(b) Let $f(x)=\dfrac{2x}{x+2}$, $x\neq -2$, $x\in\mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y=f(x)$ is $\dfrac{1}{4}$.

And sketch this function.

Slope $= f'(x)$

Quotient Rule: $\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$f'(x) = \frac{(x+2)(2)-(2x)(1)}{(x+2)^2}$$

$$= \frac{2x+4-2x}{(x+2)^2} = \frac{4}{(x+2)^2}$$

If $f'(x)=\frac{1}{4}$ $\Rightarrow$ $\dfrac{4}{(x+2)^2} = \dfrac{1}{4}$ $\Rightarrow$ $16=(x+2)^2$

Find pts.?

$\pm 4 = x+2$ $\Rightarrow$ $x = 4-2 = 2$ or $x = -4-2 = -6$

$x=2$ $\Rightarrow$ $f(2) = \dfrac{2(2)}{2+2} = \dfrac{4}{4} = 1$ pt $(2,1)$

$x=-6$ $\Rightarrow$ $f(-6) = \dfrac{2(-6)}{-2-6} = \dfrac{-12}{-4} = +3$ pt $(-6,3)$

Sketch

$$f(x) = \frac{2x}{x+2}$$

Horizontal Assymptote

ie.. x value when denominator = 0

If $x=-2$ then $f(x)$ doesn't exist

Assymptote : $x=-2$

Vertical Assymptote

Values of function when x is very large or very small

$$f(-999) = \frac{2(-999)}{(-999)+2} \approx 2$$

$$f(999) = \frac{2(999)}{(999)+2} \approx 2$$

Assymptote : $y=2$

Pt. of inflection?

ie.. When $f''(x)=0$ what is x and y value

$$f'(x) = \frac{4}{(x+2)^2} = 4(x+2)^{-2}$$

(don't need quotient rule!)

$$f''(x) = -8(x+2)^{-3}(1) = \frac{-8}{(x+2)^3}$$

If $f''(x)=0$ $\Rightarrow$ $\dfrac{-8}{(x+2)^3} = 0$ not possible! $\Rightarrow$ no pt. of inflection

Sketch