Chapter 5 Financial Maths

Exercise 5.1

Q**1**. $P = €3000$

$t = 10$ years

$r = 3\% = 0.03$

$$\begin{aligned}\text{Future value} &= €P(1+i)^t \\ &= €3000(1+0.03)^{10} \\ &= €4031.75\end{aligned}$$

Q**2**. $P = €5000$

$t = 8$ years

$r = 2.5\% = 0.025$

$$\begin{aligned}\text{Future value} &= €P(1+i)^t \\ &= €5000(1+0.025)^8 \\ &= €6092.014 \\ &= €6092.01\end{aligned}$$

$$\begin{aligned}\text{Interest paid} &= \text{Future value} - \text{sum invested } (€P) \\ &= €6092.01 - €5000 \\ &= €1092.01\end{aligned}$$

Q**3**. $(1+r)^{12} = 1+i$

$$1+r = (1+i)^{\frac{1}{12}}$$

$$r = (1+i)^{\frac{1}{12}} - 1$$

Q**4**. (i) $i = 6\% = 0.06 = \text{annual rate}$

$r = \text{monthly rate}$

$$\begin{aligned}\Rightarrow (1+r)^{12} &= 1+0.06 = 1.06 \\ 1+r &= (1.06)^{\frac{1}{12}} \\ r &= (1.06)^{\frac{1}{12}} - 1 = 0.004868 \\ &= 0.49\%\end{aligned}$$

(ii) $2.5\% = 0.025 = \text{annual rate}$

$r = \text{monthly rate}$

$$\begin{aligned}\Rightarrow (1+r)^{12} &= 1+0.025 = 1.025 \\ 1+r &= 1.025^{\frac{1}{12}} \\ r &= 1.025^{\frac{1}{12}} - 1 \\ &= 0.0020598 \\ &= 0.21\%\end{aligned}$$

(iii) $4\% = 0.04 =$ annual rate

$r =$ monthly rate

$$\Rightarrow (1+r)^{12} = 1+0.04 = 1.04$$

$$1+r = 1.04^{\frac{1}{12}}$$

$$r = 1.04^{\frac{1}{12}} - 1$$

$$= 0.0032737$$

$$= 0.33\%$$

Q**5**.

$P = €4500$

$\text{F.V.} = €5607.82$

$t = 5$ years

$$\Rightarrow \quad \text{F.V.} = P(1+i)^t$$

$$€5607.82 = €4500(1+i)^5$$

$$\Rightarrow \quad (1+i)^5 = \frac{5607.82}{4500} = 1.24618$$

$$1+i = (1.24618)^{\frac{1}{5}}$$

$$i = (1.24618)^{\frac{1}{5}} - 1$$

$$= 0.045$$

$$= 4.5\%$$

Q**6**.

$P = €15,000$

$r = 3.5\% = 0.035$

$I_1 = 15,000 \cdot 0.035 = 525$

$I_2 = 15,525 \cdot 0.035 = 543.83$

$I_3 = 16,068.38 \cdot 0.035 = 562.39$

$I_4 = 16,630.77 \cdot 0.035 = 582.07$

$I_5 = 17,212.85 \cdot 0.035 = 602.45$

Year	Principal	Interest
1	15,000	525
2	15,525	543.83
3	16,068.38	562.39
4	16,630.77	582.07
5	17,212.85	602.45

Q**7**.

$i = 4\% = 0.04$

$$\Rightarrow (1+r)^2 = 1+i$$

$$(1+r)^2 = 1+0.04 = 1.04$$

$$1+r = (1.04)^{\frac{1}{2}}$$

$$r = (1.04)^{\frac{1}{2}} - 1$$

$$= 0.0198 = 1.98\%$$

Q**8**.

$P = €6500$

$r = 1.932\% = 0.01932$

$t = 6$ years 4 months

$= 76$ months

$$\text{F.V.} = P(1+r)^t$$
$$= €6500(1+0.01932)^{76}$$
$$= €27{,}830.10$$

Q**9**.

$P = €12,000$

$i = 3.5\% = 0.035$

(i) $t = 5$ years 3 months

$= 63$ months

$$\therefore (1+r)^{12} = 1+i$$
$$(1+r)^{12} = 1+0.035 = 1.035$$
$$r = (1.035)^{\frac{1}{12}} - 1$$
$$= 0.0028709$$
$$= 0.28709\%$$
$$\text{F.V.} = P(1+r)^t$$
$$\text{F.V.} = 12{,}000(1+0.0028709)^{63}$$
$$= €14{,}375.34$$

(ii) 8 years 2 months

$= 98$ months

$$\text{F.V.} = 12{,}000(1.0028709)^{98}$$
$$= €15{,}892.57$$

(iii) 10 years 6 months

$= 126$ months

$$\text{F.V.} = 12{,}000(1.0028709)^{126}$$
$$= €17{,}220.86$$

Q**10**.

$i = 4.2\% = 0.042$

$\text{F.V.} = €10,000$

$t = 10$ years

$$\text{Present value} = \frac{\text{Future value}}{(1+i)^t}$$
$$= \frac{€10{,}000}{(1+0.042)^{10}}$$
$$= €6,627.09$$

Q**11**. $\text{F.V.} = €25,000$

$t = 9$ years

$i = 4.5\% = 0.045$

$$\text{Present value} = \frac{\text{F.V.}}{(1+i)^t}$$

$$= \frac{25,000}{(1+0.045)^9}$$

$$= €16,822.61$$

Q**12**. $P = €50,000$

$\text{F.V.} = €100,000$

$i = 3.5\% = 0.035$

$$\text{F.V.} = P(1+i)^t$$

$$\Rightarrow €100,000 = €50,000(1+0.035)^t$$

$$\Rightarrow \quad 2 = (1.035)^t$$

$$\Rightarrow \quad \log 2 = t \log 1.035$$

$$t = \frac{\log 2}{\log 1.035}$$

$$= 20.15 \text{ years}$$

Q**13**. $P = €175,000$

$i = 4.5\% = 0.045$

$t = 20$ years

$$\text{F.V.} = P(1+i)^t$$

$$\text{F.V.} = 175,000(1+0.045)^{20}$$

$$= €422,049.95$$

Q**14**. $P = €1130$

$\text{F.V.} = €3000$

$i = 5\% = 0.05$

(a)

$$\text{F.V.} = P(1+i)^t$$

$$3000 = 1130(1+0.05)^t$$

$$(1.05)^t = 2.6549$$

when $t = 20$ $\quad (1.05)^{20} = 2.6533$

$t = 20.1$ $\quad (1.05)^{20.1} = 2.6663$

$t = 20.05$ $\quad (1.05)^{20.05} = 2.6597$

$\Rightarrow \quad t = 20$ years

(b)

$$(1.05)^t = 2.6549$$
$$\Rightarrow \quad t\log(1.05) = \log 2.6549$$
$$t = \frac{\log 2.6549}{\log(1.05)}$$
$$= 20.0124$$
$$\therefore \quad t = 20 \text{ years.}$$

Q**15**. $i = 6\% = 0.06$

$r =$ monthly rate

(i)

$$\Rightarrow (1+r)^{12} = 1+i$$
$$(1+r)^{12} = 1+0.06$$
$$1+r = (1.06)^{\frac{1}{12}}$$
$$r = (1.06)^{\frac{1}{12}} - 1$$
$$= 0.0048676$$
$$= 0.48676\%$$
$$= 0.4868\%$$

(ii)

$$r = \frac{i}{12} = \frac{0.06}{12}$$
$$= 0.005$$
$$= 0.5\%$$

$t = 3$ years

$= 36$ months

$\therefore$ F.V. $= 10,000(1+0.005)^{36}$ F.V. $= 10,000(1.00486)^{36}$

$= €11,966.81$ $= €11,910.35$

$\Rightarrow$ difference $= €56.46$

(iii) $r = 0.0048676$

Correct to 3 places of decimals $= 0.005$

Correct to 4 places of decimals $= 0.0049$

$\therefore$ A minimum of 4 places of decimals are needed to create a difference.

Q**16**. $P = €15,000$

$i = 3\% = 0.03$

$t = 2$ years

$$\text{F.V.} = €P(1+i)^t$$
$$= 15,000(1+0.03)^2$$
$$= €15,913.50$$

Withdraws €2000 $\Rightarrow$ $P = €13,913.50$

$t = 3$ years F.V. $= 13,913.5(1.03)^3$

$= €15,206.66$

Exercise 5.2

Q**1**. Present value $(P) = €30,000$

depreciation $(i) = 15\% = 0.15$

(i) $t = 5$ years

$$\text{F.V.} = P(1-i)^t$$
$$= €30,000(1-0.15)^5$$
$$= €13,311.16$$

(ii) $t = 10$ years

$$\text{F.V.} = €30,000(1-0.15)^{10}$$
$$= €5,906.23$$

Q**2**. $P = €1,400$

$i = 8\% = 0.08$ per month

$t = 15$ months

$$\text{F.V.} = P(1-i)^t$$
$$= 1,400(1-0.08)^{15}$$
$$= €400.82$$

Q**3**. $P = €44,000$

$i = 20\% = 0.2$ for first year

$i = 15\% = 0.15$ after first year.

(i) 3 years $\Rightarrow$

$$\text{F.V.} = P(1-i)^t$$
$$= €44,000(1-0.2)^1$$
$$= €35,200 \quad \text{after 1 year}$$

$\Rightarrow$

$$\text{F.V.} = €35,200(1-0.15)^2$$
$$= €25,432$$

(ii) 6 years $\Rightarrow$

$$\text{F.V.} = €35,200(1-0.15)^5$$
$$= €15,618.43$$

Q**4**. $P = €140,000$

$i = 20\% = 0.2$

$t = 4$ years

(a)(i) $\Rightarrow$

$$\text{F.V.} = P(1-i)^t$$
$$= €140,000(1-0.2)^4$$
$$= €57,344$$

(ii) $P = €25,000$

$i = 3.5\% = 0.035$

$$\text{F.V.} = P(1+i)^t$$
$$= 25,000(1+0.035)^4$$
$$= €28,688.08$$

(b) Inflation 2% per annum

(i)

$$\text{F.V.} = 140,000(1+0.02)^4$$
$$= €151,540.50$$

(ii) Funds shortfall $= €151,540.50 - €(28,688.08 + 57,344)$

$= €65,508.42$

Q**5**.

$P = €175,000$

$\text{F.V.} = €73,187.09$

$i = 16\% = 0.16$

$$\text{F.V.} = P(1-i)^t$$

$$\Rightarrow 73,187.09 = 175,000(1-0.16)^t$$

$$\Rightarrow \quad 0.41821 = (0.84)^t$$

(i) at $t = 7$: $(0.84)^7 = 0.29509$

$t = 6$: $(0.84)^6 = 0.35129$

$t = 5$: $(0.84)^5 = 0.41821$

$\Rightarrow \quad t = 5$ years.

(ii)

$$\log(0.41821) = t\log(0.84)$$

$$t = \frac{\log(0.41821)}{\log 0.84}$$

$t = 5$ years.

Q**6**.

$\text{P.V.} = 60,000$ kg

$i = 15\% = 0.15$ per month

$t =$ end of Jan 2004

beginning of April 2005

$= 14$ months

$$\text{F.V.} = \text{P.V.}(1-i)^t$$

$$\text{F.V.} = 60,000(1-0.15)^{14}$$

$$= 6,166 \text{ kg}$$

Q**7**.

$\text{P.V.} = €180,000$

$\text{F.V.} = €80,000$

$t = 10$ years

$$\text{F.V.} = \text{P.V.}(1-i)^t$$

$$€8\cancel{0},\cancel{000} = 18\cancel{0},\cancel{000}(1-i)^{10}$$

(i)

$$\Rightarrow \quad 0.44\dot{4} = (1-i)^{10}$$

$$1-i = (0.44\dot{4})^{\frac{1}{10}}$$

$$1-i = 0.9221$$

$$\Rightarrow \quad i = 0.0778$$

$$= 7.8\%$$

(ii) $\text{F.V.} = €60,000 \Rightarrow 6\cancel{0},\cancel{000} = 18\cancel{0},\cancel{000}(1-0.0778)^t$

$$0.3\dot{3} = (0.922)^t$$

$$\Rightarrow \log(0.3\dot{3}) = t\log(0.922)$$

$$\Rightarrow \quad t = \frac{\log(0.3\dot{3})}{\log(0.922)}$$

$$= 13.53 \text{ years}$$

Q**8**. $P = €2500$

(a) €550 per year

(b) $35\% = 0.35$ per year

$t = 4$ years

(a) €550 per year for 4 years = €2200

$\Rightarrow \quad \text{F.V.} = €2500 - €2200 = €300$

(b) $\text{F.V.} = \text{P.V.}(1-i)^t$

$\text{F.V.} = 2500(1-0.35)^4$

$\text{F.V.} = €446.27$

Q**9**. $\text{P.V.} = €23,500$

$i = 28\% = 0.28$

(i) $t = 2$ years $\quad \text{F.V.} = \text{P.V.}(1-i)^t$

$= 23,500(1-0.28)^2$

$= €12,182.40$

(ii) $t = 5$ years $\quad \text{F.V.} = 23,500(1-0.28)^5$

$= €4,547.06$

(iii) $t = 7$ years $\quad \text{F.V.} = 23,500(1-0.28)^7$

$= €2,357.19$

Q**10**. $\text{P.V.} = €8000$

$\text{F.V.} = €1$

$t = 20$ years

(i) $\Rightarrow \quad \text{F.V.} = \text{P.V.}(1-i)^t$

$$1 = 8000(1-i)^{20}$$

$$\Rightarrow \left(\frac{1}{8000}\right)^{\frac{1}{20}} = (1-i)$$

$$i = 0.3619$$

$$i = 36.2\%$$

(ii) For $\text{F.V.} = 0 \Rightarrow (1-i)^t = 0$

$\therefore$ once i is not equal to 1, i.e. 100%.

$(1-i)^t \neq 0$ for all values of t.

(iii) $\text{slope} = \dfrac{-8000}{5} = €1,600 \text{ per year}$

(iv) $t = 4.2$ years: $\text{F.V.} = €1,300$

(v) $t = 5$ years : $\text{F.V.} = 8000(1-0.362)^5$

$= €845.66$

(vi) Reducing balance method.
Once a system is still operating, it has value, no matter what its age.

Eercise 5.3

Q**1**. $P = €20$

$i = 0.5\% = 0.005$ per month

$t = 36$ months

Future value $= 20(1.005)^1 + 20(1.005)^2 + \cdots 20(1.005)^{36}$

instalment left in for 1 month

instalment left in for 36 months

$$\therefore \qquad S_n = \frac{a(1-r^n)}{1-r} \qquad a = 20(1.005)$$

$r = 1.005$

$n = 36$

$$= 20(1.005)\frac{(1-(1.005)^{36})}{(1-(1.005))}$$

$= €790.66$

Total interest $= €790.66 - (36 \times €20)$

$= €70.66.$

Q**2**. $P = €30.00$ per month

$i = 4\%$ per annum

$= 0.04$

(i)

$$(1+r)^{12} = 1+i$$

$$(1+r)^{12} = 1+0.04$$

$$1+r = (1.04)^{\frac{1}{12}}$$

$$r = (1.04)^{\frac{1}{12}} - 1 = 0.00327$$

$$= 0.33\%.$$

(ii) 18 years $\rightarrow$ 21 years

$= 36$ months

$= 30(1.0033) + 30(1.0033)^2 + \cdots 30(1.0033)^{36}$

$$\Rightarrow \qquad \text{F.V.} = 30(1.0033)\left[\frac{1-(1.0033)^{36}}{1-1.0033}\right]$$

$= €1148.55$

Q**3**. $P = €2000$

$i = 4\% = 0.04$

$t = 5$ years

$$\text{F.V.} = 2000(1.04) + 2000(1.04)^2 + \cdots 2000(1.04)^5$$

$$= 2000(1.04)\left[\frac{1-(1.04)^5}{1-(1.04)}\right]$$

$= €11,265.95$

Q**4**. n payments of €P

i %

$$\text{F.V.} = P(1+i)^1 + P(1+i)^2 + \cdots P(1+i)^n$$

i.e. the sum of a geometric series with $a = P(1+i),\ \ r = (1+i)$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$= P(1+i)\left[\frac{1-(1+i)^n}{1-(1+i)}\right]$$

$$= P(1+i)\left[\frac{1-(1-i)^n}{-i}\right]$$

$$= P(1+i)\left[\frac{(1-i)^n-1}{i}\right]$$

Q**5**. n payments of €P

i %

$$\text{P.V.} = \frac{€P}{1+i} + \frac{P}{(1+i)^2} + \cdots \frac{P}{(1+i)^n}$$

i.e. the sum of a geometric series with $a = \left(\frac{P}{1+i}\right),\ r = \left(\frac{1}{1+i}\right)$

$$S_n = a\left[\frac{1-r^n}{1-r}\right]$$

$$\text{P.V.} = \frac{P}{(1+i)}\left[\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right]$$

$$= \frac{P}{(1+i)}\left[\frac{1-\frac{1}{(1+i)^n}}{\frac{\cancel{1}+i-\cancel{1}}{1+i}}\right]$$

$$= \frac{P}{(\cancel{1+i})}\left(\frac{\cancel{1+i}}{i}\right)\left[\frac{(1+i)^n-1}{(1+i)^n}\right]$$

$$= \frac{P}{(1+i)^n}\left[\frac{(1+i)^n-1}{i}\right]$$

Q**6**. F.V. = €6523.23

$i = 9\% = 0.09$

$t = 5$ years

(i) $$\text{F.V.} = A(1.09) + A(1.09)^2 + \cdots A(1.09)^5$$

(ii) $$€6523.23 = a\left(\frac{1-r^n}{1-r}\right)$$

$$= A(1.09)\left(\frac{1-(1.09)^5}{1-1.09}\right)$$

$$€6523.23 = A(6.52323)$$

$$\Rightarrow \quad A = €1000$$

Q**7**. $P = €200$

$t = 2$ years $= 24$ months

$i = 9\%$ per annum

$= 0.09$

$$(1+r)^{12} = 1+i$$

$$r = (1+i)^{\frac{1}{12}} - 1$$

$$= (1+0.09)^{\frac{1}{12}} - 1$$

$$= 0.007207$$

$$\text{F.V.} = \text{P.V.}(1+r)\left[\frac{(1+r)^n - 1}{r}\right]$$

$$= 200(1.007207)\left[\frac{(1.007207)^{24} - 1}{0.007207}\right]$$

$$= €5257.31$$

Q**8**. $\text{F.V.} = €10,000$

$t = 7$ years

$i = 8.5\% = 0.085$

$P = ?$

$$\text{F.V.} = P(1.085) + P(1.085)^2 + \cdots P(1.085)^7$$

$$\therefore €10,000 = P(1.085)\left[\frac{1-(1.085)^7}{1-(1.085)}\right]$$

$$10,000 = P(9.8306)$$

$$\Rightarrow \quad P = €1017.23$$

Q**9**. F.V. = €5000

$t = 3$ years $= 12$ quarters

$i = 7.2\% = 0.072$

$P = ?$

$$(1+r)^4 = 1+i$$
$$(1+r)^4 = 1.072$$
$$r = (1.072)^{\frac{1}{4}} - 1$$
$$= 0.01753$$

$$\text{F.V.} = €5000 = P(1.01753) + P(1.01753)^2 + \cdots P(1.01753)^{12}$$
$$= P(1.01753)\left[\frac{1-(1.01753)^{12}}{1-(1.01753)}\right]$$
$$= P(13.4592)$$
$$\Rightarrow \quad P = €371.49$$

Q**10**. Let €P be the instalment at the <u>start</u> of each period at i%.

$$\Rightarrow \text{Present value} = P + \frac{P}{1.0i} + \frac{P}{(1.0i)^2} + \frac{P}{(1.0i)^3} + \cdots \frac{P}{(1.0i)^n}$$
$$= P\left(\frac{1-\left(\frac{1}{1.0i}\right)^n}{1-\frac{1}{1.0i}}\right) = P\left[\frac{\frac{(1.0i)^n-1}{(1.0i)^n}}{\frac{1.0i-1}{1.0i}}\right]$$
$$= P\left[\frac{1.0i^n-1}{(1.0i)^n}\right]\cdot\frac{1.0i}{0.0i}$$
$$= \frac{P}{(1.0i)^{n-1}}\left[\frac{1.0i^n-1}{0.0i}\right]$$

$$\text{Future value} = P(1.0i) + P(1.0i)^2 + \cdots P(1.0i)^n$$
$$= P(1.0i)\left[\frac{1-(1.0i)^n}{1-1.0i}\right]$$
$$= P(1.0i)\left[\frac{1-(1.0i)^n}{-0.0i}\right]$$
$$= P(1.0i)\left[\frac{(1.0i)^n-1}{0.0i}\right]$$

$$\Rightarrow \quad \frac{\text{Future value}}{(1+i)^n} = \frac{P(1.0i)}{(1.0i)^n}\left(\frac{1.0i^n-1}{0.0i}\right)$$
$$= \frac{P}{(1.0i)^{n-1}}\left(\frac{1.0i^n-1}{0.0i}\right) = \text{P.V.}$$

Q**11**. $P = €3000$

$t = 6$ years

$i = 8\% = 0.08,$

assuming payment is made at the start of each year.

(i) $$\text{P.V.} = 3000 + \frac{3000}{1.08} + \frac{3000}{(1.08)^2} + \cdots \frac{3000}{(1.08)^6}$$

$$= 3000\left[\frac{1-\left(\frac{1}{1.08}\right)^6}{1-\frac{1}{1.08}}\right]$$

$$= €14,978.13$$

(ii) $$\text{F.V.} = 3000(1.08) + 3000(1.08)^2 + \cdots 3000(1.08)^6$$

$$= 3000(1.08)\left[\frac{1-(1.08)^6}{1-1.08}\right]$$

$$= €23,768.41$$

(iii) $$A = P(1+i)^n$$

$$= €14,978(1+0.08)^6$$

$$= €23,768.41$$

Exercise 5.4

Q**1**. Mortgate €200,000

$t = 30$ years $= 360$ months

$i = 6\% = 0.06$ per year

$$\begin{aligned}(1+r)^{12} &= 1+i = 1.06\\ 1+r &= (1.06)^{\frac{1}{12}}\\ r &= (1.06)^{\frac{1}{12}} - 1\\ &= 0.004867\end{aligned}$$

$$\begin{aligned}\text{Payment} &= \frac{M(i)(1+i)^n}{(1+i)^n - 1}\\ &= \frac{200,000(0.004867)(1.004867)^{36}}{(1.004867)^{36} - 1}\\ &= €1178.66\end{aligned}$$

Q**2**. $t = 20$ years $= 240$ months

$i = 8\% = 0.08$ per year

repayment = €850 per month

$$\begin{aligned}(1+r)^{12} &= 1+i = 1.08\\ 1+r &= (1.08)^{\frac{1}{12}}\\ r &= (1.08)^{\frac{1}{12}} - 1\\ &= 0.006434\end{aligned}$$

$$\begin{aligned}\text{Payment} &= \frac{M(i)(1+i)^n}{(1+i)^n - 1}\\ €850 &= \frac{M(0.006434)(1.006434)^{240}}{(1.006434)^{240} - 1}\\ €850 &= M(0.0081914)\\ \Rightarrow \quad M &= €103,766\\ &= €103,800\end{aligned}$$

Q**3**. €75,000

$i = 8\% = 0.08$

(a) $t = 20$ years

$= 240$ months

$$(1+r)^{12} = 1+i = 1.08$$

$$1+r = (1.08)^{\frac{1}{12}}$$

$$r = (1.08)^{\frac{1}{12}} - 1$$

$$= 0.006434$$

$$P = \frac{75,000(0.006434)(1.006434)^{240}}{(1.006434)^{240} - 1}$$

$$= €614$$

$$I = (€614 \times 240) - €75,000 = €72,360$$

(b) $t = 25$ years

$= 25 \times 12 = 300$ months

$$P = \frac{75,000(0.006434)(1.006434)^{300}}{(1.006434)^{300} - 1}$$

$$= €565$$

$$I = (€565 \times 300) - €75,000 = €94,500$$

(c) $t = 30$ years

$= 30 \times 12 = 360$ months

$$P = \frac{75,000(0.006434)(1.006434)^{360}}{(1.006434)^{360} - 1}$$

$$= €536$$

$$I = (€536 \times 360) - €75,000 = €117,960$$

Q**4**. $M = €15,000$

Plan A 10% discount, 9% for 5 years.

$$10\% \text{ discount} = 15000 \times (0.1) = €1,500$$

$$\Rightarrow M = €13,500$$

$$\text{Repayments} = \frac{M(i)(1+i)^t}{(1+i)^t - 1}$$

$$= \frac{13,500(0.09)(1.09)^5}{(1.09)^5 - 1}$$

$$= €3469 \text{ per year}$$

Plan B 3% for 5 years.

$$\text{Repayments} = \frac{15,000(0.03)(1.03)^5}{(1.03)^5 - 1}$$
$$= €3275 \text{ per year}$$

$\therefore$ *Plan B* is better

Q**5**. Fund $= €250,000$

$t = 25$ years

$i = 5\% = 0.05$

$$\text{Payment} = \frac{M(i)(1+i)^t}{(1+i)^t - 1}$$
$$= \frac{250,000(0.05)(1.05)^{25}}{(1.05)^{25} - 1}$$
$$= €17,738.11$$

Q**6**. Option 1: €200,000 invested for 25 years at 5%

$$A = P(1+i)^t$$
$$= 200,000(1+0.05)^{25}$$
$$= €677,270$$

Option 2:

$$A = 15,000(1.05) + 15,000(1.05)^2 + \cdots 15,000(1.05)^{25}$$
$$A = 15,000(1.05)\left[\frac{1-(1.05)^{25}}{1-1.05}\right]$$
$$= €751,701.81$$

Option 2 is better.

Q7. $i = 6.6\% = 0.066$

$t = 3$ years $= 36$ months

Payment $= €400$

$$(1+r)^{12} = 1+i = 1.066$$
$$1+r = (1.066)^{\frac{1}{12}}$$
$$r = (1.066)^{\frac{1}{12}} - 1$$
$$= 0.005340$$

$$\therefore \quad \text{Payment} = \frac{\text{Mortgage}(i)(1+i)^t}{(1+i)^t - 1}$$
$$400 = \frac{M(0.00534)(1.00534)^{36}}{(1.00534)^{36} - 1}$$
$$= M(0.030607)$$
$$\Rightarrow \qquad M = €13,068.78$$

Revision Exercise (Core)

Q1. Instalment = €1000

$i = 8\% = 0.08$

$t = 5$ years

$$\begin{aligned}
\text{F.V.} &= €1000(1.08) + 1000(1.08)^2 + \cdots 1000(1.08)^5 \\
&= €1000(1.08)\left[\frac{1-(1.08)^5}{1-(1.08)}\right] \\
&= 1000(1.08)\left[\frac{1.08^5 - 1}{0.08}\right] \\
&= €6335.93
\end{aligned}$$

Q2. Instalment = €300

$i = 6\% = 0.06$ per year

$t = 8$ years $= 96$ months

$$\begin{aligned}
(1+r)^{12} &= 1 + i = 1.06 \\
1 + r &= (1.06)^{\frac{1}{12}} \\
r &= (1.06)^{\frac{1}{12}} - 1 \\
&= 0.004867
\end{aligned}$$

$$\begin{aligned}
\text{F.V.} &= 300(1.004867)\left[\frac{(1.004867)^{96} - 1}{0.004867}\right] \\
&= €36,778.58
\end{aligned}$$

Q3. Car loan = €20,000

$i = 2\% = 0.02$

$t = 25$ instalments

$$\begin{aligned}
\text{Payment} &= \frac{M(i)(1+i)^t}{(1+i)^t - 1} \\
&= \frac{€20,000(0.02)(1.02)^{25}}{(1.02)^{25} - 1} \\
&= €1024
\end{aligned}$$

Q**4**. $t = 3$ years

$= 36$ months

Payment = €600

$M = ?$

$i = 4\% = 0.04$

$$(1+r)^{12} = 1+i = 1.04$$
$$1+r = (1.04)^{\frac{1}{12}}$$
$$r = (1.04)^{\frac{1}{12}} - 1$$
$$= 0.003274$$

$$P = \frac{M(i)(1+i)^t}{(1+i)^t - 1}$$
$$€600 = \frac{M(0.003274)(1.003274)^{36}}{(1.003274)^{36} - 1}$$
$$€600 = M(0.0294923)$$
$$M = €20,344.29$$

Q**5**. Introductory: 1.25% = 0.0125 per month

$$(1+r)^{12} = 1+i \qquad r\% \text{ per month}$$
$$i\% \text{ per year}$$

$$\Rightarrow \quad 1+i = (1+0.0125)^{12}$$
$$i = (1.0125)^{12} - 1$$
$$= 0.1607$$
$$= 16.07\%$$
$$= 16.1\%$$

Regular: $2.5\% = 0.025$ per month

$$1+i = (1.025)^{12}$$
$$i = (1.025)^{12} - 1$$
$$= 34.5\%$$

Q**6**. Savings $= 200(1.0075) + 200(1.0075)^2 + 200(1.0075)^3$
$+200(1.0075)^4 + 200(1.0075)^5$

$a = 200(1.0075)$

$r = 1.0075$

$$S_n = 200(1.0075)\left[\frac{1-(1.0075)^5}{1-(1.0075)}\right]$$

Q**7**. $$\text{F.V.} = P(1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\left.\begin{array}{l} P = €1600 \\ i = 6\% = 0.06 \\ n = 5 \text{ years} \end{array}\right\} \quad \text{F.V.} = €1600(1.06)\left[\frac{(1.06)^5 - 1}{0.06}\right]$$

$$= €9560.51$$

Q**8**. $P = €3000$ per year [start of]

$i = 7.3\% = 0.073$

$t = 8$ years

(i) $$\text{P.V.} = \frac{P}{(1+i)^{n-1}}\left[\frac{(1+i)^n - 1}{i}\right]$$ when investment is made at the <u>start</u> of each term.

$$= \frac{3000}{(1.073)^7}\left[\frac{1.073^8 - 1}{0.073}\right]$$

$$= €19,000.13$$

(ii) Future value $= \text{P.V. } (1+i)^t$

$$= 19,000.13(1.073)^8$$

$$= €33,385.23$$

(iii) €3000 invested per year at 7.3% for 8 years.

$$\text{F.V.} = P(1+i)\left[\frac{(1+i)^t - 1}{i}\right]$$

$$= €3,000(1.073)\left[\frac{(1.073)^8 - 1}{0.073}\right]$$

$$= €33,385.22$$

Revision Exercise (advanced)

Q**1**.

$$t = 10 \text{ years}$$
$$\text{F.V.} = €500,000$$
$$i = 9\% = 0.09$$
$$\text{P.V.} = ?$$

(i)

$$\text{P.V.} = \frac{\text{F.V.}}{(1+i)^t}$$
$$= \frac{€500,000}{(1+0.09)^{10}}$$
$$= €211,205.40$$

(ii)

$$\text{F.V.} = P + P(1+i) + P(1+i)^2 + \cdots P(1+i)^{10}$$
$$€500,000 = P + P(1.09) + P(1.09)^2 + P(1.09)^{10}$$
$$= P\left[\frac{1-(1.09)^{10}}{1-(1.09)}\right]$$
$$€500,000 = P(15.1929)$$
$$\Rightarrow \quad P = \frac{€500,000}{15.1929}$$
$$= €32910$$

Q**2**.

$$t = 20 \text{ years}$$
$$= 240 \text{ months}$$
$$i = 12\% = 0.12$$

$$(1+r)^{12} = 1+i = 1.12$$
$$1+r = (1.12)^{\frac{1}{12}}$$
$$r = (1.12)^{\frac{1}{12}} - 1$$
$$= 0.00948$$

(i) €100,000 invested $\Rightarrow \text{F.V.} = P(1+i)^t$

$$= €100,000(1.00948)^{240}$$
$$= €964,629.31$$

(ii) €1000 per month

$$\text{F.V.} = P(1+i)\left[\frac{(1+i)^t - 1}{i}\right]$$
$$= 1000(1.00948)\left[\frac{1.00948^{240} - 1}{0.00948}\right]$$
$$= €919,857.64$$

$\therefore$ option (i) : Invest €100,000 at 12% AER for 20 years.

Q**3**. $account\ D: €638.14 = €500(1+i)^5$

$$\Rightarrow (1+i)^5 = 1.27628$$
$$1+i = (1.27628)^{\frac{1}{5}}$$
$$i = (1.27628)^{\frac{1}{5}} - 1$$
$$= 4.9\dot{9}\%$$
$$= 5\%$$

account C has money invested each year (€500).
account D has one investment only (€500).

$$\text{F.V.} = P(1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 500(1.05)\left[\frac{(1.05)^6 - 1}{i}\right]$$

Note: After 5 years, there has been 6 payments,
the first payment was at year 0.
$= €3571.$

Q**4**. $i = 10\% = 0.10$
$t = 20$ years $= 240$ months
repayments $= €700$

$$(1+r)^{12} = 1+i = 1.1$$
$$1+r = (1.1)^{\frac{1}{12}}$$
$$r = (1.1)^{\frac{1}{12}} - 1$$
$$= 0.007974$$

$$\text{repayments} = \frac{€M(i)[(1+i)^t]}{(1+i)^t - 1}$$
$$\Rightarrow \quad M = \frac{\text{repayment}[(1+i)^t - 1]}{i[(1+i)^t]}$$
$$= \frac{700[(1.007974)^{240} - 1]}{(0.007974)(1.007974)^{240}}$$
$$= €74,736$$

Q**5**.

$$t = 25 \text{ years}$$
$$= 300 \text{ months}$$
$$€M = €100,000$$
$$\text{repayments} = €800$$

$$\Rightarrow 800 = \frac{100,000(i)(1+i)^{300}}{(1+i)^{300}-1}$$

$$\text{let } i = 0.008 \quad : \quad \frac{100,000(0.008)(1.008)^{300}}{(1.008)^{300}-1} = €880 > 800$$

$$\text{let } i = 0.007 \quad : \quad \frac{100,000(0.007)(1.007)^{300}}{(1.007)^{300}-1} = €798.49 < 800$$

$$\text{let } i = 0.0072 \quad : \quad \frac{100,000(0.0072)(1.0072)^{300}}{(1.0072)^{300}-1} = €814 > 800$$

$$\text{let } i = 0.0071 \quad : \quad \frac{100,000(0.0071)(1.0071)^{300}}{(1.0071)^{300}-1} = €806 > 800$$

$$\text{let } i = 0.00702 \quad : \quad \frac{100,000(0.00702)(1.00702)^{300}}{(1.00702)^{300}-1} = €800.11 > 800$$

$\therefore \quad i_m = 0.00702$ per month.

$$(1+i_m)^{12} = 1+i_{\text{year}}$$
$$\Rightarrow \quad (1.00702)^{12} = 1+i$$
$$\Rightarrow \quad i = 0.0875$$
$$= 8.75\%$$

Q**6**.

Year	Pension Fund	Interest added	Payment
One	€ 127,953	€ 3838.59	€ 15000
Two	€ 116,791.59	€ 3503.75	€ 15000
Three	€ 105,295.38	€ 3158.86	€ 15000
Four	€ 93,454.24	€ 2803.63	€ 15000
Five	€ 81,257.87	€ 2437.74	€ 15000
Six	€ 68,695.61	€ 2060.87	€ 15000
Seven	€ 55,756.48	€ 1672.69	€ 15000
Eight	€ 42,429.17	€ 1272.58	€ 15000
Nine	€ 28,702.01	€ 861.06	€ 15000
Ten	€ 14,563.07	€ 436.87	€ 15000

Revision Exercise (Extended-Response Questions)

Q**1**.

Year	Principal	Interest	Payment	Year	Principal	Interest	Payment
1	100,000	+5000	–6,000	1	100,000	+5000	–12,000
2	99,000	+4950	–6,000	2	93,000	+4650	–12,000
3	97,950	+4897.5	–6,000	3	85,650	+4282.5	–12,000
4	96,847.5	+4842.38	–6,000	4	77,932.5	+3896.63	–12,000
5	95,689.88	+4784.49	–6,000	5	69,829.13	+3491.46	–12,000
6	94,474.37	+4723.72	–6,000	6	61,320.59	+3066.03	–12,000
7	93,198.09	+4659.90	–6,000	7	52,386.62	+2619.33	–12,000
8	91,857.99	+4592.90	–6,000	8	43,005.95	+2150.3	–12,000
9	90,450.89	+4522.54	–6,000	9	33,156.25	+1657.81	–12,000
10	88,973.43	+4448.67	–6,000	10	22,814.06	+1140.70	–12,000
	€ 87,422.10				€ 11,954.76		

Q**2**. (i) 65 years $\rightarrow$ 100 years = 35 years.

Pension of €20,000 at the end of each of the 35 years

$$= \text{Present value} = \frac{\text{Pension}}{(1+i)^n}\left(\frac{(1+i)^n - 1}{i}\right)$$

4% = 0.04

$$= \frac{20,000}{(1.04)^{35}}\left(\frac{(1.04)^{35} - 1}{0.04}\right)$$

$$= €373,292.26 \quad \text{pension}$$

$$\underline{€300,000.00} \quad \text{relocation}$$

$$€673,292.26 \quad \text{Total}$$

(ii) €40,000 invested for 30 years at 5%

$$\text{F.V.} = \text{P.V.}(1+i)^t$$

$$= €40,000(1+0.05)^{30}$$

$$= €172,877.69$$

$\therefore$ amount needed to be saved $= (673,292.26 - 172,877.69)$

$$= €500,414.57.$$

Let instalment = €P

$\Rightarrow$ Future value of 30 years of €P yearly instalments at 5%

$$\Rightarrow \; €500,414.57 = P(1+0.05) + P(1+0.05)^2 + \cdots P(1+0.05)^{30}$$

$$= P(1.05)\left(\frac{1-(1.05)^{30}}{1-(1.05)}\right)$$

$$= P(1.05)\left(\frac{(1.05)^{30} - 1}{0.05}\right)$$

$$€500,414.57 = €P(69.7608)$$

$\Rightarrow$ Instalment €P per year = €7173.30

(iii) $\therefore$ a total of €673,292.26 needed in 25 yrs at 5%.

$$€673,292.26 = \frac{P(1.05)(1.05^{25}-1)}{0.05}$$

$$€673,292.26 = P\ (50.1135)$$

$$\Rightarrow \qquad P = €13,435.36$$

Q**3**. (i) $$€P = \frac{€M(i)(1+i)^n}{(1+i)^n - 1}$$

$€P =$ repayments on loan

$€M =$ value of loan/mortgage

$i =$ interest rate

$n =$ number of repayments (months/years)

(ii) $$i = 9\% = 0.09$$

$$\Rightarrow \quad (1+r)^{12} = 1+i = 1.09$$

$$1+r = (1.09)^{\frac{1}{12}}$$

$$r = (1.09)^{\frac{1}{12}} - 1$$

$$= 0.007207$$

$$= 0.72\% \text{ monthly.}$$

(iii) $€M = €100,000$

$€P = €800$

$i = 9\%$ per annum

$= 0.72\%$ per month

$n =$ number of payments

$$\therefore \quad P = \frac{M(i)(1+i)^n}{(1+i)^n - 1}$$

$$800 = \frac{100,000(0.007207)(1.007207)^n}{(1.007207)^n - 1}$$

$$\frac{800}{720.7} = \frac{(1.007207)^n}{(1.007207)^n - 1}$$

$$1.11\left((1.007207)^n - 1\right) = (1.007207)^n$$

$$1.11(1.007207)^n - (1.007207)^n = 1.11$$

$$0.11(1.007207)^n = 1.11$$

$$(1.007207)^n = \frac{1.11}{0.11} = 10.\dot{0}\dot{9}$$

$$\Rightarrow \quad n\log(1.007207) = \log(10.0909)$$

$$n = \frac{\log(10.0909)}{\log(1.007207)} = 321$$

(iv) 26 years, 9 months (i.e. 321 monthly repayments)

Q**4**. (i) Formula for $B2: \ = A2 * 0.006628$

$$\left.\begin{array}{l}\text{Note}: i = 8.25\% = 0.0825 \\ \Rightarrow \qquad r = 0.006628\end{array}\right\}(1+r)^{12} = 1+i$$

Formula for $D2: \ = A2 + B2 - C2$

$\Rightarrow$ Balance after 5 years $= €149,310.2$

(ii) After 155 payments, the balance remaining on the spreadsheet is €964.58.

Using mortgage formula:

$$€P = \frac{€M(i)(1+i)^n}{(1+i)^n - 1}$$

Mortgage remaining is $€149,310.2 - 40,000 = €109,310.2$

$$\Rightarrow \quad 1127 = \frac{109,310(0.006628)(1.006628)^n}{(1.006628)^n - 1}$$

$$\Rightarrow \quad 1.5555 = \frac{(1.006628)^n}{(1.006628)^n - 1}$$

$$\Rightarrow \quad 1.5555(1.006628)^n - 1.5555 = (1.006628)^n$$

$$\Rightarrow \quad 0.5555(1.006628)^n = 1.5555$$

$$1.006628^n = 2.80025$$

$$\Rightarrow \quad n \ \frac{\log 2.80025}{\log 1.006628} = 155.87$$

$\therefore$ number of months $= 155.87$

$= 12$ years, 11.87 months.

Q**5**. (i) 1st payment $= A$

2nd payment $= A(1.04)$

3rd payment $= A(1.04)^2$

4th payment $= A(1.04)^3$

(ii) $S_{26} = A + A(1.04) + A(1.04)^2 + A(1.04)^3 + \quad A(1.04)^{25}$

(iii) number of terms

$$S_{26} = \frac{A(1-(1.04)^{26})}{(1-1.04)}$$

$$= \frac{A((1.04)^{26}-1)}{0.04}$$

$$\Rightarrow \quad \$21,500,000.00 = A(44.311)$$

$$\Rightarrow \quad A = \$485,199$$

(iv)

payment	1	2	3	4
actual amount	$485,199	$504,607	$524,791	$545,783

(v)

payment	1	2	3	4
present value	$485,199	$481,587	$478,002	$474,443.85

(vi) $\text{P.V.} = \dfrac{\text{F.V.}}{(1+i)^{n-1}}$: $(n-1)$ is used since payments were paid immediately.

$$\Rightarrow \quad = \frac{\$485,199(1.04)^{n-1}}{(1+0.0478)^{n-1}}$$

(vii)

$$\Rightarrow \quad \text{P.V.} = 485199 + \frac{485199(1.04)}{(1.0478)} + \frac{485199(1.04)}{1.0478} + \cdots \frac{485199(1.04)^{25}}{(1.0478)^{25}}$$

$$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}$$

$$= 485199\left[\frac{1-(\frac{1.04}{1.0478})^{26}}{1-(\frac{1.04}{1.0478})}\right]$$

$$= \$11,500,000.00$$

(viii) $\text{Tax} = \$11.5\text{ m} - \$7.9\text{ m} = \$3.6\text{ m}$

$$\Rightarrow \quad \frac{3.6\text{ m}}{11.5\text{ m}} \times \frac{100}{1} = 31.3\%\text{ tax.}$$