3. Simultaneous equations in context

Example 4

An opera was attended by 240 people. Two ticket prices, €31 and €16, were available. If the total takings on the night were €5595, find using this data

(i) two linear equations connecting the two types of tickets sold
(ii) the number of €31 tickets sold
(iii) the number of €16 tickets sold.

let x = the no. of €31 tickets
let y = the no. of €16 tickets

$x + y = 240$ (1)

$31x + 16y = 5595$ (2)

Eliminate ys: (2) -16 (1)

$31x + 16y = 5595$
$-16x - 16y = -3840$
$15x = 1755$

$\Rightarrow x = \frac{1755}{15} \Rightarrow \boxed{x = 117}$

Sub $x = 117$ into (1)

$117 + y = 240 \Rightarrow \boxed{y = 123}$

Example 5

Fifty, twenty and ten cent coins are collected from a coin machine and counted. The total value of the coins is €32. When counting, the cashier noted that twice the number of twenty cent coins, added to the number of ten cent coins, equalled three times the number of fifty cent coins. She then noticed that four times the number of fifty cent coins, added to the number of ten cent coins, equalled six times the number of twenty cent coins.
Find the number of each type of coin in the machine.

x = no. of 50c coins
y = no. of 20c coins
z = no. of 10c coins

$50x + 20y + 10z = 3200$ (in cents)
$5x + 2y + z = 320$ (1)

$2y + z = 3x$
$-3x + 2y + z = 0$ (2)

$4x + z = 6y$
$4x - 6y + z = 0$ (3)

Eliminate zs: (1) $-$ (2)

$5x + 2y + z = 320$
$+3x - 2y - z = 0$
$8x = 320 \Rightarrow \boxed{x = 40}$

(1) $-$ (3)

$5x + 2y + z = 320$
$-4x + 6y - z = 0$
$x + 8y = 320$ (4)

Sub $x = 40$ into (4)

$40 + 8y = 320 \Rightarrow 8y = 280 \Rightarrow \boxed{y = 35}$

Sub $x = 40$, $y = 35$ into (3)

$4(40) - 6(35) + z = 0$

5. Solve the following equations with three unknowns. (i)

$$2x + y + z = 8 \quad (1)$$
$$5x - 3y + 2z = 3 \quad (2)$$
$$7x + y + 3z = 20 \quad (3)$$

let's eliminate the ys

(2) $5x - 3y + 2z = 3$

$+3(1)$ $+6x + 3y + 3z = 24$

$11x + 5z = 27$ (4)

(3) $7x + y + 3z = 20$

$-(1)$ $-2x - y - z = -8$

$5x + 2z = 12$ (5)

Solve (3) & (4) Eliminate zs

$2(4)$ $22x + 10z = 54$

$-5(5)$ $-25x - 10z = -60$

$-3x = -6$ $\Rightarrow x = 2$

Sub $x = 2$ into (5)

$5(2) + 2z = 12$

$10 + 2z = 12$ -10

$2z = 2$ $\Rightarrow z = 1$

Sub $x = 2$, $z = 1$ into (1)

$2(2) + y + 1 = 8$

$4 + y + 1 = 8$

$5 + y = 8$ $\Rightarrow y = 3$