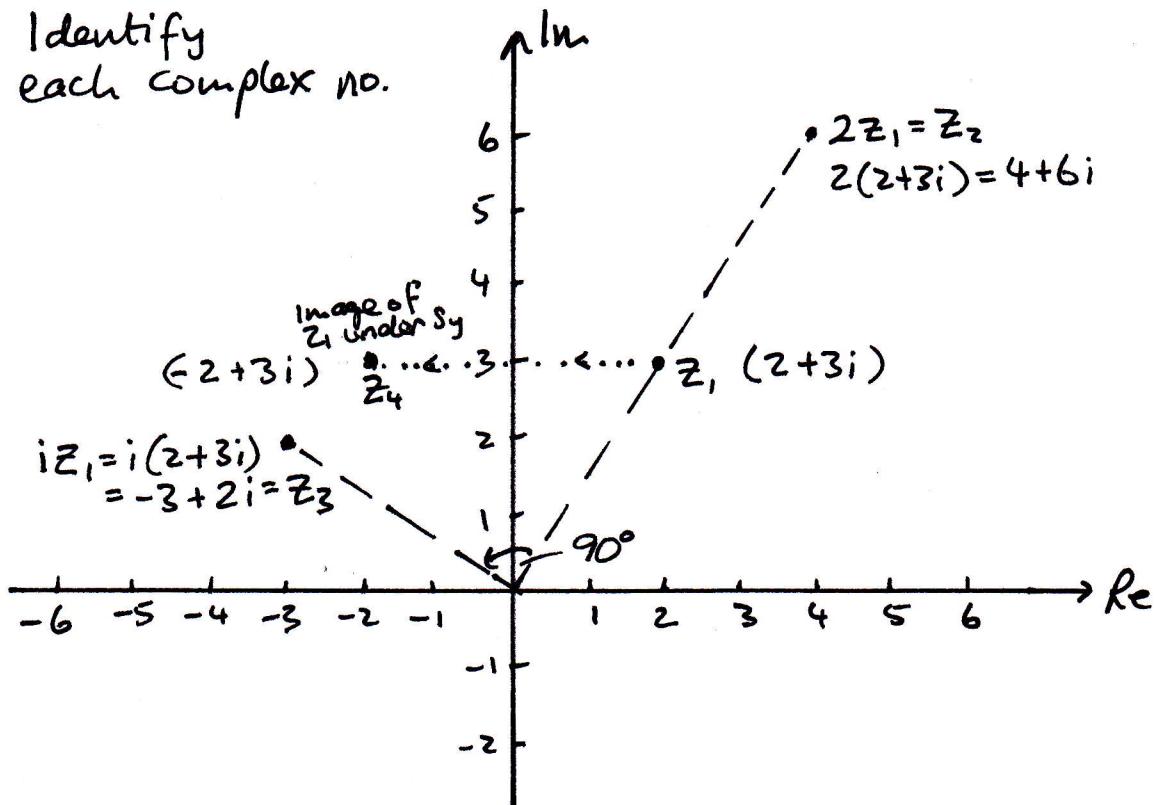


Q.1

- a) If $z = a+bi$, where a and $b \in \mathbb{R}$,
Show that $z \cdot \bar{z}$ simplifies to a constant.

$$\begin{aligned} z \cdot \bar{z} &= (a+bi)(a-bi) \\ &= a^2 - abi + abi - b^2 i^2 \\ &= a^2 + b^2 \end{aligned} \quad (15 \text{ marks})$$

- b) Identify each complex no.



Given: $z_2 = 2z_1$, $z_3 = iz_1$, $z_4 = \text{image of } z_1 \text{ under } S_y$

(10 marks)

Note: S_y = Axial Symmetry in the y-axis.

Q.2

a) Show $a^2 + 16 + b^2 \geq 2ab$ for all $a, b \in \mathbb{R}$

If $a^2 + 16 + b^2 \geq 2ab$

$$a^2 - 2ab + b^2 + 16 \geq 0$$

$$(a-b)^2 + 16 \geq 0 \quad \text{true}$$

Since $(a-b)^2 \geq 0$

QED

(10 marks)

b) Solve: $x + 3y = -5$

$$2x^2 + y^2 = 41$$

where $x, y \in \mathbb{Z}$.

(Rewrite linear) $x = -5 - 3y$

(Sub into quadratic) $2(-5-3y)^2 + y^2 = 41$

(Solve for y) $2(25 + 30y + 9y^2) + y^2 = 41$

$$50 + 60y + 18y^2 + y^2 = 41$$

$$19y^2 + 60y + 9 = 0$$

$$(19y + 3)(y + 3) = 0$$

$y = -\frac{3}{19} \times \text{X}, y = -3 \checkmark$
(reject: not $\in \mathbb{Z}$) (accept: $-3 \in \mathbb{Z}$)

(Sub into linear.
to solve for x.)

$$x = -5 - 3(-3) = 4$$

Solution: $(4, -3)$

(15 marks)

Q3

- a) Given that $x^2 + x - 6$ is a factor of $2x^3 - px^2 + qx - 6$ find the value of p and q.

(It will divide with no remainder)

$$\begin{array}{r} 2x + 1 \\ \hline x^2 + x - 6) 2x^3 - px^2 + qx - 6 \\ \cancel{- 2x^3} \cancel{- 2x^2} + 12x \\ \hline (-p-2)x^2 + (q+12)x - 6 \\ \quad \quad \quad \cancel{- x^2} \quad \cancel{+ x} \quad \cancel{- 6} \\ \hline 0x^2 + 0x + 0 \end{array}$$

(x^2 coefficients)

$$\begin{aligned} \Rightarrow -p - 2 - 1 &= 0 \\ -p - 3 &= 0 \\ p &= -3 \end{aligned}$$

(x coefficients)

$$\begin{aligned} q + 12 - 1 &= 0 \\ q &= -11 \end{aligned}$$

(10 marks)

- b) Prove $f(x) : x^3 + 7x^2 + 17x + 15$ has only one real root.

$$f(-3) = 0 \Rightarrow x = -3 \text{ is root \& } (x+3) \text{ is factor}$$

(divide by $x+3$)

$$\begin{array}{r} x^2 + 4x + 5 \\ \hline x+3) x^3 + 7x^2 + 17x + 15 \\ \cancel{- x^3} \cancel{- 3x^2} \\ \hline 4x^2 + 17x \\ \quad \quad \quad \cancel{- 4x} \cancel{- 12x} \\ \hline 5x + 15 \\ \quad \quad \quad \cancel{- 5x} \cancel{- 15} \end{array}$$

(Show $x^2 + 4x + 5 = 0$ has 2 imaginary roots)

$$\Delta = b^2 - 4ac < 0 \text{ if imaginary roots } a = 1, b = 4, c = 5$$

$$\Delta = 4^2 - 4(1)(5) = 16 - 20 = -4 < 0 \quad \text{QED.} \quad (15 \text{ marks})$$

Q4 Solve for x

a)

$$3^{2x+1} + 26(3^x) - 9 = 0$$

$$\text{note: } 3^{2x+1} = (3)3^{2x}$$

$$\text{let } 3^x = y \Rightarrow (3)3^{2x} = 3y^2$$

Solve

$$3y^2 + 26y - 9 = 0$$

$$(3y - 1)(y + 9) = 0$$

$$\begin{array}{l|l} 3y = 1 & y + 9 = 0 \\ y = \frac{1}{3} & y = -9 \end{array}$$

But $y = 3^x$

$$y = \frac{1}{3} = 3^{-1} ; y = -9 = 3^? \\ \text{not possible!}$$

$$\Rightarrow x = -1$$

Check: $3^{2(-1)+1} + 26(3)^{-1} - 9$

$$= 3^{-1} + \frac{26}{3} - 9 = \frac{1}{3} + \frac{26}{3} - 9$$

$$= 9 - 9 = 0 \quad \checkmark$$

(10 marks)

Q4 b) Sean wants to save €50,000 over a ten year period. If the expected interest rate is 2.5% how much does Sean need to invest at the beginning of each year to save €50,000.

let P = amount Sean invests each year
now consider future values of these 10 investments.

$$F_1 = P(1.025)^{10}$$

$$F_2 = P(1.025)^9$$

:

$$F_{10} = P(1.025)^1$$

The sum of F_1 to F_{10} = €50,000

Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$a = P(1.025)$$

$$r = 1.025$$

$$n = 10$$

$$S_{10} = \frac{P(1.025)(1 - 1.025^{10})}{1 - 1.025}$$

$$S_{10} = 11.483466 P$$

$$\Rightarrow 11.483466 P = 50000$$

$$P = 50000 / 11.483466 = €4,354.09$$

(10 marks)

Q5

- a) The probability distribution of the no. of students who forgot their book on a given day is shown below:

X	1	2	3	4	5
$P(X)$	0.1	0.2	0.3	0.3	0.1

- (i) Find the probability that at least three students forgot their text books on a given day.

$$\begin{aligned} P(\text{at least } 3) &= P(3 \text{ or } 4 \text{ or } 5) \\ &= (0.3) + (0.3) + (0.1) = 0.7 \end{aligned}$$

- (ii) Find the mean number of students who forgot their books on a given day.

X	1	2	3	4	5	Σ
$P(X)$	0.1	0.2	0.3	0.3	0.1	1
$X \cdot P(X)$	0.1	0.4	0.9	1.2	0.5	3.1

$$\bar{x} = \frac{3.1}{1} = 3.1$$

(10 marks)

Q5

- b) A group of 300 people surveyed re: an EU treaty. Complete the table

Gender	For	Against	Total
male	58	85	143
female	84	73	157
Total	142	158	300

- i) Find probability that a person is male and against the treaty.

$$P(M \cap Ag.) = \frac{85}{300} = \frac{17}{60}$$

- ii) Find the probability that the person is female or is for the treaty.

$$P(F \cup For) = \frac{157 + 58}{300} = \frac{43}{60}$$

(we could also do this another way)

$$P(F \cup For) = 1 - P(M \cap Ag) = 1 - \frac{17}{60} = \frac{43}{60}$$

- iii) Given that a person is for the treaty what is the probability that the person is male.

$$P(M | For) = \frac{58}{142} = \frac{29}{71}$$

(15 marks)

Q6.

(70 marks)

The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

- a) Complete the table below and hence calculate the expected value of X , $E(X)$.

X	13	14	15	16
$P(X)$	0.383	0.575	0.038	0.004
			↗	

$$1 - (0.383 + 0.575 + 0.004) = 0.038$$

($E(X)$ is like the mean value)

X	13	14	15	16	\leq
$P(X)$	0.383	0.575	0.038	0.004	1
$X \cdot P(X)$	4.979	8.05	0.57	0.064	13.663

$$E(X) = \frac{\sum x \cdot P(x)}{\sum P(x)} = \frac{13.663}{1} = 13.663$$

- ii) If X is age in complete years on 1 Jan. 2013 of a second year student selected at random from all Irish schools, explain what $E(X)$ represents.

The expected age of the selected person.
It is a mean value of all students.

(15 marks)

Q6 (i) What is a prime number?

(b)

A prime number is a natural number that has no natural number factors except 1 and itself.

(ii) Two dice A, B are rolled. Complete the table Sample Space for the sum of values on the faces.

		Die A						
		1	2	3	4	5	6	
Die B		1	2	3	4	5	6	7
2	3	4	5	6	7	8		
3	4	5	6	7	8	9		
4	5	6	7	8	9	10		
5	6	7	8	9	10	11		
6	7	8	9	10	11	12		

(iii) What is the probability that the sum is a prime number?

Primes are: 2, 3, 5, 7, 11

$$P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 11) = \frac{15}{36} = \frac{5}{12}$$

(iv) Probability sum is not a prime number?

$$P(\text{not prime}) = 1 - P(\text{prime}) = 1 - \frac{5}{12} = \frac{7}{12}$$

(20 MARKS)

Q6

- (C) Bob and Peter play a game based on rolling two dice. Bob pays Peter €3 if the sum is not prime and Peter pays Bob €3 if the sum is prime.

Complete the Table

Outcome	Not Prime	Prime
P	$\frac{7}{12}$	$\frac{5}{12}$
Net income to Bob	- €3	+ €3
$x \cdot P(x)$	€ 1.75	€ 1.25

(10 MARKS)

- (d) What is the expected value of the game
(i) to Bob? What does this value mean?

$$\text{Expected value} = 1.75 - 1.25 = € 0.50$$

This means on average Bob will win € 0.50 every time he plays

- (ii) If the game is played 30 times what are Bob's expected winnings?

$$\text{Expected Winnings} = \text{Expected Value} \times \text{no. trials.}$$

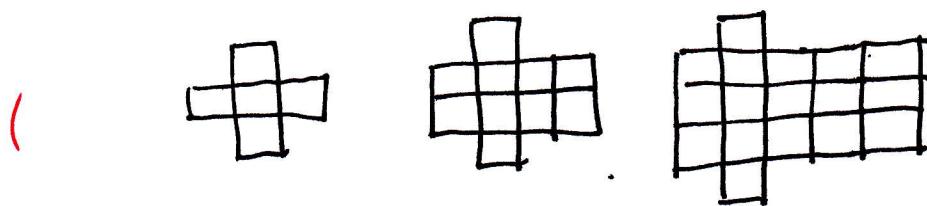
$$= (0.50)(30)$$

$$= € 15.00$$

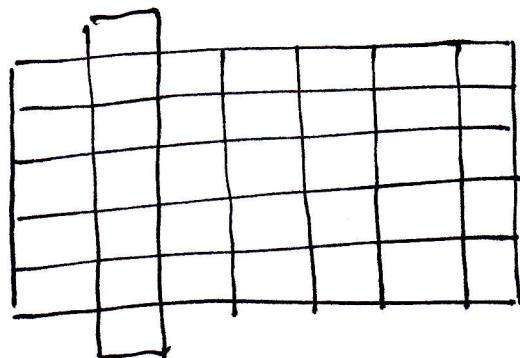
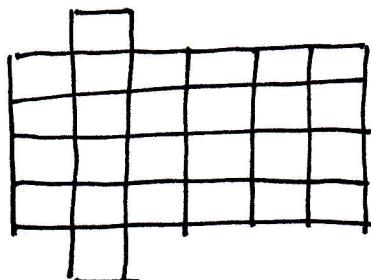
(10 MARKS)

Q7

Paul is creating designs using green blocks



a) Draw the next two patterns in the sequence



b) John states the pattern is linear. Mary disagrees. Would you agree with John or Mary?

(Assuming the pattern is referring to the number of blocks in each pattern)

Stage	1	2	3	4	5
# Blocks	5	10	17	26	37
1st Difference		5	7	9	11
2nd Difference			2	2	2

Because the 2nd difference is constant
the sequence is quadratic.

(C) The general term for the sequence is given by $T_n = an^2 + bn + c$. find the values of a, b and c .

$$T_1 = 5 \Rightarrow a(1)^2 + b(1) + c = 5 \\ a + b + c = 5 \quad \textcircled{1}$$

$$T_2 = 10 \Rightarrow a(2)^2 + b(2) + c = 10 \\ 4a + 2b + c = 10 \quad \textcircled{2}$$

$$T_3 = 17 \Rightarrow a(3)^2 + b(3) + c = 17 \\ 9a + 3b + c = 17 \quad \textcircled{3}$$

Solve equations:

$$\textcircled{2} - \textcircled{1} \Rightarrow 3a + b = 5 \quad \textcircled{4}$$

$$\textcircled{3} - \textcircled{2} \Rightarrow 5a + b = 7 \quad \textcircled{5}$$

$$\textcircled{5} - \textcircled{4} \Rightarrow 2a = 2$$

$$\boxed{a = 1}$$

Sub $a=1$ into $\textcircled{4}$ $3(1) + b = 5$

$$\boxed{b = 2}$$

sub $a=1$ and $b=2$ into $\textcircled{1}$

$$1 + 2 + c = 5 \Rightarrow \boxed{c = 2}$$

$$\rightarrow \boxed{T_n = n^2 + 2n + 2}$$

Check: $T_4 = 4^2 + 2(4) + 2 = 26 \quad \checkmark$

Q7

(d) What pattern will contain 122 green blocks.

$$T_n = n^2 + 2n + 2 = 122$$

$$n^2 + 2n - 120 = 0$$

$$(n + 12)(n - 10) = 0$$

$$\begin{aligned} n &= -12 \times, \quad n = 10 \\ (\text{no sense}) &\quad \checkmark \end{aligned}$$

$$\Rightarrow T_{10} = 122$$

Q8

Richter Scale formula : $M = \log_{10} \left[\frac{I}{I_0} \right]$

M = magnitude in mm

$I_0 = 10^{-3}$ mm

I = Intensity in mm

- (a) Calculate the magnitude of an earthquake with an intensity of 500 mm.

$$M = \log_{10} \left[\frac{500}{10^{-3}} \right] = 5.6989 \approx 5.7 \text{ mm}$$

- (b) In 2005 an earthquake measured 7.5 on the Richter Scale. Calculate the Intensity of the earthquake.

$$I = ? \text{ if } M = 7.5$$

$$M = \log_{10} \left[\frac{I}{I_0} \right]$$

$$7.5 = \log_{10} \left[\frac{I}{10^{-3}} \right]$$

$\boxed{\text{If } x^n = y \Leftrightarrow \log_x y = n}$

$$\begin{aligned} 10^{7.5} &= \frac{I}{10^{-3}} \Rightarrow I = (10^{7.5})(10^{-3}) = 10^{4.5} \\ &= 31,622.7766 \text{ mm} \end{aligned}$$

Q8

(c) A lorry on a road can get a reading of 3.3 on the Richter Scale. An at home seismograph records an intensity of 40,000 I₀ while the owner is away. Was the reading likely caused by an earthquake or a lorry passing the house?

$$M = 3.3 \text{ mm for lorry.}$$

Compare this with m of quake with intensity of 40,000 I₀.

$$M = \log_{10} \left[\frac{40,000 I_0}{I_0} \right] = \log_{10}(40,000)$$

$$M \approx 4.6 \text{ mm.}$$

This could not have been caused by a lorry so more likely to have been caused by an earthquake.