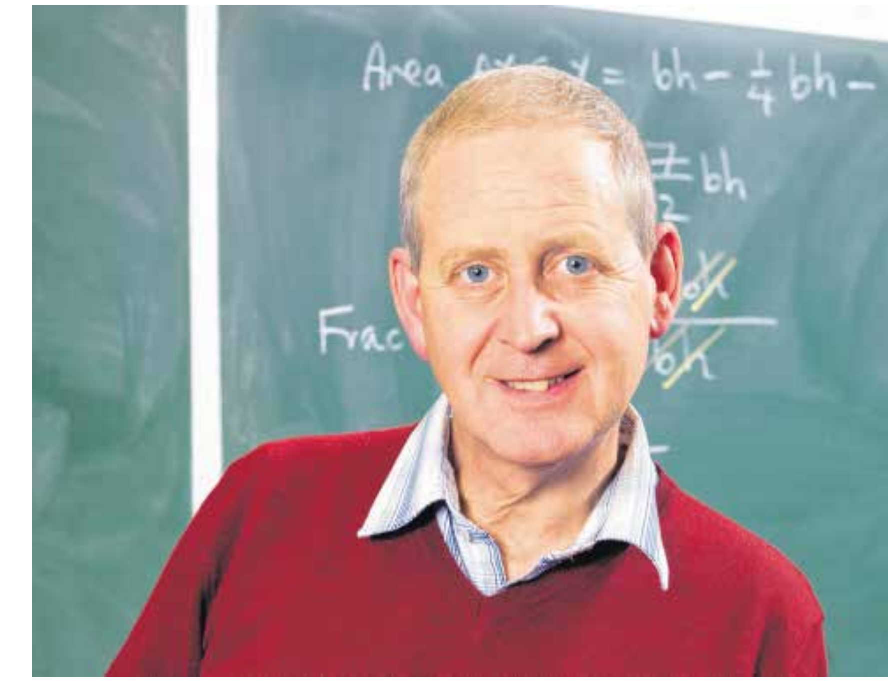


(b) (i)
$$\frac{z+w}{\bar{z}} = \frac{(2+i)+(1+3i)}{2-i}$$
$$= \frac{3+4i}{2-i}\times\frac{2+i}{2+i}$$
$$= \frac{3(2+i)+4i(2+i)}{2(2+i)-i(2+i)}$$
$$= \frac{6+3i+8i-4}{4+2i-2i+1}$$
$$= \frac{2+11i}{5} = \frac{2}{5}+\frac{11}{5}i$$
(ii) $|z| = |2+i|$
$= \sqrt{2^2+1^2} = \sqrt{5} = 2\cdot24$
$|w| = |1+3i|$
$= \sqrt{1^2+3^2} = \sqrt{10} = 3\cdot16$
$|z+w| = |3+4i|$
$= \sqrt{3^2+4^2} = \sqrt{25} = 5$
Thus
$|z+w|-|z|-|w|$
$= 5-2\cdot24-3\cdot16 = -0\cdot4$
(c) (i) $k(4-i) = t(2+3i)+8-9i$
$4k-ki = (2t+8)+(3t-9)i$
Thus
1. $4k = 2t+8$
$2k-t = 4$
2. $-k = 3t-9$
$-k-3t = -9$
Then
1: $2k-t = 4$
2×2: $-2k-6t = -18$
$-7t = -14$
Thus $t = 2$ and $k = 3$.
(ii) $z^2+10z+29 = 0$
$$z = \frac{-10\pm\sqrt{10^2-4(1)(29)}}{2}$$
$$z = \frac{-10\pm\sqrt{-16}}{2}$$
$$z = \frac{-10\pm 4i}{2}$$
$z = -5\pm 2i$
The solutions are $-5+2i$ and $-5-2i$.

3. Financial Maths

Question
(a) Eithne has a gross income of €84000 and an income after tax of €60212 for a certain year. The standard rate cut off point is €37200 and the standard and higher rates of tax are 20% and 41% respectively.
(i) Calculate her gross tax for the year.
(ii) What are Eithne's tax credits for the year?
(b) What sum of money, invested now at 3% per annum compound interest, will grow to €4919·50 in seven years? Give your answer correct to the nearest euro.
(c) The value of a car depreciates at the rate of 18% per annum. Express the value of the car after 5 years as a percentage of the original value.

Solution
(a) (i) Gross income: €84000
Standard rate:
€37200@20% = €7440
Higher rate:
€46800@41% = <u>€19188</u> (+)
Gross tax: = €26628
(ii) Gross income: = €84000
Net income: = <u>€60212</u> (−)
Net tax: = €23788



Then
Gross tax: = €26628
Net tax: = <u>€23788</u> (−)
Tax credits: = €2840
(b) $F = 4919\cdot50$ $P = ?$ $i = 0\cdot03$ $t = 7$
Then
$F = P(1+i)^t$
$4919\cdot50 = P(1\cdot03)^7$
$4919\cdot50 = P\times1\cdot229873865$
$$P = \frac{4919\cdot50}{1\cdot229873865}$$
$P = 4000$
The sum of money was €4000.
(c) $F = ?$ $P = P$ $i = 0\cdot18$ $t = 5$
Then
$F = P(1-i)^t$
$F = P(1-0\cdot18)^5$
$F = 0\cdot3707P$
and so F is $37\cdot07\%$ of P.

4. Functions, Graphs and Differentiation

Question
(a) $f:\mathbb{R}\to\mathbb{R}^+: x\to 3(2)^x$.
(i) Complete the table of values:

x	-1	0	1	2
$y = f(x)$				

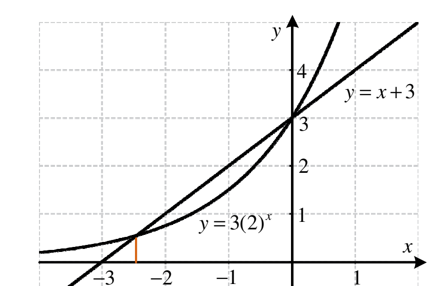
and hence construct a graph of the curve $y = f(x)$.
(ii) Using the same axes and the same scales, construct a graph of the function
$g: x\to x+3$.
(iii) Use your graph to find the solutions of the equation
$3(2)^x = x+3$.
(b) Let $f(x) = x^3-9x^2+15x+3$, $x\in\mathbb{R}$.
Find the co-ordinates of the local maximum and local minimum points of the curve $y = f(x)$.

Solution
(a) (i) $f(x) = 3(2)^x$
Table:

x	-2	-1	0	1	2
$y = f(x)$	$\frac{3}{4}$	$\frac{3}{2}$	3	6	12

■ Aidan Roantree, senior maths teacher at the Institute of Education. PHOTOGRAPH: BRENDAN DUFFY

The graph is constructed below.



(ii) $g(x) = x+3$
The linear graph $y = x+3$ contains the points $(-3,0)$ and $(0,3)$, and is shown above.
(iii) $3(2)^x = x+3$
$f(x) = g(x)$
The solutions are the x co-ordinates of the points of intersection of $y = f(x)$ and $y = g(x)$.
From the graph, the solutions are approximately $-2\cdot5$ and 0.
(b) Let $y = f(x) = x^3-9x^2+15x+3$.
$$\frac{dy}{dx} = 3x^2-18x+15$$
Put $\frac{dy}{dx} = 0$:
$3x^2-18x+15 = 0$
$x^2-6x+5 = 0$
$(x-1)(x-5) = 0$
$x-1 = 0$ or $x-5 = 0$
$x = 1$ or $x = 5$
$x = 1$: $y = 1-9+15+3 = 10$
$(1,10)$
$x = 5$: $y = 125-225+75+3 = -22$
$(5,-22)$
Then $\frac{d^2y}{dx^2} = 6x-18$
$x = 1$: $\frac{d^2y}{dx^2} = -12 < 0$
Thus $(1,10)$ is a local max.
$x = 5$: $\frac{d^2y}{dx^2} = 12 > 0$
Thus $(5,-22)$ is a local min.

Exam Times

Maths Ordinary Level

All in the method

Paper 2

Take advantage of the past two years of exam papers for this part of the exam, noting the topics asked and question style

By now, we have two year's experience of similar exams, and a further year when the exam was largely the same. You should study these papers closely, looking at the topics that were asked and the way questions were asked. Although you cannot expect to get exactly the same this year, you should be able to learn some things to help you with the exam you will be facing.

You should also be aware that there are a couple of changes in the offing for this year's exam. First of all, the course on probability and statistics has been extended to include material that was originally deferred. This material has been examined before on the Higher Level papers for the past two years. Looking at these questions will provide some insight to what you can expect.

Secondly, the choice of question in Question 6 on geometry, i.e. the choice between 6A and 6B, is now gone. This was also only for the interim period. From now on, we will have to answer all questions on the paper, and we have no guarantee about where a question on geometry will occur.

Strand 1, on probability and statistics, and Strand 2, on geometry, co-ordinate geometry and trigonometry will form the backbone of Paper 2. However, areas and volumes, from Strand 3, will also be examined on the paper.

As with Paper 1, Paper 2 will contain two sections, both of which you must attempt. Section A is called 'Concepts and Skills' and it is important to recognise that both will be examined in this section. The 'Skills' will ask you to show that you know the techniques that you have learned in school: solve equations, use formulae, make calculations, etc.

The 'Concepts' refers to questions where you have to show that you understand what you are doing. This is designed to reward students who can show that they have a grasp of the methods they are using. Students who rely on memory alone are going to find such questions difficult.

An example of a concepts question is being asked to match a number of lines on a supplied diagram with their equations, which examines an understanding of slope and the *y*-intercept. Here you would not have the opportunity to use a formula, and would not be able to tackle the question unless you fully understood the concept of slope in particular.

Another example of a concept question is where you are given a number of histograms, without data or markings, and asked to state which has the greatest standard deviation. Again only a proper understanding of standard deviation will enable you to answer.

Although we cannot be sure what questions will cover what topics, from the past papers and sample papers, it is likely that the first two questions will deal with probability and statistics, with Question 1 on some aspect of probability and counting techniques.

For questions 3 to 5, it is fairly definite that at least one of them will deal with co-ordinate geometry, with possibly two questions on the topic. The remaining question, or questions, will probably involve areas and volumes, or trigonometry. In any case, you cannot afford to omit any of these topics.

Section B will also account for 150 marks, half the total for Paper 2. As with Paper 1, all we know is that the number of questions will lie between two and four. The number of marks for each individual question in this section is impossible to predict: you will have to wait until you open the paper next June.

However, on the basis of the likely split of marks between the strands, it is probable that half the marks in Section B will cover probability and statistics, with statistics due to dominate. Also, if trigonometry does not appear in Section A, then it will be prominent in Section B.

Section B is called 'Contexts and Applications' which accurately reflects the idea that the questions will involve real life situations to which you will have to apply the maths you have learned. For the questions in this section there is only one thing you can rely on: they will be unlike anything you have seen before.

This means that one of the greatest difficulties you will have with these questions lies in deciding what is the best approach. It goes without saying that you should attempt each part of these long questions, even if you think you haven't got a clue. The very worst that can happen is that you are completely wrong, and will get no marks from that part.

You can also expect to see a few question parts asking for reasons or an opinion, in line with the spirit of the new course. These parts are not likely to be worth many marks, but answering them correctly could make the difference between one grade and another.

In preparation for Paper 2 you should naturally master all the techniques and concepts for the topics on the paper. You should also examine the past papers, for all schools in 2012, 2013 and 2014, and for the Pilot Schools for the two years before that. However, you should only use these papers as a guide to question types and lengths, and not as a plan for the paper. Many changes have occurred.

If these papers frighten you, go to the examinations.ie website and look at the marking schemes. You will be reassured by how many marks were obtained for making relatively minor progress with questions. Another reassuring fact to bear in mind is that the difficulties you are experiencing with novel questions will be shared by the vast majority of students. Thus the marking will be adjusted to take these difficulties into account. If you make every effort, you will be surprised how well you will be rewarded.

Probability

As stated above, there is likely to be at least one question in Section A on probability, and probability will perhaps appear as part in one or more of the questions in Section B.

You should ensure that you fully understand the concept of probability of an event as a number between 0 and 1, with more likely events having a probability nearer to one. It is also necessary to understand how to count arrangements, what equally likely events are, how to recognise mutually exclusive events, how to use and interpret Venn diagrams when tackling probability questions and how to set up a tree diagram as a method of calculation.

There are certain formulae that you must be able to use. Unfortunately, these are not contained in the *Formulae and Tables*: you will have to remember them.

* For equally likely events,
$$P(E)=\frac{\#(E)}{\#(S)},$$
where S is the sample space and E is the event.

* For mutually exclusive events,
$$P(A \text{ or } B)=P(A)+P(B)$$

* For independent events:
$$P(A \text{ and } B)=P(A).P(B)$$

It is also important to be aware that many probability questions can be tackled in a number of different ways.

Calculating the expected value of a random variable is also on the course, as is calculating probability in Bernoulli trials.

1. **Fundamental principle of counting**
   e.g. a code consists of two letters followed by two different digits, e.g. TP04.
   (i) How many such codes are possible?
   (ii) How many of these codes contain the letter X?
2. **Arrangements (permutations)**
   e.g. The letters of the word CHEMISTRY are all arranged at random.
   (i) How many arrangements are possible?
   (ii) How many of these arrangements contain both the letter groups HEM and TRY each in that order?
3. **Concept of probability**
   e.g. Four cards are chosen from a standard pack of cards. We want to find the probability of getting exactly three '7's.
   (i) Viewing the sample space as the number of '7's we can get, list the outcomes in the sample space.
   (ii) Are these outcomes equally likely? Explain.
4. **Listing outcomes**
   e.g. Five discs each have one of the letters S, P, A, D, E written on them. The discs are placed in a bag. One disc is drawn out and not replaced. Another disc is then drawn out of the bag.
   (i) List all the possible outcomes in the form (S, P).
   (ii) Find the probability that the letter D is one of those picked.
   (iii) Find the probability that at least one of the letters D and E are picked.
5. **Equally likely outcomes**
   e.g. Two different letters are picked at random from the 26 letters of the alphabet. What is the probability that we get two vowels (A, E, I, O and U)?
6. **Probability and set theory**
   e.g. A and B are two events for an experiment.
   $P(A)=\frac{3}{9}$ and $P(A\cup B)=\frac{5}{6}$.
   If A and B are mutually exclusive, calculate $P(B)$.
7. **Multiplication law**
   e.g. A bag contains 6 blue discs and 7 white discs. One disc is picked from the bag, its colour noted and returned to the bag. A second disc is then chosen. What is the probability that both discs are blue?
8. **Success and failure**
   e.g. A spinner has four equal sides, coloured black, green, blue and yellow. Three people each spin the spinner once and the colour each gets is recorded. What is the probability that at least one person gets a green?
9. **Independent events**
   e.g. Let A and B be events such
   $P(A)=\frac{3}{5}$ and $P(A \text{ and } B)=\frac{1}{15}$.
   If A and B are independent, find $P(B)$.
10. **Expected value**
    e.g. Mary and Paul play a game. A die is thrown. If the result is a 1, a 3 or a 5, Mary gives Paul €2, €5 or €8 respectively. If the result is a 2, a 4 or a 6, Paul gives Mary €1, €4 or €9.
    (i) If the game is played once, calculate Mary's expected value.
    (ii) Is this a fair game? Give a reason.
11. **Bernoulli trials**
    e.g. when a die is thrown, we consider getting a 1 or a 2 a success. The die is thrown three times.
    (i) What is the probability of getting exactly two successes?
    (ii) What is the probability of getting the first success on the third throw?

Statistics

As far as our course is concerned, statistics concerns the scientific gathering of data, representing the collected data graphically and then analysing the data so that conclusions can be drawn.

There are many terms in statistics whose definitions and meaning must be known. These include categorical and numerical data, discrete and continuous data, population, sample, random sample, observational study and designed experiment.

There are also concepts that you have to understand and be able to explain, such as that for a sample to be a good approximation of the population from which it is drawn, it should be a random sample. You also have to be aware of the essential features of an observational study and a designed experiment, and when each type is appropriate.

Once the data has been gathered, or given to us in the exam, we can be asked to represent it graphically, by means of pie charts, line plots, bar charts, histograms and stemplots, including back to back stemplots.

After that, you have to be able to calculate the middle of the data by using the mean, the median and the mode. You also must be able to describe the spread of the data by identifying the range, interquartile range and the standard deviation.

You also have to know how to interpret given graphs (histograms and other graphs) in terms of skewness and the position of the mean.

Next, you have to be able to use and interpret scatterplots for determining the relationship, if any, between two variables. This extends to understanding the concept of correlation, and the ability to interpret the correlation coefficient, *r*, from supplied graphs.

You also have to be aware of the idea of the normal distribution and how to use the empirical rule.
Finally, you also have to be able to calculate the margin of error for a population proportion, and use it to conduct a hypothesis test on the population proportion.

1. **Data types**
   e.g. A number of registered voters are polled and asked to select which candidate they intend to vote for in an upcoming election. They are also asked for their age in years.
   (i) What type of data is the candidate that they intend to vote for?
   (ii) What type of data is their age in years?
2. **Populations and samples**
   e.g. Helen wants to determine the average amount spent on mobile phones per year by all the people in Ireland.
   (i) Describe the population and the population parameter.
   (iii) She then uses the data from her sample to draw conclusions about the mobile phone expenses of the entire population of Ireland. What type of statistics is this?
3. **Questionnaires and questions**
   e.g. Describe what is unsuitable about each of the following questions. Suggest what improvements could be made.
   (i) What is your opinion of our new hi-speed connection?
   (ii) The government appears to have addressed the current financial crisis very effectively. Do you agree?
   (iii) How would you rate the service of our mechanics?
   Excellent ☐
   Very good ☐
   Above average ☐
   Average ☐
   Below average ☐
   (iv) How often do you take a go to your doctor?
4. **Study types**
   e.g. A study in 2009 concluded that people with a thigh measurement of more than $46\cdot5$ have a smaller risk of heart problems or an early death than those with a thigh measurement less than this (yes really!). Suppose you wish to conduct your own study to see whether or not this is valid.
   (i) Why is a survey or a controlled experiment not appropriate? Give reasons.
   (ii) What type of observational study would you suggest? Give reasons.
5. **Representation of data**
   e.g. Customers who bought new cars at two car salesrooms, A and B, were queried as to the amount of the loan they had taken out to buy their new car. The results, to the nearest €1000, are given in the list below.

A:	8	25	33	48	22	17	56
	9	15	44	35	52		
B:	12	7	6	10	15	22	9
	15	14	23	8	13		

   (i) Draw a back to back stem plot to represent this data.
   (ii) One of these car salesrooms specialised in luxury cars. Can you guess which one? Give a reason.
6. **Central tendency and spread**
   e.g. In a class test, the percentage marks of a number of students are given below.

45	68	65	72	39	77	56
64	89	44	67	72	68	48
61	90	78	52	82	43	45
70	59	81	41	42	88	79.

   (i) Use a stemplot to arrange the data in ascending order.
   (ii) State the median of the data.
   (iii) Calculate the interquartile range.
   (iv) If Sean says that he is in the top 15% of the class, what is the least score that he could have achieved?
7. **Shape of distributions**
   e.g. Which of the following distributions do you think would be (a) symmetric, (b) skewed to the right, (c) skewed to the left? Give a reason for your answer in each case.
   (i) The ages of all the people in Ireland.
   (ii) The prices of all new cars sold in Ireland last year.
   (iii) The number of fingers on the left hand of everybody in the world.
   (iv) The heights of all the students in your class.
8. **Scatterplots and correlation**
   e.g. The results achieved by a number of students in their fifth year exams in English and history are shown below.

English	History
80	75
90	80
65	55
75	80
55	65
95	85
100	85
45	50
85	80
70	85

   (i) Represent this information on a scatterplot.
   (ii) Estimate the correlation coefficient giving reasons for your choice.
9. **Empirical rule**
   e.g. The times spent playing computer games per week by 500 students is normally distributed with a mean of 10 hours and a standard deviation of 2 hours. Use the empirical rule to
   (i) find the percentage of students who spent between 8 and 12 hours,
   (ii) estimate the number of students who spend between 6 and 12 hours.
10. **Population proportion, margin of error and hypothesis testing**
    e.g. The manufacturers of a new treatment claim that it cures hiccups within 20 seconds for 90% of people. An independent body decides to check this claim. The new treatment is tested on a random sample of 4000 hiccups sufferers, and cures the hiccups of 3540 of these.
    (i) State the hypothesis that should be made by the independent body.
    (ii) On what basis should the independent body reject or not reject the hypothesis?
    (iii) Determine if the independent body should have concern about the claim made by the manufacturers.

Synthetic Geometry

As mentioned above, we no longer have any guarantee about where questions on geometry are going to occur in this paper. There is likely to be at least one question on geometry in Section A, possibly two.

From the syllabus, we have to be able to use 10 theorems and 1 corollary to solve problems. Also, there are 6 constructions on the course that you will need to practise.

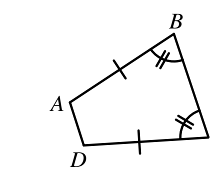
In addition, there are six terms from the language of proof: theorem, proof, axiom, corollary, converse, implies, that you need to understand exactly. You may be asked to explain one or more of these terms, and possibly give an example.

Geometry may also be a topic in one of the questions in Section B. Even if geometry does not appear to be examined directly, it is often required in questions involving trigonometry.

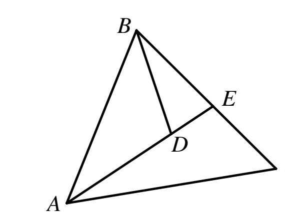
With this in mind, you have to practise tackling geometry problems, often referred to as 'cuts'. Learn to identify whether you are dealing with triangles, parallelograms or circles. Notice identical angles, parallel lines, lines parallel to the base of a triangle, right angled triangles, etc, and think about what the theorems you have learned say. Write down as many true statements as you can, realising that you are piling up marks as you do.

An idea connected to Theorems 12 and 13 is that of enlargements. You must be able to construct the image of a set of points under an enlargement, locate the centre of enlargement, calculate the scale factor, and lengths and areas of objects and their images.

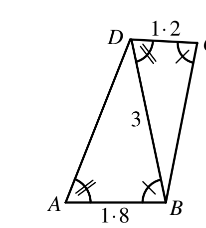
1. **Language of proof**
   e.g. Explain what is meant by an 'axiom'. Give an example of an axiom.
2. **Congruent triangles**
   e.g. $ABCD$ is a quadrilateral (four sided figure) in which $|AB|=|CD|$ and $|\angle ABC|=|\angle BCD|$.



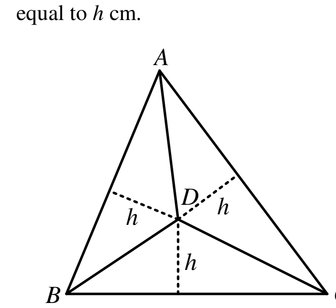
   By considering the triangles ABC and BCD, show that $|AC|=|BD|$.
3. **Other triangle theorems**
   e.g. Let D be any point on the inside of the triangle ABC. Let AD meet BC at E between B and C.



   (i) Prove that $|\angle ADB|>|\angle AEB|$.
   (ii) Prove that $|\angle ADB|>|\angle ACB|$.
4. **Quadrilaterals and parallelograms**
   e.g. One angle between the diagonals of a rectangle is $72°$. Find the angle between a diagonal and one of the longer sides of the rectangle.
5. **Ratio theorems**
   e.g. $ABCD$ is a quadrilateral with $|\angle ABD|=|\angle BCD|$, $|\angle DAB|=|\angle CDB|$, $|AB|=1\cdot8$ cm, $|DB|=3$ cm and $|DC|=1\cdot2$ cm.



   (i) Explain why the triangles ABD and BCD are similar.
   (ii) Calculate $|BC|$.
   (iii) Calculate $|AD|$.
6. **Area theorems**
   e.g. In the diagram, the heights of the three triangles ABD, BCD and ACD are each equal to h cm.



Continued on page 22

Maths Paper 2 (Ordinary Level)

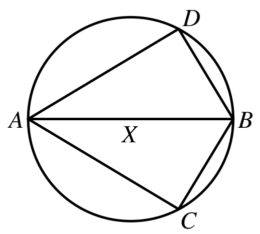
Continued from page 21

Given that the area of ΔABC is 5 cm^2,

and the perimeter of ΔABC is $\frac{25}{2}$ cm,

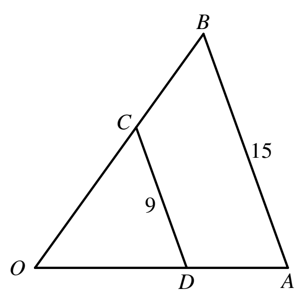
find the value of h.

7. **Circle theorems**
   e.g. $[AB]$ is a diameter of a circle which also contains the points C and D. The centre of the circle is X. $|BC|=|BD|$.



   (i) Prove that the triangles ABC and ABD are congruent.
   (ii) Deduce that $|AC|=|AD|$.
   (iii) Explain why BC and BD are tangents to a circle with centre A containing the points C and D.

8. **Enlargements**
   e.g. The triangle ODC is the image of the triangle OAB under an enlargement, centre O.
   $|CD|=9$ and $|AB|=15$.



   (i) Find the scale factor of the enlargement.
   (ii) If the area of the triangle OAB is $87\cdot5$ square units, find the area of the triangle ODC.
   (iii) Write down the area of the region $ABCD$.

9. **Constructions**
   e.g. (i) Construct the triangle ABC such that $|\angle ABC|=70°$, $|\angle ACB|=50°$ and $|BC|=8$ cm.
   (ii) Show how to construct the incentre of this triangle.

Areas and Volumes

Areas and volumes may be examined in a dedicated question, or may form an element of a question on trigonometry or geometry. You will need to be able to calculate plane areas, including a trapezium and a sector. The trapezoidal rule, as a way of approximating a plane area, should also be studied.

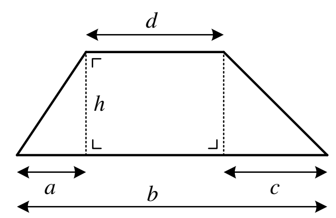
It will also be necessary to calculate volumes and surface areas of solid objects such as cylinders, cones and spheres. Being able to construct and interpret nets of certain solid objects is also important.

Many of the formulae we require are in the *Formulae and Tables*.

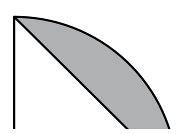
It will also be necessary to calculate volumes and surface areas of solid objects such as cylinders, cones and spheres. Being able to construct and interpret nets of certain solid objects is also important.

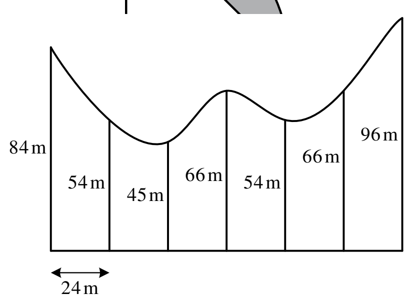
Many of the formulae we require are in the *Formulae and Tables*.

1. **Areas**
   e.g. in the diagram below, show that the area of the trapezium shown is
   $$\frac{1}{2}(b+d)h.$$



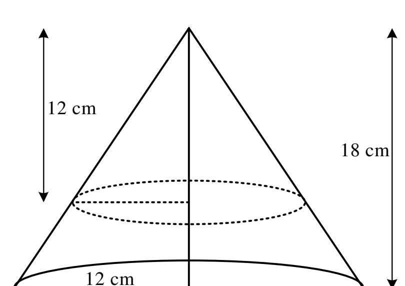
2. **Arcs and sectors**
   e.g. A right-angled triangle lies inside a sector as shown.



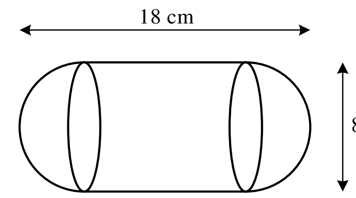


   At equal intervals of 24 m perpendicular measurements of 84 m, 54 m, 45 m, 66 m, 54 m, 66 m and 96 m are made to the stream.
   Use the trapezoidal rule to estimate the area of the field.

4. **Prisms**
   e.g. the triangular base of a prism has area 24 cm^2 and the height of the prism is 15 cm. Calculate the volume of the prism.
5. **Spheres and hemispheres**
   e.g. a sphere has diameter 18 cm. Express its volume in terms of π.
6. **Cylinders**
   e.g. calculate, in terms of π, the volume of a cylinder which has a base of radius 12 cm and a height of 18 cm
7. **Cones**



   e.g. a cone has a height of 18 cm and a radius of 12 cm. Express the volume of the cone in terms of π.
   A smaller cone of height 12 cm is cut from the top of the solid cone. Find the volume of the smaller cone.
8. **Compound volumes**
   e.g. a solid consists of a cylinder with a hemisphere at each end, as shown.
   The length of the solid is 18 cm and its height is 8 cm.



   (i) Find the radius of the hemispheres and the cylinder.
   (ii) Find the length of the cylinder.
   (iii) Find the volume of the solid, in terms of π.
9. **Forming equations**
   e.g. the volume of a sphere is $2304\pi\text{ cm}^3$. Find the radius of the sphere, and the volume of the smallest cube that the sphere fits into.
10. **Equal volumes**
   e.g. find, in terms of π, the volume of a hemisphere of radius length 12 cm.
   This hemisphere is fully submerged in water in a cylindrical container.
   If, as a result, the water level rises by 4 cm, find the radius of the cylinder, correct to two decimal places.

Trigonometry

Whatever about Section A, i.e. the short questions, it is highly likely that trigonometry will form a major part of at least one of the long questions in Section B. This question is likely to be a real life situation where trigonometry can be used.

If a question on trigonometry does occur in Section A, it is probable that it will be a theoretical or understanding based question, e.g. showing an understanding of the unit circle.

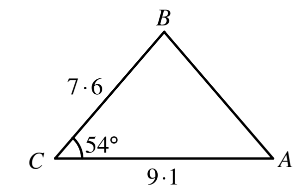
It is important to note that many of the formulae you will require are present in the *Formulae and Tables*, on pages 9 to 16. You should become familiar with these pages of the tables, and which formulae you may require.

Of course, you must completely master using your calculator to work out trig functions, e.g. $\sin 57°$, and the inverse trig functions, e.g. to find the angle A such that $\cos A = 0\cdot45$.

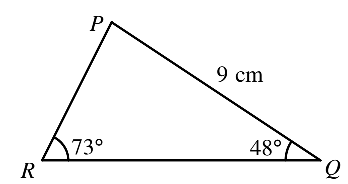
For right angled triangles, you will need to be able to use the definitions of the trig ratios and Pythagoras' theorem. For general triangles, learn how to use the Sine Rule, the Cosine Rule and the Area of a Triangle formula.

In more complicated questions, where there is more than one triangle, you will have to learn how to decide which rule to use and how to proceed.

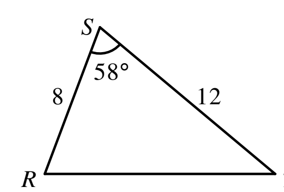
1. **Using a calculator**
   e.g. use your calculator to find
   (i) $\sin 37°$
   (ii) the angle $0° < A < 90°$, to the nearest degree, if $\cos A = 0\cdot6157$
2. **Connected ratios**
   e.g. if $\tan A = \frac{8}{15}$, for $0° \le A \le 90°$, express $\sin A$ in the form $\frac{p}{q}$, where $p, q \in \mathbb{Z}$
3. **Area of a triangle**
   e.g. in the triangle ABC, $|BC|=7\cdot6$, $|AC|=9\cdot1$ and $|\angle ACB|=54°$.



   Find the area of the triangle ABC, correct to one decimal place.
4. **Sine Rule**
   e.g. in the triangle PQR, $|PQ|=9$ cm, $|\angle PRQ|=73°$ and $|\angle PQR|=48°$.



   Find $|PR|$, correct to one decimal place.
5. **Cosine Rule**
   e.g. in the triangle RST, $|RS|=8$, $|ST|=12$ and $|\angle RST|=58°$. Find $|RT|$, correct to one decimal place.



6. **Solving triangles**
   e.g. in the triangle RST, $|RS|=7\cdot6$, $|ST|=12\cdot9$ and $|\angle SRT|=82°$. Find:
   (i) $|\angle STR|$, correct to the nearest degree,
   (ii) $|RT|$, correct to one decimal place.
7. **Arcs and sectors**
   e.g. a disc has radius of length 12 cm. A sector of this disc has area $24\pi\text{ cm}^2$.
   Find the measure of the angle at the centre of the disc, and the length of the arc of the sector.
8. **Trig functions**
   e.g. if
   $$\cos 115° = -\cos A,$$
   where $0° < A < 90°$, find the value of A. Use the unit circle to explain your reasoning.

Co-ordinate Geometry

Co-ordinate geometry will probably be examined in one, perhaps two, of the short questions in Section A. It may also appear in one of the long questions in Section B.

A. The Line

In co-ordinate geometry of the line, it is important to remember all the methods and formulae from Junior Cert. You should have revised all these throughout your Leaving Cert studies, but it is important not to underestimate these. Basic formulae such as the distanc between two points, the midpoint of a lin segment and especially the slope of a line, wi undoubtedly be required in some form. You ar also likely to be asked to demonstrat understanding of the concept of the slope of line.

Another important topic is the equation of line. You must know how to interpret th equation of a line, and not just find the equatio given a point and a slope. This means that yo have to know how to find different things fro the equation of a line, e.g. the slope of the lin which points lie on the line, where the lin crosses the x-axis and where it crosses the y axis.

The formula for the area of a triangle is new t Leaving Cert. You need to be able to find th area of a triangle using the formula in *Formul and Tables*, and be able to translate a triangle s that this formula can be used.

1. **Distance and midpoint**
   e.g. find the co-ordinates of M, the midpoint of $[AB]$, if $A=(3,-4)$ and $B=(9,2)$. Verify that $|AM|=|MB|$.
2. **Idea of slope**
   e.g. The line l intersects the x-axis on the right of the origin and intersects the y-axis above the origin. State, giving a reason, whether the slope of l is positive, zero or negative.
3. **Slope**
   e.g. $P=(1,-3)$ and $Q=(3,1)$. Find the slope of the line PQ and investigate if PQ is perpendicular to the line with equation $x+2y=9$.
4. **Plotting lines**
   e.g. plot the line $3x-2y=12$ and verify that the line contains the point $(16,18)$
5. **Equation of a line**
   e.g. find the equation of the line AB if $A=(-5,1)$ and $B=(-1,7)$
6. **Connected lines**
   e.g. find the equation of the line through $A=(-2,5)$ which is perpendicular to the line $3x+y=11$
6. **Translations**
   e.g. $A=(-3,4)$, $B=(-1,2)$, $C=(2,7)$.
   Find the co-ordinates of D if $ABCD$ is a parallelogram.
7. **Area of a triangle**
   e.g. find the area of the triangle with vertices $(-4,3)$, $(2,-1)$, $(5,-2)$.

B. The Circle

In co-ordinate geometry of the circle, the most important thing is to be able to find the equation of a circle. This means identifying the centre, in co-ordinate form, and being able to determine the radius of the circle. Once this has been done, the equation can be written down using the formula
$$x^2+y^2=r^2$$
for circles centre $(0,0)$, and the formula
$$(x-h)^2+(y-k)^2=r^2$$
for circles with other centres.

You also have to know how to find out whether a point lies inside, on or outside a circle. In addition, using algebra to find the points of intersection of a given circle, with centre $(0,0)$, and a given line is essential study.

1. **The equation** $x^2+y^2=r^2$
   e.g. find the equation of the circle with centre $(0,0)$ and which contains the point $(5,-2)$

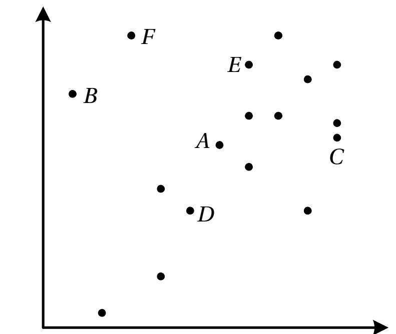
The Institute of Education, Lower Leeson Street. PHOTOGRAPH: ALAN BETSON

2. **The equation** $(x-h)^2+(y-k)^2=r^2$
   e.g. find the equation of the circle which has $[PQ]$ as a diameter, where $P=(-3,6)$ and $Q=(5,-2)$
3. **Point on a circle**
   e.g. the point $(4,k)$ belongs to the circle
   $$(x-1)^2+(y+4)^2=25.$$
   Find the values of the real number k.
4. **Finding the centre and the radius**
   e.g. find the co-ordinates of the centre and the length of the radius of the circles
   (i) $x^2+y^2=17$
   (ii) $(x+2)^2+(y-3)^2=53$
5. **Position of a point relative to a circle**
   e.g. investigate if the point $(-1,-4)$ is outside the circle
   $$(x-3)^2+(y+2)^2=17$$
6. **Different circles**
   e.g. c is a circle with centre $(0,3)$ and radius length 5.
   (i) Find the equation of c.
   (ii) c intersects the x-axis at the points A and B. Find the co-ordinates of A and B, if A lies to the right of the origin.
   (iii) s is a circle with radius 2 which intersects c at a single point. Find the equation of one possible circle s.
   (iv) k is a circle with centre A which intersects c at a single point. Find the equation of k.
7. **Intersection of a line and a circle**
   e.g. find the co-ordinates of the points of intersection of the line $x+2y=5$ and the circle $x^2+y^2=13$.

Paper 2: Sample Questions

1. Statistics

Question
The scatterplot shown below represents bivariate data. Some of the points have been given the names A, B, C, D, E and F.

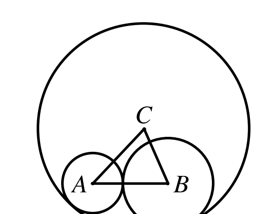


(i) Is there a positive or negative correlation between the variables represented on the two axes? Give a reason.
(ii) Would you say that the correlation is strong, moderate or weak? Give a reason.
(iii) The correlation coefficient, r, is one of the following values:
$0\cdot88,\ 0\cdot75,\ 0\cdot34,\ 0\cdot02,\ -0\cdot48,\ -0\cdot79$
Which do you think is the correct value of r? Give a reason.
(iv) If two of the named points are removed, the value of r, the correlation coefficient, changes to $0\cdot72$. Which two named points are removed?

Solution
(i) There is positive correlation between the variables. This is because the trend in the points is generally upwards.
(ii) There appears to be moderate correlation, as there are quite a few points a long way off the line through the middle that best fits the data.
(iii) $r=0\cdot34$, as this reflects moderate positive correlation.
(iv) B and F.

2. Synthetic Geometry

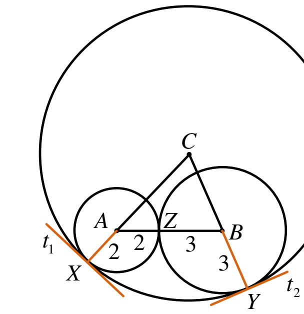
Question
A circle with centre A and radius 2 touches externally a circle with centre B and radius 3. Another circle with centre C and radius 7 touches the first two circles internally.



Show that the triangle ABC is isosceles and find the lengths of its sides.

Solution
Let t_1 be the tangent to the circle centre C at the point X. Thus t_1 is perpendicular to CX. As t_1 is also a tangent to the circle centre A at the point X, XC contains the point A.

Likewise, if t_2 is the tangent to the circle centre C at the point Y, then it is also a tangent to the circle centre B at the point Y.



Then $|AB|=2+3=5$

and $|AC|=|CX|-|AX|$
$=7-2=5$

As $|AB|=|AC|$, the triangle ABC is isosceles.

Also, $|CB|=|CY|-|BY|$
$=7-3=4.$

3. Co-ordinate Geometry

Question
(a) Determine, by calculation, if the point $(3,-3)$ is inside the circle $x^2+y^2=20$.
(b) $A(0,3)$, $B(4,-3)$ and $C(-2,-7)$ are the vertices of a triangle.
  (i) Verify that AB is perpendicular to BC.
  (ii) The circle s contains all three points A, B and C. Find the centre of s.
  (iii) Find the equation of the circle s.

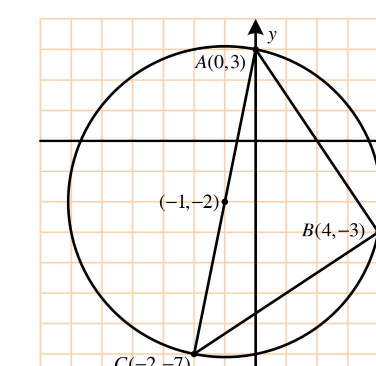
Solution
(a) $x^2+y^2=20$
$(3,-3):\ (3)^2+(-3)^2 \quad 20$
$9+9 \quad 20$
$18<20$
Thus $(3,-3)$ is inside the circle.

(b) **(i)** Let m_1 be the slope of AB, where $A=(0,3)$ and $B=(4,-3)$.
$$m_1=\frac{-3-3}{4-0}=\frac{-6}{4}=\frac{-3}{2}$$
Let m_2 be the slope of BC, where $B=(4,-3)$ and $C=(-2,-7)$.
$$m_2=\frac{-7+3}{-2-4}=\frac{-4}{-6}=\frac{2}{3}$$
As $m_1.m_2=\left(\frac{-3}{2}\right)\left(\frac{2}{3}\right)=-1,$

$AB \perp BC$.

(ii) The centre of s is the midpoint of $[AC]$. (This is because of the Junior Cert theorem you studied which states that an angle subtended by a diameter at the circumference is a right angle.)

Thus
$$(h,k)=\left(\frac{0-2}{2},\frac{3-7}{2}\right)$$
$$=(-1,-2)$$



Radius of s is the distance from $(-1,-2)$ to $A(0,3)$.
$$r=\sqrt{(0+1)^2+(3+2)^2}$$
$$=\sqrt{1^2+5^2}$$
$$=\sqrt{26}$$

Thus the equation of s is
$$(x-h)^2+(y-k)^2=r^2$$
$$(x+1)^2+(y+2)^2=26.$$

Exam Times

ACHIEVE YOUR POTENTIAL IN A SCHOOL FOCUSED ON YOU

Pictured: Some of the students from the Institute who were awarded the UCD Entrance Scholarship 2014/2015.

In 2014 The Institute of Education sent more students to universities and colleges than any other school in Ireland; 152 of our students achieved a place in UCD, 119 in Trinity College Dublin and 53 in DCU.

Learn more about our unique approach to preparing for the Leaving Certificate, meet some of our teachers and discuss school life with current and past pupils at one of our upcoming Information Evenings:

5th Year: Tuesday, 20 January 2015 at 6.00 PM in the school
6th Year: Tuesday, 10 February 2015 at 6.00 PM in the school
4th Year: Tuesday 24 February 2015 at 6.00 PM in the school

For more information visit **instituteofeducation.ie**, call **01 661 3511** or email **info@instituteofeducation.ie**.

The Institute of Education
79-85 Lower Leeson Street, Dublin 2

46 Years of Academic Excellence