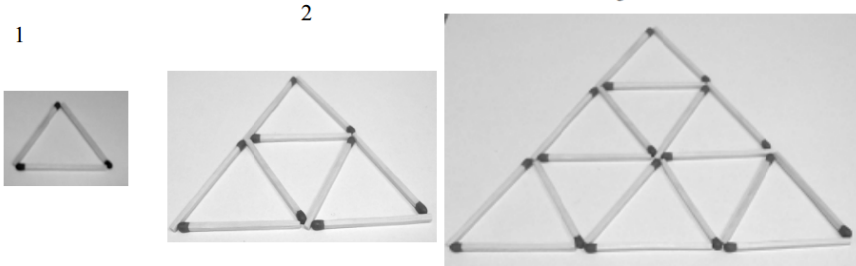
Question 9 **(50 marks)**

Shapes in the form of small equilateral triangles can be made using matchsticks of equal length. These shapes can be put together into patterns. The beginning of a sequence of these patterns is shown below.

1 2 3



(ii) The table below shows the number of small triangles in each pattern and the number of matchsticks needed to create each pattern. Complete the table.

Pattern	1st	2nd	3rd	4th
Number of small triangles	1	4	9	16
Number of matchsticks	3	9	18	30

(b) Write an expression in n for the number of triangles in the n^{th} pattern in the sequence.

n^2

(c) Find an expression, in n, for the number of matchsticks needed to turn the $(n-1)^{th}$ pattern into the n^{th} pattern.

$T_1 \quad T_2 \quad T_3 \quad T_4$

$3, \; 9, \; 18, \; 30, \; \ldots$

extra $\quad 6 \quad 9 \quad 12$

$3 \quad 3$

$T_2 - T_1 = 9 - 3 = 6 = 3(2)$

$T_3 - T_2 = 18 - 9 = 9 = 3(3)$

$T_4 - T_3 = 30 - 18 = 12 = 3(4)$

$T_n - T_{n-1} = 3(n)$

$T_n - T_{n-1} = 3n$

(d) The number of matchsticks in the n^{th} pattern in the sequence can be represented by the function $u_n = an^2 + bn$ where $a, b \in \mathbb{Q}$ and $n \in \mathbb{N}$. Find the value of a and the value of b.

$T_1 = a(1)^2 + b(1) = 3 \Rightarrow a + b = 3 \quad ①$

$T_2 = a(2)^2 + b(2) = 9 \Rightarrow 4a + 2b = 9 \quad ②$

② $\quad 4a + \cancel{2b} = 9$

$-2$① $\quad -2a - \cancel{2b} = -6$

$2a = 3$

$\boxed{a = 3/2}$

①

$3/2 + b = 3$

$b = 3 - 3/2 = 3/2$

$\boxed{b = 3/2}$

$u_n = \frac{3}{2}n^2 + \frac{3}{2}n$

(e) One of the patterns in the sequence has 4134 matchsticks. How many small triangles are in that pattern?

matchsticks

$U_n = \frac{3}{2}n^2 + \frac{3}{2}n$

$\frac{3}{2}n^2 + \frac{3}{2}n = 4134$

$3n^2 + 3n = 8268$

$n^2 + n - 2756 = 0$

$(n - 52)(n + 53) = 0$

$n = 52$ ✓ | $n = -53$ ✗

triangles $= n^2$

$= (52)^2$

$= 2704$