

Differentiation	
1st Principles - quadratic	
<p>Differentiate <math>f(x) = 2x^2 - 3x + 2</math> from 1st principles w.r.t. x</p> $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	

- ①  $f(x) = 2x^2 - 3x + 2$
- ②  $f(x+h) = 2(x+h)^2 - 3(x+h) + 2$   
 $= 2[x^2 + 2xh + h^2] - 3x - 3h + 2$   
 $= 2x^2 + 4xh + 2h^2 - 3x - 3h + 2$
- ③  $\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2 - 3h}{h}$   
 $= 4x + 2h - 3$
- ④  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 4x + 2(0) - 3$
- ⑤  $f'(x) = 4x - 3$

## Differentiation

$f(x)$	$f'(x)$
$x^n$	$nx^{n-1}$
$\ln x$	$\frac{1}{x}$
$e^x$	$e^x$
$e^{ax}$	$ae^{ax}$
$a^x$	$a^x \ln a$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\cos^{-1} \frac{x}{a}$	$-\frac{1}{\sqrt{a^2 - x^2}}$
$\sin^{-1} \frac{x}{a}$	$\frac{1}{\sqrt{a^2 - x^2}}$
$\tan^{-1} \frac{x}{a}$	$\frac{a}{a^2 + x^2}$

## Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

## Differentiate

- (i)  $y = (2x+3x^2)^2$   
 $\frac{dy}{dx} = 2(2x+3x^2)(2+6x)$
- (ii)  $y = \cos^2(2x+3) = (\cos(2x+3))^2$   
 $\frac{dy}{dx} = 2(\cos(2x+3))(-\sin(2x+3))(2)$   
 $= -4 \cos(2x+3)\sin(2x+3)$
- (iii)  $y = e^{2x^2}$   
 $\frac{dy}{dx} = (4x)e^{2x^2}$
- (iv)  $y = x \sin x$   
 $\frac{dy}{dx} = x(\cos x) + \sin x$   
 $= x \cos x + \sin x$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

(v)  $y = \frac{\ln 2x}{x^2}$

$u = \ln 2x$	$v = x^2$
$\frac{du}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$	$\frac{dv}{dx} = 2x$

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$$\frac{dy}{dx} = \frac{x^2 \left( \frac{1}{x} \right) - \ln 2x (2x)}{x^4}$$

$$\frac{dy}{dx} = \frac{1/x - 2x \ln 2x}{x^4} = \frac{1 - 2 \ln 2x}{x^3}$$

(vi)  $y = \sin^{-1} (2x) = \sin^{-1} \left( \frac{x}{\frac{1}{2}} \right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - x^2}} = \frac{1}{\sqrt{\frac{1}{4} - x^2}}$$

OR  
(vi)  $y = \sin^{-1} (2x) = \sin^{-1} \left( \frac{2x}{1} \right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1^2 - (2x)^2}} \cdot (2) = \frac{2}{\sqrt{1 - 4x^2}}$$

$$y = \ln(4x)$$

$$\ln x \rightarrow \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{4x} \cdot (4) = \frac{1}{x}$$

$f(x) \rightarrow f'(x)$	$y = \tan^{-1} x^2 = \tan^{-1} \left( \frac{x^2}{1} \right)$
$\tan^{-1} \frac{x}{a} \rightarrow \frac{a}{a^2 + x^2}$	$\begin{aligned}\frac{dy}{dx} &= \frac{1}{1^2 + (x^2)^2} \cdot (2x) \\ &= \frac{2x}{1 + x^4}\end{aligned}$