Section 2.1 Quadratic equations

2. Use the quadratic formula to solve each of the following, giving your answers correct to one place of decimals:

(a) (i) $x^2 - 2x - 2 = 0$

3. Use the quadratic formula to solve each of the following, leaving your answers in surd form:

(a) (i) $3x^2 + 4x - 5 = 0$

4. Solve the following equations:

(a) (i) $\frac{x+7}{3} + \frac{2}{x} = 4$

5. By finding a suitable substitution, solve each of the following:

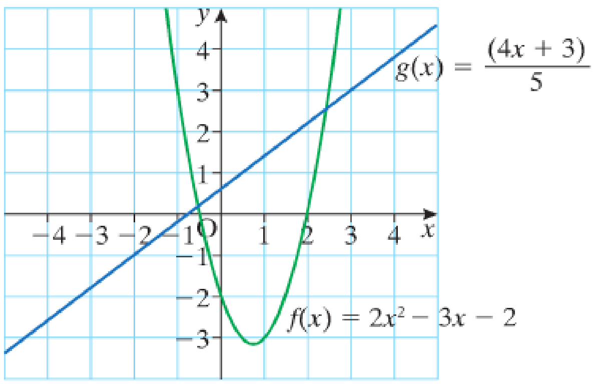
(c) $\left(y + \frac{4}{y}\right)^2 - 9\left(y + \frac{4}{y}\right) + 20 = 0$

10. The graphs of the functions

$f(x) = 2x^2 - 3x - 2$ and $g(x) = \dfrac{4x + 3}{5}$

are drawn as shown. Using the graphs, estimate the solutions of the following equations

(a) $f(x) = 0$
(b) $g(x) = 0$
(c) $f(x) = g(x)$.



Section 2.2 Nature of quadratic roots

9. Prove that the equation $(k - 2)x^2 + 2x - k = 0$ has real roots, whatever the value of k.

3. Find the discriminant of each of the following equations and state if the roots are

(a) real and different (b) real and equal (c) imaginary.

(i) $2x^2 + x + 5 = 0$ (ii) $-2x^2 + 3x + 1 = 0$ (iii) $3x^2 + 2x - 1 = 0$

(iv) $-3 + 2x - x^2 = 0$ (v) $x^2 + 8x + 16 = 0$ (vi) $25 - 10x + x^2 = ($

10. Find the value of k for which the equation $(k-2)x^2 + x(2k+1) + k = 0$ has equal roots.

13. Show that the equation $x^2 - 2px + 3p^2 + q^2 = 0$ cannot have real roots for $p, q \in R$.

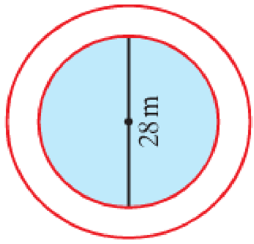
Section 2.3 Solving quadratic and linear equations

Solve: **12.** $x^2 + y^2 + 2x - 4y + 3 = 0$
$x - y + 3 = 0$

Section 2.4 Quadratic and linear equations in context

10. The hypotenuse of a right-angled triangle is 6 cm longer than the shortest side. The third side is 3 cm longer than the shortest side. Find the length of the shortest side.

13. A circular swimming pool with a diameter of 28 metres has a wooden deck around its edge.
If the deck has an area of $60\pi\,m^2$, find the width of the deck.



Section 2.5 Forming quadratic equations from their roots

1. State (i) the sum and (ii) the product of the roots of each of the following quadratic equations.

(a) $x^2 + 9x + 4 = 0$

(b) $x^2 - 2x - 5 = 0$

3. Find the quadratic equations that have the following pairs of roots (r_1, r_2).

(iv) $(\sqrt{5}, 4)$

(viii) $\left(\frac{5}{2}, \frac{3}{5}\right)$

Section 2.6 Max and Min of Quadratic graphs

3. Write each of the following in the form $(x - p)^2 + q = 0$.

(i) $x^2 + 4x - 6 = 0$

4. The graph of $y = a(x - p)^2 + q$ has a minimum point (p, q).
By completing the square, find the minimum point of each of the following quadratic equations:

(ii) $3x^2 - 6x - 1 = 0$

9. If $f(x) = x^2 + 4x + 7$, find

(i) the smallest possible value of $f(x)$

(ii) the value of x at which this smallest value occurs

(iii) the greatest possible value of $\frac{1}{(x^2 + 4x + 7)}$.

Section 2.7 Surds

2. Express each of the following in its simplest form:

(i) $2\sqrt{2} + 6\sqrt{2} - 3\sqrt{2}$ (ii) $2\sqrt{2} + \sqrt{18}$

5. By rationalising the denominator, express each of the following in its simplest form.

(ii) $\dfrac{12}{3 - \sqrt{2}}$

Section 2.8 Algebraic surd equations

4. Show that $\dfrac{-1+\sqrt{3}}{1+\sqrt{3}} = 2-\sqrt{3}$.

7. Solve the following equations and check your solutions in each case:

(iv) $\sqrt{3x-5} = x-1$

8. Solve each of these equations and check each solution:

(iv) $\sqrt{3x - 2} = \sqrt{x - 2} + 2$

Section 2.9 The factor theorem

6. Show that $(2x - 1)$ is a factor of $2x^3 + 7x^2 + 2x - 3$.

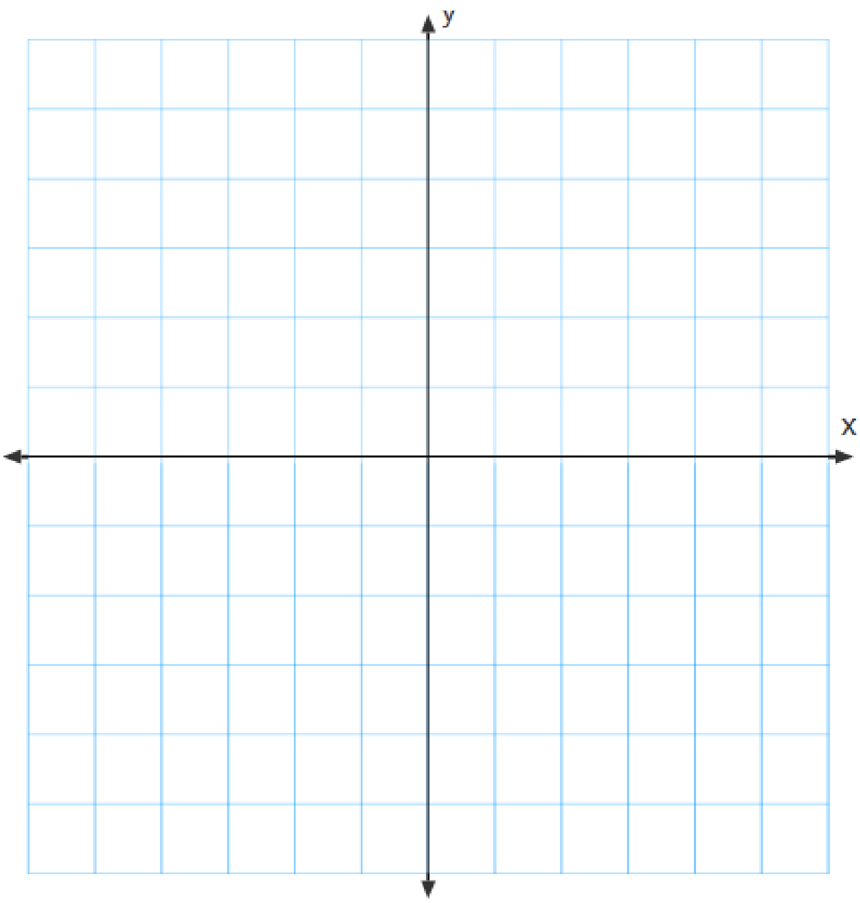
15. Factorise fully $x^3 - x^2 - 14x + 24$.
Hence solve the equation $x^3 - x^2 - 14x + 24 = 0$.

20. If $(x + 2)$ and $(x - 3)$ are both factors of $2x^3 + ax^2 - 17x + b$, find the values of a and b.

Hence find the third factor.

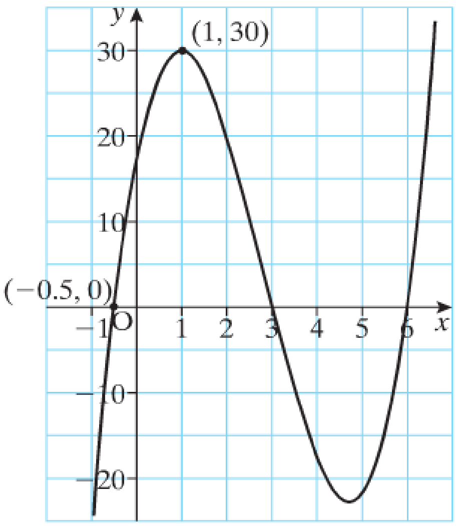
Section 2.10 Graphs of cubic polynomials

7. Given $f(x) = (x + 2)(x - 1)(x - 3)$, find the values of $f(0), f(\frac{1}{2})$ and $f(2)$. Hence draw a rough sketch of the curve.

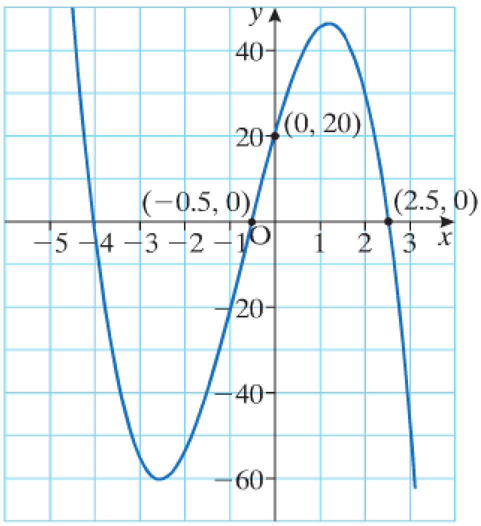


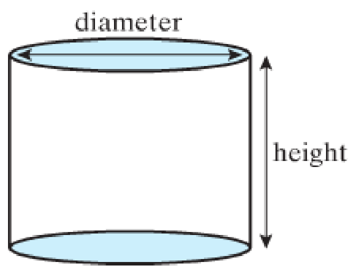
11. Find a cubic expression for each of the following curves.

(i)



(ii)





15. The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height.
Given that the diameter is equal to the height, show that the volume can be written as

$$V = ah^3.$$

Taking $\pi = 3.14$, find the value of a correct to two places of decimals.
Using this function, calculate the volume of a cylinder with a diameter of 11 cm.
Find the diameter of a cylinder whose volume is 215.58 cm^3, correct to one place of decimals.