

Algebra Revision Worksheet

Leaving Cert. Higher Level
Project Maths



FACTORS & FRACTIONS

Factorise:	
HCF	i $3p^2 + 6pq = 3p(p + 2q)$
Group	ii $6ab + 12bc - 8ac - 9b^2$ $= 3b(2a - 3b) + 4c(3b - 2a) = (3b - 4c)(2a - 3b)$
HCF / DOTS	iii $3x^2 - 12y^2 = 3(x^2 - 4y^2) = 3(x + 2y)(x - 2y)$
QUADRATIC	iv $2x^2 - 7x - 15 = (2x + 3)(x - 5)$
Difference of 2 cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$64 - 27x^3 = (4 - 3x)(16 + 12x + 9x^2)$
HCF / Sum of 2 cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$4a^3 + 32b^3 = 4(a^3 + 8b^3) = 4(a + 2b)(a^2 - 2ab + 4b^2)$

Simplify : Factorise: $\frac{x^3 + 7x^2 + 12x}{x^2 + 2x - 3}$ $\frac{\text{HCF}}{\text{QUADRATIC}}$ $\frac{\text{QUADRATIC}}{\text{SIMPLIFY}}$	$\frac{x^3 + 7x^2 + 12x}{x^2 + 2x - 3}$ $= \frac{x(x^2 + 7x + 12)}{(x+3)(x-1)}$ $= \frac{x(x+3)(x+4)}{(x+3)(x-1)}$ $= \frac{x(x+4)}{(x-1)}$
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Simplify. : Notice: $2x^2 - 3x = x(2x-3)$ $\Rightarrow \text{LCD} = x(2x-3)$	$\frac{5}{2x-3} - \frac{3}{2x^2-3x} - \frac{1}{x}$ $= \frac{5(x) - 3(1) - 1(2x-3)}{x(2x-3)}$ $= \frac{5x - 3 - 2x + 3}{x(2x-3)}$ $= \frac{3x}{x(2x-3)}$ $= \frac{3}{2x-3}$
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<p>Simplify :</p> <p>divide by fraction → invert and multiply</p>	$\frac{a+b}{\frac{1}{a} + \frac{1}{b}}$ $= \frac{a+b}{\left(\frac{b+a}{ab} \right)}$ $= (a+b) \left(\frac{ab}{b+a} \right)$ $= ab$
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<p>$f(x) = \frac{x^3-1}{x^2-1}, x \neq 1$</p> <p>$g(x) = \frac{x^2+x+1}{x^2-x-2}, x \neq -1, 2$</p> <p>If $f(x) \div g(x) = ax+b$, find the value of a and b.</p> <p>Invert denominator and multiply</p> <p>FACTORISE EACH PART AND SIMPLIFY</p>	$\frac{f(x)}{g(x)} = \frac{\left(\frac{x^3-1}{x^2-1} \right)}{\left(\frac{x^2+x+1}{x^2-x-2} \right)}$ $= \frac{(x^3-1)}{(x^2-1)} \cdot \frac{(x^2-x-2)}{(x^2+x+1)}$ <p style="color: red;">DOTC QUADRATIC DOTS QUADRATIC</p> $= \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} \cdot \frac{(x-2)(x+1)}{(x^2+x+1)}$ $= x-2$ $\therefore a=1, b=-2$
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MANIPULATING FORMULAE

If $c = \frac{b^2 - ac}{b+a}$, express a in terms of the other variables.

$$\times (b+a)$$

$$c(b+a) = b^2 - ac$$

$$cb + ac = b^2 - ac$$

$$+ac, -cb$$

$$ac + ac = b^2 - cb$$

$$2ac = b^2 - cb$$

$$\div 2c$$

$$a = \frac{b^2 - cb}{2c}$$

$\times \text{LCD}$ ie fuv

$$fuv \left(\frac{1}{f} \right) = fuv \left(\frac{1}{u} \right) + fuv \left(\frac{1}{v} \right)$$

$$uv = fv + fu$$

$$-fv$$

$$uv - fv = fu$$

$$\text{HCF}$$

$$v(u-f) = fu$$

$$\div (u-f)$$

$$v = \frac{fu}{(u-f)}$$

Sub IN $f=15$, $u=20$
and evaluate

$$v = \frac{(15)(20)}{(20-15)} = \frac{300}{5} = 60$$

	$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$ express r in terms of the other variables
Square both sides	$T^2 = \frac{4\pi^2 r^3}{GM}$
$\times GM$	$T^2 GM = 4\pi^2 r^3$
$\div 4\pi^2$	$\frac{T^2 GM}{4\pi^2} = r^3$
Cube root both sides	$r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$

POLYNOMIAL IDENTITIES

	$x^2 - 6x + t = (x+k)^2$ where t and k are constants. Find the value of k and t .
expand LHS	$x^2 - 6x + t = x^2 + 2kx + k^2$
equate coefficients	$-6 = 2k \quad \quad t = k^2 \quad ②$ $-3 = k \quad ①$
$① \sim ②$	$t = (-3)^2 = 9$
\therefore	$k = -3, t = 9$

<p>expand LHS and Simplify</p> <p>equate coefficients</p>	$ay^2 + by(y-4) + c(y-4) = y^2 + 13y - 20$ <p>find. the value of constants a, b and c.</p> $\begin{aligned} \text{LHS} &= ay^2 + by^2 - 4by + cy - 4c \\ &= (a+b)y^2 + (c-4b)y - 4c \end{aligned}$ $\Rightarrow a+b = 1 \quad (1) \quad c-4b = 13 \quad (2) \quad -4c = -20$ $c = 5 \quad (3)$ $\begin{aligned} (3) \rightarrow (2) \quad 5 - 4b &= 13 \Rightarrow -4b = 8 \Rightarrow b = -2 \quad (4) \\ (4) \rightarrow (1) \quad a - 2 &= 1 \Rightarrow a = 3 \end{aligned}$ $\therefore a = 3, b = -2, c = 5$
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SURDS

<p>Express in its Simplest Surd form :</p>	$\begin{aligned} \text{i)} \quad \sqrt{99} &= \sqrt{9} \sqrt{11} = 3\sqrt{11} \\ \text{ii)} \quad \frac{1}{2}\sqrt{11}2 &= \frac{\sqrt{16 \times 7}}{2} = \frac{4\sqrt{7}}{2} = 2\sqrt{7} \\ \text{iii)} \quad \sqrt{2\frac{1}{4}} &= \sqrt{\frac{9}{4}} = \frac{3}{2} \\ \text{iv)} \quad \sqrt{180} + \sqrt{20} - \sqrt{125} &= \sqrt{36(5)} + \sqrt{4(5)} - \sqrt{25(5)} \\ &= \frac{6\sqrt{5}}{2} + 2\sqrt{5} - 5\sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$
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Express $\frac{1}{3\sqrt{5}} - \frac{1}{2\sqrt{20}}$ in the form $k\sqrt{5}$

$2\sqrt{20} = 2\sqrt{4(5)}$
 $= 2(2)\sqrt{5}$
 $= 4\sqrt{5}$

Write as single fraction

express in form $k\sqrt{5}$

$$\begin{aligned} &= \frac{1}{3\sqrt{5}} - \frac{1}{4\sqrt{5}} \\ &= \frac{4\sqrt{5} - 3\sqrt{5}}{(3\sqrt{5})(4\sqrt{5})} = \frac{\sqrt{5}}{12(5)} = \frac{\sqrt{5}}{60} \\ &= \frac{1}{60}\sqrt{5} \end{aligned}$$

Express $\frac{-4}{\sqrt{5}-3}$ in the form $\sqrt{5} + a$

multiply above and below by the Conjugate of the denominator

$$\begin{aligned} &= \frac{-4(\sqrt{5}+3)}{(\sqrt{5}-3)(\sqrt{5}+3)} \quad \text{DOTS} \\ &= \frac{-4\sqrt{5}-12}{5-9} \\ &= \frac{-4\sqrt{5}-12}{-4} \\ &= \sqrt{5} + 3 \end{aligned}$$

<p style="margin: 0;">multiply above and below by the conjugate of the denominator</p>	<p>Express $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3} - b$</p> $ \begin{aligned} &= \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} \text{ DOTS} \\ &= \frac{1 - 2\sqrt{3} + 3}{1 - 3} \\ &= \frac{4 - 2\sqrt{3}}{-2} \\ &= -2 + \sqrt{3} \end{aligned} $
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QUADRATIC EQUATIONS & FUNCTIONS

<p style="color: blue; margin: 0;">FACTORISE</p>	<p>Solve $3x^2 - 5x - 12 = 0$</p> $ \begin{aligned} &(3x + 4)(x - 3) = 0 \\ \Rightarrow \quad &3x + 4 = 0 \quad \quad x - 3 = 0 \\ &3x = -4 \\ &x = -\frac{4}{3} \quad \quad x = 3 \end{aligned} $
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<p>Solve : $2x^2 - 7x + 4 = 0$, leave your answer in surd form</p> <p>use formula</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>$a = 2$ $b = -7$ $c = 4$</p>	$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$ $= \frac{7 \pm \sqrt{49 - 32}}{4}$ $= \frac{7 \pm \sqrt{17}}{4}$ $x_1 = \frac{7 - \sqrt{17}}{4}$ $x_2 = \frac{7 + \sqrt{17}}{4}$
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<p>form a quadratic equation with roots $\frac{1}{2}$ and 3</p> <p>either use</p> $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$ $\Rightarrow x^2 - \left(\frac{1}{2} + 3\right)x + \left(\frac{1}{2}\right)(3) = 0$ $x^2 - 3.5x + 1.5 = 0$ <p>or</p> $2x^2 - 7x + 3 = 0$ <hr/> <p>or get factors and expand</p>	<p>Roots : $x = \frac{1}{2}, x = 3$</p> <p>factors : $(x - \frac{1}{2}), (x - 3)$</p> $\Rightarrow (x - \frac{1}{2})(x - 3) = 0$ $x^2 - 3x - \frac{1}{2}x + 1.5 = 0$ $x^2 - 3.5x + 1.5 = 0$ <p>or</p> $2x^2 - 7x + 3 = 0$
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	Verify $2x^2 - x + 3 = 0$ has no real roots
If no real roots	
$\Rightarrow \Delta = b^2 - 4ac < 0$	$\Delta = (-1)^2 - 4(2)(3)$ = 1 - 24 = -23 < 0
$a = 2$ $b = -1$ $c = 3$	\Rightarrow it has no real roots. QED

	Solve $\frac{1}{x+1} + \frac{4}{2x-1} = \frac{5}{3}$
multiply by LCD ie.. $(x+1)(2x-1)(3)$	$\Rightarrow 1(x+1)(2x-1)(3) + 4(x+1)(2x-1)(3) = 5(x+1)(2x-1)(3)$
expand	$6x-3 + 12x+12 = 5(2x^2 - x + 2x - 1)$
simplify	$18x + 9 = 5(2x^2 + x - 1)$
	$18x + 9 = 10x^2 + 5x - 5$
$-18x, -15$	$10x^2 - 13x - 14 = 0$
	$(10x + 7)(x - 2) = 0$
	$x = -\frac{7}{10}, x = 2$
	r term

MODULUS EQUATIONS

	Solve $ 3x-2 =4$
Square	$(3x-2)^2 = (4)^2$
Solve quadratic	$9x^2 - 12x + 4 = 16$ $9x^2 - 12x - 12 = 0$ $3x^2 - 4x - 4 = 0$ $(3x+2)(x-2) = 0$
	$x = -\frac{2}{3}, x = 2$
Method 2: Consider both options	If $ 3x-2 =4$ either $3x-2=4$ or $3x-2=-4$ $3x=6$ $3x=-2$ $x=2$ $x=-\frac{2}{3}$

	Solve $2 x+1 - x+3 = 0$
bring moduli to opposite sides square	$2 x+1 = x+3 $ $(2 x+1)^2 = (x+3)^2$ $4(x^2 + 2x + 1) = x^2 + 6x + 9$ $4x^2 + 8x + 4 = x^2 + 6x + 9$ $3x^2 + 2x - 5 = 0$ $(3x+5)(x-1) = 0$
Solve quadratic	$3x+5=0 \quad \quad x-1=0$ $x = -\frac{5}{3} \quad \quad x=1$
check: $x = -\frac{5}{3}$	$2 \left -\frac{5}{3} + 1 \right \stackrel{?}{=} \left -\frac{5}{3} + 3 \right $ $2 \left -\frac{2}{3} \right \stackrel{?}{=} \left \frac{4}{3} \right $
$x = 1$	$2 \left 1 + 1 \right \stackrel{?}{=} \left 1 + 3 \right $ $2 \cdot 2 \stackrel{?}{=} 4$

SURD EQUATION

	Solve $x = \sqrt{3x+7} - 1$
+1	$x + 1 = \sqrt{3x+7}$
Square	$(x+1)^2 = 3x+7$
	$x^2 + 2x + 1 = 3x + 7$
	$x^2 - x - 6 = 0$
	$(x-3)(x+2) = 0$
	$x = 3, x = -2$
check:	$3 \stackrel{?}{=} \sqrt{3(3)+7} - 1 = \sqrt{16} - 1 = 4 - 1 = 3 \checkmark$
	$-2 \stackrel{?}{=} \sqrt{3(-2)+7} - 1 = \sqrt{1} - 1 = 0 \times$
	SOLUTION: $x = 3$

COMPLETE SQUARE (VERTEX FORM) OF A QUADRATIC EQUATION

	Express $x^2 - 6x - 12 = 0$ in the form $(x-p)^2 + q = 0$. Hence solve for x .
$\begin{array}{cc} x & -3 \\ \times & \boxed{\begin{array}{ c c } \hline x^2 & -3x \\ \hline -3x & +9 \\ \hline \end{array}} \\ -3 & \end{array}$	$x^2 - 6x + 9 - 9 - 12 = 0$ $(x-3)^2 - 21 = 0$ Hence solve $(x-3)^2 = 21$ $x-3 = \pm\sqrt{21}$ $x = 3 \pm \sqrt{21}$

Express $-x^2 + 11x + 15 = 0$ in the form $q - (x-p)^2 = 0$ and hence solve.

$$-\left[x^2 - 11x - 15 \right] = 0$$

$$-\left[\left(x - \frac{11}{2} \right)^2 - \frac{121}{4} - 15 \right] = 0$$

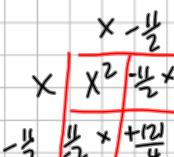
$$-\left[\left(x - \frac{11}{2} \right)^2 - \frac{181}{4} \right] = 0$$

$$\frac{181}{4} - \left(x - \frac{11}{2} \right)^2 = 0$$

Hence:

$$\frac{181}{4} = \left(x - \frac{11}{2} \right)^2$$

$$\pm \sqrt{\frac{181}{4}} = x - \frac{11}{2}$$

$$x = \frac{11}{2} \pm \sqrt{\frac{181}{4}}$$


CUBIC EQUATIONS & FUNCTIONS

$f(x) = 3x^3 + mx^2 - 17x + n$
where m and n are constants.

Given that $x-3$ and $x+2$ are factors of $f(x)$. find the value of m and n .

If $(x-3)$ is a factor
 $\Rightarrow f(3) = 0$

$$3(3)^3 + m(3)^2 - 17(3) + n = 0$$

$$81 + 9m - 51 + n = 0$$

$$9m + n = -30 \quad \textcircled{1}$$

If $(x+2)$ is a factor
 $\Rightarrow f(-2) = 0$

$$3(-2)^3 + m(-2)^2 - 17(-2) + n = 0$$

$$-24 + 4m + 34 + n = 0$$

$$4m + n = -10 \quad \textcircled{2}$$

Solve equations
 $\textcircled{1} - \textcircled{2}$

$$5m = -20 \Rightarrow m = -4$$

Sub into $\textcircled{2}$

$$4(-4) + n = -10$$

$$-16 + n = -10 \Rightarrow n = 6$$

	Solve $4x^3 + 10x^2 - 7x - 3 = 0$
find integer root	$f(-3) = 4(-3)^3 + 10(-3)^2 - 7(-3) - 3 = -108 + 90 + 21 - 3 = 0$
write related factor and divide	$\Rightarrow x = -3$ is a root & $(x+3)$ is a factor
	$\begin{array}{r} 4x^2 - 2x - 1 \\ \hline x+3) 4x^3 + 10x^2 - 7x - 3 \\ \underline{+ 4x^3 + 12x^2} \\ \underline{- 2x^2 - 7x} \\ \underline{\pm 2x^2 \pm 6x} \\ \underline{- x - 3} \\ \underline{\pm x \pm 3} \end{array}$
SOLVE QUADRATIC FACTOR = 0	$4x^2 - 2x - 1 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)} = \frac{2 \pm \sqrt{20}}{8}$ $= \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4}$
SOLUTIONS:	$x = -3, x = \frac{1+\sqrt{5}}{4}, x = \frac{1-\sqrt{5}}{4}$

	$x^2 - t$ is a factor of $x^3 - px^2 - qx + r$
i)	Show that $pq = r$
ii)	Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q .
let	$\begin{aligned} x^3 - px^2 - qx + r &= (x^2 - t)(x + k) \\ &= x^3 + kx^2 - tx - tk \end{aligned}$
equate coefficients	$\begin{array}{l l l} -p = k & -q = -t & r = -tk \quad (3) \\ p = -k \quad (1) & q = t \quad (2) & \end{array}$
i) Show $pq = r$	$pq = (-k)(t) = -tk = r \quad \text{QED}$
ii) Roots in terms of p and q	factors are: $(x^2 - t)$ and $(x + k)$ $\Rightarrow x^2 = t$ $x = \pm \sqrt{t}$ $t = q$
Roots:	$\Rightarrow x = \pm \sqrt{q}$
	$x = p$

Inequalities

Solve $11 - 3x \geq 2$, $x \in \mathbb{N}$
and show the solution set on a number line.

$$11 - 3x \geq 2$$

$$-11$$

$$-3x \geq -9$$

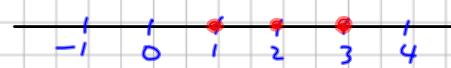
Change signs
and inequality

$$3x \leq 9$$

$$\div 3$$

$$x \leq 3$$

$$x \in \mathbb{N}$$



i) Solve Set A

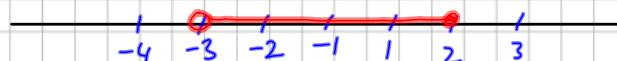
$$\begin{aligned} \text{Set } A : 2x + 7 &\leq 11, \quad x \in \mathbb{R} \\ 2x &\leq 4 \\ x &\leq 2 \end{aligned}$$

ii) Solve Set B

$$\begin{aligned} \text{Set } B : 4 - 2x &< 10, \quad x \in \mathbb{R} \\ -2x &< 6 \\ 2x &> -6 \\ x &> -3 \end{aligned}$$

iii)

Find $A \cap B$ and show on a no. line.



$$A \cap B : -3 < x \leq 2$$

<p><i>Consider if quadratic = 0</i></p> <p>outside or inside? zero test</p> <p>SOLUTION:</p>	<p>Solve $15 + 2x \geq x^2$, $x \in \mathbb{R}$</p> $x^2 - 2x - 15 \leq 0$ <p>If $x^2 - 2x - 15 = 0$ $(x + 3)(x - 5) = 0$ $x = -3, x = 5$</p> $(0)^2 - 2(0) - 15 \leq 0$ \Rightarrow zero value works zero is inside $-3 \leq x \leq 5$ <p>$-3 \leq x \leq 5$</p> 
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<p><i>Consider if quadratic = 0</i></p> <p>INSIDE / OUTSIDE values?</p> <p>SOLUTION</p>	<p>Solve $12 < x^2 + 4x$, $x \in \mathbb{R}$</p> $0 < x^2 + 4x - 12$ $x^2 + 4x - 12 > 0$ <p>If $x^2 + 4x - 12 = 0$ $(x - 2)(x + 6) = 0$ $x = 2, x = -6$</p> <p>$(0)^2 + 4(0) - 12$ is <u>not</u> > 0 \Rightarrow 0 value doesn't work 0 is between 2 and -6 \Rightarrow outside values work</p> <p>$-6 < x < 2$</p> 
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Solve $\frac{3x+1}{x-1} \geq 2, x \in \mathbb{R}, x \neq 1$

multiply by $(x-1)^2$ $\frac{(3x+1)(x-1)^2}{(x-1)} \geq 2(x-1)^2$

$3x^2 - 3x + x - 1 \geq 2[x^2 - 2x + 1]$

$3x^2 - 2x - 1 \geq 2x^2 - 4x + 2$

$x^2 + 2x - 3 \geq 0$

Consider if $= 0$ If $x^2 + 2x - 3 = 0$
 $(x+3)(x-1) = 0$
 $x = -3, x = 1$

Inside / outside? $(0)^2 + 2(0) - 3 \text{ is } \underline{\text{not}} \geq 0$
 $\Rightarrow 0 \text{ doesn't work}$
 $0 \text{ is between } -3 \text{ and } 1$
 $\Rightarrow \text{outside values work}$

Solution: $-3 > x > 1$

$f(x) = x^2 - 7x + 12$

i) Show if $f(x+1) \neq 0$ then

$f(x) = \frac{x-4}{x-2}$
 $f(x+1) = \frac{x^2 - 7x + 12}{(x+1)^2 - 7(x+1) + 12} = \frac{x^2 - 7x + 12}{x^2 + 2x + 1 - 7x - 7 + 12}$
 $= \frac{x^2 - 7x + 12}{x^2 - 5x + 6} = \frac{(x-4)(x-3)}{(x-2)(x-3)}$ QED

ii) find the range of values of x for which

SOLVE INEQUALITY $\frac{f(x)}{f(x+1)} > 3 \Rightarrow \frac{x-4}{x-2} > 3$

MULTIPLY by $(x-2)^2$ $\frac{(x-4)(x-2)^2}{(x-2)} > 3(x-2)^2$
 $x^2 - 2x - 4x + 8 > 3(x^2 - 4x + 4)$
 $x^2 - 6x + 8 > 3x^2 - 12x + 12$
 $0 > 2x^2 - 6x + 4$
 $x^2 - 3x + 2 < 0$

$\frac{1}{2}$ If quadratic = 0 OUTSIDE/INSIDE?
 $\text{If } x^2 - 3x + 2 = 0, (x-2)(x-1) = 0, x=2, x=1$
 $(0)^2 - 3(0) + 2 \text{ is } \underline{\text{not}} < 0 \text{ since } 0 \text{ is Outside}$
 $1 \text{ and } 2 \Rightarrow \text{Inside Works} \therefore 1 < x < 2$

Solve $|5 - 2x| \leq 3$

Square

Solve quadratic inequality

If $\Delta = 0$

$$\begin{aligned} 25 - 20x + 4x^2 &\leq 9 \\ 4x^2 - 20x + 16 &\leq 0 \\ 2x^2 - 10x + 8 &\leq 0 \\ x^2 - 5x + 4 &\leq 0 \end{aligned}$$

If $x^2 - 5x + 4 = 0$

$$(x - 1)(x - 4) = 0$$

$$x = 1, x = 4$$

OUTSIDE/INSIDE?

If $x = 0 \Rightarrow (0)^2 - 5(0) + 4 \text{ is not } \leq 0$
 $\Rightarrow 0 \text{ doesn't work, } 0 \text{ is outside}$
 $\Rightarrow \text{Inside work}$

$\therefore 1 \leq x \leq 4$



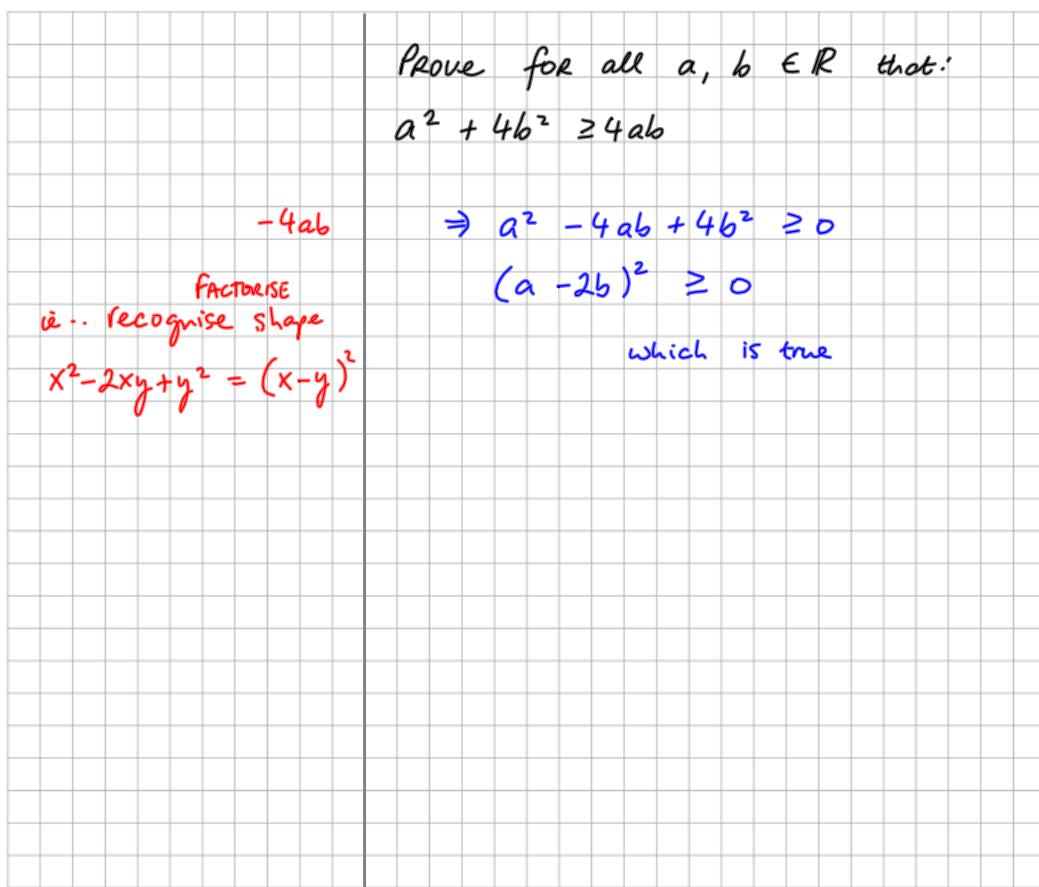
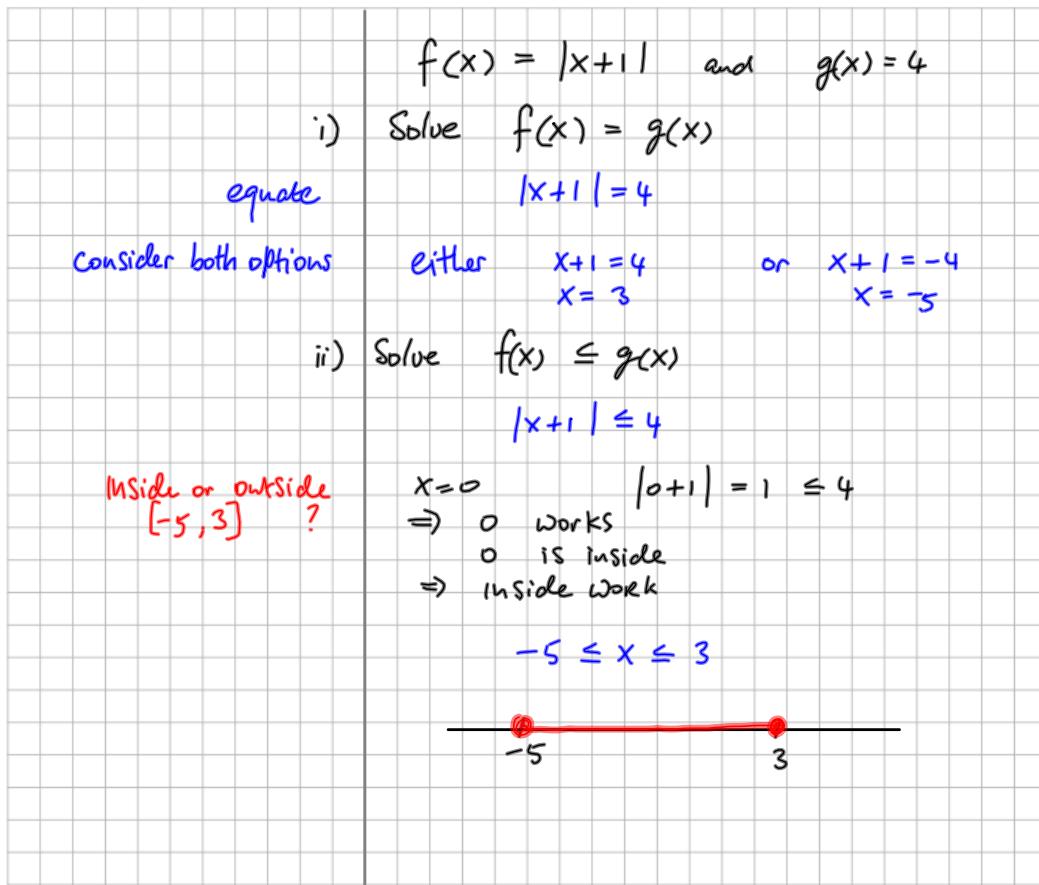
Method 2: Write as double inequality and solve

$$\begin{aligned} |5 - 2x| &\leq 3 \Rightarrow -3 \leq 5 - 2x \leq 3 \\ -8 &\leq -2x \leq -2 \Rightarrow -4 \leq -x \leq -1 \\ (\text{change signs}) &\Rightarrow 4 \geq x \geq 1 \Rightarrow 1 \leq x \leq 4 \quad (\text{reorder}) \end{aligned}$$

Solve $|4x + 7| > 1$

Easiest to write as double inequality and solve

$$\begin{aligned} &\Rightarrow -1 > 4x + 7 > 1 \\ &-7 &-7 &-7 \\ &-8 > 4x > -6 \\ &\frac{-8}{4} &\frac{4x}{4} &\frac{-6}{4} \\ &-2 > x > -\frac{3}{2} \\ &-2 > x > -1.5 \end{aligned}$$

	Show $\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}$ if $a+b \geq 0$ and $a, b \neq 0$
multiply by LCD i.e. a^2b^2	$a^2b^2\left(\frac{a}{b^2}\right) + a^2b^2\left(\frac{b}{a^2}\right) \geq a^2b^2\left(\frac{1}{a}\right) + a^2b^2\left(\frac{1}{b}\right)$ $a^3 + b^3 \geq ab^2 + a^2b$ $a^3 + b^3 - ab^2 - a^2b \geq 0$ $a(a^2 - b^2) + b(b^2 - a^2) \geq 0$ $(a-b)(a^2 - b^2) \geq 0$ $(a-b)(a-b)(a+b) \geq 0$ $(a-b)^2(a+b) \geq 0$ which is true
factorise by Grouping	
DOTS	

SIMULTANEOUS EQUATIONS

Solve	$\frac{2x-5}{3} + \frac{y}{5} = 6 \quad ①$
	$\frac{3x}{10} + 2 = \frac{3y-5}{2} \quad ②$
$15 \ ①$	$5(2x-5) + 3y = 6 \quad ⑤$ $10x - 25 + 3y = 90$ $10x + 3y = 115 \quad ③$
$10 \ ②$	$3x + 20 = 5(3y-5)$ $3x + 20 = 15y - 25$ $3x - 15y = -45 \quad ④$
eliminate y's	$50x + 15y = 575$ $3x - 15y = -45$ $53x = 530 \Rightarrow x=10$
Sub $x=10$ into ③	$10(10) + 3y = 115$ $100 + 3y = 115$ $3y = 15 \Rightarrow y=5$

Solve	$\begin{array}{l} 3x + y + z = 0 \quad (1) \\ x - y + z = 2 \quad (2) \\ 2x - 3y - z = 9 \quad (3) \end{array}$
Eliminate z	$\begin{array}{l} (1) \\ + (3) \\ \hline 3x + y + z = 0 \\ 2x - 3y - z = 9 \\ \hline 5x - 2y = 9 \quad (4) \end{array}$
	$\begin{array}{l} (2) \\ + (3) \\ \hline x - y + z = 2 \\ 2x - 3y - z = 9 \\ \hline 3x - 4y = 11 \quad (5) \end{array}$
Eliminate y	$\begin{array}{l} (4) \\ - (5) \\ \hline 10x - 4y = 18 \\ -3x + 4y = -11 \\ \hline 7x = 7 \end{array} \Rightarrow x = 1$
Sub $x=1$ into (4)	$\begin{array}{l} 5(1) - 2y = 9 \\ 5 - 2y = 9 \\ -2y = 4 \end{array} \Rightarrow y = -2$
Sub $x=1, y=-2$ into (1)	$\begin{array}{l} 3(1) + (-2) + z = 0 \\ 3 - 2 + z = 0 \\ 1 + z = 0 \end{array} \Rightarrow z = -1$

<p>Craig invested €50,000 in three funds paying 6%, 8% and 10% interest p.a. The total earned in year 1 was €3700. Twice as much was invested at 6% as was 10%. How much was invested in each fund?</p> <p>Let :</p> <p>x = amount invested @ 6% y = amount invested @ 8% z = amount invested @ 10%</p>	
Total invested is €50,000	$x + y + z = 50,000 \quad (1)$
Total interest is €3,700	$x(0.06) + y(0.08) + z(0.1) = 3700$ $3x + 4y + 5z = 185,000 \quad (2)$
Amount invested @ 6% was twice amount @ 10%	$x = 2z \quad (3)$
Sub (3) into (1)	$2z + y + z = 50,000 \Rightarrow 3z + y = 50,000 \quad (4)$
Sub (3) into (2)	$3(2z) + 4y + 5z = 185,000 \Rightarrow 11z + 4y = 185,000 \quad (5)$
	$\begin{array}{r} 12z + 4y = 200,000 \\ -11z - 4y = -185,000 \\ \hline z = 15,000 \end{array}$
Sub $z = 15,000$ into (3)	$x = 2(15,000) \Rightarrow x = 30,000$
Sub $z = 15,000$ & $x = 30,000$ into (1)	$30,000 + y + 15,000 = 50,000 \Rightarrow y = 5,000$

	Solve : $\begin{aligned} 2x + y &= 3 \\ x^2 + xy + y^2 &= 3 \end{aligned}$
①	rewrite linear $y = 3 - 2x$
②	Sub into quadratic solve $\begin{aligned} x^2 + x(3 - 2x) + (3 - 2x)^2 &= 3 \\ x^2 + 3x - 2x^2 + 9 - 12x + 4x^2 &= 3 \\ 3x^2 - 9x + 6 &= 0 \\ x^2 - 3x + 2 &= 0 \\ (x - 1)(x - 2) &= 0 \end{aligned}$
	$x = 1, x = 2$
③	Sub back into linear $y = 3 - 2(1) = 1$ $y = 3 - 2(2) = -1$
	Solution couples : $(1, 1)$ and $(2, -1)$

Indices, Index Equations

Simplify	i) $125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = 25$
	ii) $32^{\frac{2}{5}} - 81^{\frac{1}{4}} = (\sqrt[5]{32})^2 - \sqrt[4]{81}$ $= 2^2 - 3 = 4 - 3$ $= 1$
DIVIDE FRACTION ⇒ INVERT DENOMINATOR and multiply	iii) $\frac{4^{-\frac{1}{2}}}{64^{\frac{1}{3}}} = \frac{\left(\frac{1}{\sqrt{4}}\right)}{\left(\sqrt[3]{64}\right)^2} = \frac{\left(\frac{1}{2}\right)}{(4)^2}$ $= \frac{\left(\frac{1}{2}\right)}{\left(\frac{16}{1}\right)} = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right)$ $= \frac{1}{32}$

Solve: $\begin{array}{l} 3^2 = 9 \\ 3^3 = 27 \\ 3^4 = 81 \\ 3^5 = 243 \end{array}$	$27^{4+3x} = 243^{1+2x}$ <i>rewrite with same base</i> $(3^3)^{4+3x} = (3^5)^{1+2x}$ $3^{12+9x} = 3^{5+10x}$ <i>equate powers</i> $12+9x = 5+10x$ $7 = x$
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Solve $8 = 2^3$	$2^{x^2} = 8^{2x+9}$ <i>write with same base</i> $2^{x^2} = (2^3)^{2x+9}$ $2^{x^2} = 2^{6x+27}$ <i>equate powers</i> $x^2 = 6x + 27$ $x^2 - 6x - 27 = 0$ $(x + 3)(x - 9) = 0$ $x = -3, x = 9$
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Solve	$2^x = 8^{y+1}$, $3^{x-9} = 9^y$
rewrite with same base	$2^x = (2^3)^{y+1}$	$3^{x-9} = (3^2)^y$
equate powers	$2^x = 2^{3y+1}$	$3^{x-9} = 3^{2y}$
	$x = 3y + 1$	$x - 9 = 2y$
	$x - 3y = 1 \quad \textcircled{1}$	$x - 2y = 9 \quad \textcircled{2}$
Solve equations		
	$\begin{array}{r} x - 2y = 9 \\ -x + 3y = -3 \\ \hline y = 6 \end{array}$	
Sub $y = 6$ into $\textcircled{1}$	$x - 3(6) = 3$	
	$x - 18 = 3$	
	$x = 21$	
Solution:	$x = 21$, $y = 6$

Solve	$2^{2x+1} - 5(2^x) + 2 = 0$
$2^{2x+1} = 2(2^x)$	$\Rightarrow 2(2^x) - 5(2^x) + 2 = 0$
let $y = 2^x$	$\Rightarrow 2y^2 - 5y + 2 = 0$
solve quadratic	$(2y - 1)(y - 2) = 0$
	$y = \frac{1}{2}, y = 2$
Sub into $y = 2^x$	$\frac{1}{2} = 2^x \Rightarrow x = -1$
	$2 = 2^x \Rightarrow x = 1$

<p>Solve:</p> $\left. \begin{array}{l} 3^{x+2} - 82 + 3^{2-x} = 0 \\ 3^{x+2} = (3^2)(3^x) \\ = 9(3^x) \\ 3^{2-x} = \frac{(3^2)}{(3^x)} = \frac{9}{3^x} \end{array} \right\}$ <p>let $3^x = y$</p> <p><i>Multiply by y Solve quadratic</i></p>	$3^{x+2} - 82 + 3^{2-x} = 0$ $\Rightarrow 9(3^x) - 82 + \frac{9}{3^x} = 0$ $9y - 82 + \frac{9}{y} = 0$ $9y^2 - 82y + 9 = 0$ $(9y - 1)(y - 9) = 0$ $y = \frac{1}{9}, y = 9$ $\text{Sub solns into } y = 3^x \quad 3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2} \Rightarrow x = -2$ $3^x = 9 = 3^2 \Rightarrow x = 2$
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Logs and logarithmic equations

<p>Evaluate</p>	<p>i) $\log_7 343 = 3$</p> <p>ii) $\log_{27} \frac{1}{3} = -\frac{1}{3}$</p>
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Solve	$\log_2(x+6) - \log_2(x+2) = 1$
$\log a - \log b = \log \frac{a}{b}$	$\Rightarrow \log_2\left(\frac{x+6}{x+2}\right) = 1$
$\log_a a = 1$	$\Rightarrow \log_2\left(\frac{x+6}{x+2}\right) = \log_2 2$
	$\Rightarrow \frac{x+6}{x+2} = 2$
	$x+6 = 2(x+2)$
	$x+6 = 2x + 4$
	$2 = x$

Solve :	$\ln(x+1) + \ln(x-1) = \ln 3$
$\log a + \log b = \log ab$	$\ln(x+1)(x-1) = \ln 3$
DOTS	$\Rightarrow (x+1)(x-1) = 3$
	$x^2 - 1 = 3$
	$x^2 - 4 = 0$
DOTS	$(x-2)(x+2) = 0$
	$x = 2, x = -2$
	$\xrightarrow{\text{or}} x^2 = -4$
	$x = \pm 2$

Solve $\log_2 x - \log_2(x-1) = 4 \log_4 2$ $\log a - \log b = \log \frac{a}{b}$ on calculator $4 \log_4 2 = 2$	$\log_2 \left(\frac{x}{x-1} \right) = 4 \log_4 2$ $\log_2 \left(\frac{x}{x-1} \right) = 2$ $LHS = 2 (\log_2 2)$ $LHS = \log_2 2^2$ $LHS = \log_2 4$ $\therefore \frac{x}{x-1} = 4$ $x = 4(x-1)$ $x = 4x - 4$ $4 = 3x \Rightarrow x = \frac{4}{3}$
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$\log ab = \log a + \log b$ $\log_a a = 1$ $\log a^n = n \log a$	Given $p = \log_c x$ Express $\log_c \sqrt{x} + \log_c cx$ in terms of p . $\begin{aligned} & \log_c \sqrt{x} + \log_c cx \\ &= \log_c x^{\frac{1}{2}} + \log_c c + \log_c x \\ &= \frac{1}{2} \log_c x + 1 + \log_c x \\ &= \frac{3}{2} \log_c x + 1 \\ &= \frac{3}{2} p + 1 \end{aligned}$
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	<p>Loudness in dB is given by the formula $L = 10 \log I$ where I is the intensity.</p> <p>i) A whisper has the intensity of 100 what is the loudness in dB.</p> <p><i>note:</i> $\log a = \log_{10} a$</p>
	$L = 10 \log 100 = 20 \text{ dB}$ <p>ii) What is the Intensity of a sound that is 50 dB.</p> $50 = 10 \log I$ $5 = \log_{10} I$ $I = 10^5 = 100000$

	<p>The population of Snow leopards is 6000. If no conservation is put in place the population will decrease by 11% per year. find to the nearest year how long till the population is halved.</p> <p>DEPRECIATION</p> $F = P(1-i)^t$ <p>$F = 3000$ $P = 6000$ $i = 11\%$ $t = ?$</p>
	<p>When population is halved it will = 3000</p> $3000 = 6000(1 - 11\%)^t$ $\frac{3000}{6000} = (89\%)^t$ $t = \log_{89\%} \left(\frac{1}{2}\right)$ $= 5.948 \text{ years} \approx 6 \text{ years}$

Solve $\log_3 x + \log_3 y = 2$ (1)
 $\log_3(2y-3) - 2\log_3 x = 1$ (2)

$\log_a a = 1$
 $n \log a = \log a^n$ (1)
 $\log a + \log b = \log ab$

$\log_3 x + \log_3 y = 2 \log_3 3$
 $\log_3(xy) = \log_3 3^2$
 $LHS = \log_3 9 \Rightarrow xy = 9$ (3)

(2) $\log_3(2y-3) - 2\log_3 x = 1$ $y = \frac{9}{x}$
 $\log_3(2y-3) - \log_3 x^2 = 1$
 $\log_3(2y-3) - \left(\frac{\log_3 x^2}{\log_3 3}\right) = 1$
 $\log_3(2y-3) - \frac{\log_3 x^2}{2} = 1$

$2\log_3(2y-3) - \log_3 x^2 = 2$
 $\log_3(2y-3)^2 - \log_3 x^2 = 2 \log_3 3$
 $\log_3 \frac{(2y-3)^2}{x^2} = 2 \log_3 3$
 $2 \log_3 \left(\frac{2y-3}{x}\right) = 2 \log_3 3$

$\frac{2y-3}{x} = 3 \Rightarrow 2y-3 = 3x$ (4)
 $2(9/x) - 3 = 3x \Rightarrow 18 - 3x = 3x^2 \Rightarrow x^2 + x - 6 = 0$
 $(x-2)(x+3) = 0 \Rightarrow x=2, x=-3$
Reject $x=-3$ because can't get log of negative no.
 $y = 9/x = 9/2$ SOLUTION: $x=2, y=9/2$

SOLVING (3) & (4)

EXPONENTIAL FUNCTION

	In an experiment the following was recorded														
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>t</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> <tr> <td>$Q(t)$</td><td>2.920</td><td>2.642</td><td>2.391</td><td>2.163</td></tr> </table>					t	0	1	2	3	$Q(t)$	2.920	2.642	2.391	2.163
t	0	1	2	3											
$Q(t)$	2.920	2.642	2.391	2.163											
i)	<p>If $Q(t) = A e^{-bt}$</p> <p>Find the value of A and b.</p>														
$t=0$	$2.920 = A e^{-b(0)} = A(1) \Rightarrow A = 2.920$														
$t=1$	$2.642 = 2.920 e^{-b(1)}$														
	$\Rightarrow \frac{2.642}{2.920} = e^{-b} \Rightarrow -b = \ln \left(\frac{2.642}{2.920}\right)$														
	$\Rightarrow -b = -0.1 \Rightarrow b = 0.1$														
ii)	<p>Find the value of constant k for which</p> $Q(t+k) = \frac{1}{2} Q(t), t \geq 0$ $\cancel{2.92} e^{-0.1(t+k)} = \frac{1}{2} (\cancel{2.92} e^{-0.1t})$ $(e^{-0.1t})(e^{-0.1k}) = \frac{1}{2} (e^{-0.1t})$ $e^{-0.1k} = \frac{1}{2} \Rightarrow -0.1k = \ln \frac{1}{2}$ $k = \frac{\ln \frac{1}{2}}{-0.1} = 6.93$														
	$Q(t) = 2.92 e^{-0.1t}$														
	$\frac{1}{2} = e^n \Rightarrow n = \ln \frac{1}{2}$														
	equate powers														