SOLUTIONS

PROJECT MATHS

Text & Tests 5

LEAVING CERTIFICATE HIGHER LEVEL

STRAND 1

PROBABILITY & STATISTICS

FULLY WORKED SOLUTIONS TO ALL QUESTIONS

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Contents

Text of Tests 5

Chapter 1: Probability 1

Exercise 1.1

Q1.

4	3	5

= 60 ways

Q2.

6	7

= 42 ways

Q3.

26	9	8

= 1872 codes

Q4.

10	6	4

= 240 ways

Q5.

6	5	4	3	2	1

= 6! = 720 ways

Q6.

7	6	5	4	3	2	1

= 7! = 5040 ways

6! × 2! = 1440 ways

Q7.

5	4	3	2	1

= 5! = 120 ways

(i)

1	4	3	2	1

= 24

(ii)

1	3	2	1	1

= 6

Q8.

7	6	5	4	3	2	1

= 5040 arrangements

(i)

2	6	5	4	3	2	1

= 1440

(ii) 6! × 2! = 1440

Q9. (i)

4	5	4	3	2	1

= 480 arrangements

(ii)

5	4	3	2	1	2

= 240 arrangements

(iii) 5! × 2! = 240 arrangements

Q10. (i)

1	6	5	4	3	2	1

= 720

(ii) 6! × 2! = 1440

Q11.

7	6	5	4	3	2	1

= 7! = 5040

5! × 3! = 720

Q12. (i) 5! × 3! = 720 arrangements

(ii)

4	3	3	2	2	1	1

= 144 arrangements

Q13. $\boxed{7}\boxed{6}\boxed{5}\boxed{4}$ or $\left({}^{7}P_{4}\right) = 840$

Q14. $\boxed{6}\boxed{5}\boxed{4}\boxed{3}$ or $\left({}^{6}P_{4}\right) = 360$

Q15. $\boxed{8}\boxed{7}\boxed{6}$ or $\left({}^{8}P_{3}\right) = 336$ ways

Q16. $\boxed{5}\boxed{4}\boxed{9}\boxed{8}$ = 1440 codes

Q17. (i) $\boxed{9}\boxed{8}\boxed{7}$ = 504 three-digit numbers

(ii) $\boxed{9}\boxed{9}\boxed{8}$ = 648 three-digit numbers

Q18. $\boxed{4}\boxed{3}\boxed{2}\boxed{1}$ = 24 four-digit numbers

(i) $\boxed{1}\boxed{3}\boxed{2}\boxed{1}$ = 6

(ii) $\boxed{3}\boxed{2}\boxed{1}\boxed{1}$ = 6

(iii) $\boxed{2}\boxed{3}\boxed{2}\boxed{1}$ = 12

Q19. $\boxed{9}\boxed{9}\boxed{8}\boxed{7}$ = 4536 four-digit numbers

(i) $\boxed{2}\boxed{9}\boxed{8}\boxed{7}$ = 1008

(ii) $\boxed{9}\boxed{8}\boxed{7}\boxed{1}$ = 504

Q20. $\boxed{3}\boxed{4}\boxed{3}\boxed{2}$ = 72 four-digit numbers

Start with 5 $\boxed{1}\boxed{3}\boxed{2}\boxed{1}$ = 6

Start with 8 $\boxed{1}\boxed{3}\boxed{2}\boxed{2}$ = 12

Start with 9 $\boxed{1}\boxed{3}\boxed{2}\boxed{1}$ = $\underline{6}$

24 four-digit numbers

Q21. $\boxed{5}\boxed{5}\boxed{4}$ = 100 three-digit numbers

(i) $\boxed{3}\boxed{5}\boxed{4}$ = 60

(ii) $\boxed{1}\boxed{5}\boxed{4}$ = 20

Q22. (i) $\boxed{5}\boxed{5}\boxed{4}\boxed{3}$ = 300 codes

(ii) $\boxed{1}\boxed{5}\boxed{4}\boxed{3}$ = 60 codes

Q23. $\boxed{3}\boxed{2}\boxed{1}\boxed{3}\boxed{2}\boxed{1}$ or $3! \times 3! = 36$ codes

Q24.

9	8	7	1	6	5	4

= 60,480 arrangements

Q25.

7	6	5	4	3	2	1

$= 7! = 5040$ arrangements

(i) $6! \times 2! = 1440$

(ii) $5040 - 1440 = 3600$

Q26. (i)

10	9	8

= 720 ways

(ii)

8	7	6

= 336 ways

(iii)

1	1	8

$= 8 \times 3! = 48$ ways

Exercise 1.2

Q1. (i) 15

(ii) 35

(iii) 45

(iv) 66

(v) 153

Q2. (i) $\binom{12}{9} = 220, \binom{12}{8} = 495, \binom{13}{9} = 715$

Hence, $220 + 495 = 715$

(ii) $\binom{10}{2} = 45 \quad \Rightarrow \quad 8\binom{10}{2} = 360$

$\binom{10}{3} = 120 \quad \Rightarrow \quad 3\binom{10}{3} = 360$

Hence, $8\binom{10}{2} = 3\binom{10}{3}$

Q3. $\binom{8}{5} = 56$ different selections

Q4. $\binom{14}{11} = 364$ teams ; $\binom{13}{10} = 286$ teams

Q5. $\binom{9}{5} = 126$ selections

(i) $\binom{8}{4} = 70$

(ii) $\binom{7}{4} = 35$

Q**6**. $\binom{9}{5} = 126$ ways ; $\binom{8}{4} = 70$ ways

Q**7**. $\binom{52}{3} = 22{,}100$ hands ; $\binom{13}{3} = 286$ hands

Q**8**. (i) $\binom{8}{3} = 56$

(ii) $\binom{9}{5} = 126$

(iii) $\binom{7}{3} = 35$

Q**9**. $\binom{5}{3} \times \binom{4}{3} = 10 \times 4 = 40$ ways

Q**10**. (i) $\binom{10}{3} \times \binom{12}{3} = 120 \times 220 = 26,400$

(ii) $\binom{10}{2} \times \binom{12}{4} = 45 \times 495 = 22,275$

Q**11**. $\binom{6}{3} = 20$ subsets

(i) $\binom{2}{1} \times \binom{4}{2} = 2 \times 6 = 12$

(ii) No vowels $= \binom{4}{3} = 4 \Rightarrow$ at least one vowel $= 20 - 4 = 16$

Q**12**. $\binom{8}{6} = 28$ ways

$\binom{4}{4} \times \binom{4}{2} = 1 \times 6 = 6$

Q**13**. (i) $\binom{5}{4} \times \binom{3}{2} = 5 \times 3 = 15$ ways

(ii) $\quad$ 4 men and 2 women $= \binom{5}{4} \times \binom{3}{2} = 5 \times 3 = 15$

or 5 men and 1 woman $= \binom{5}{5} \times \binom{3}{1} = 1 \times 3 = \underline{\ 3\ }$

Total $=$ 18 ways

Q14. $\binom{3}{1} \times \binom{6}{3} \times \binom{4}{2} = 3 \times 20 \times 6 = 360$ teams

Q15. (i) $\binom{8}{4} = 70$ subcommittees

(ii) $\binom{6}{2} = 15$ subcommittees

(iii) $\binom{6}{4} = 15$ subcommittees

Q16. $\binom{5}{3} = 10$ triangles

[XY] as one side $= \binom{3}{1} = 3$

Q17. (i) $\binom{6}{4} = 15$ quadrilaterals

(ii) [AB] as one side $= \binom{4}{2} = 6$

Q18. (i) $\binom{7}{3} = 35$ ways

(ii) $\quad$ Ann included, Barry excluded $= \binom{7}{4} = 35$

or Barry included, Ann excluded $= \binom{7}{4} = \underline{\ 35\ }$

Total $=$ 70 ways

(iii) No restrictions : 9 people, select 5 $= \binom{9}{5} = 126$

Ann, Barry and Claire excluded $= \binom{6}{5} = 6$

Hence, at least one of Ann, Barry and Claire must be included $= 126 - 6 = 120$ ways

Q19. 3 section A and 2 section B $\Rightarrow \binom{5}{3} \times \binom{7}{2} = 10 \times 21 = 210$

or 2 section A and 3 section B $\Rightarrow \binom{5}{2} \times \binom{7}{3} = 10 \times 35 = \underline{350}$

Total = 560 ways

Q20.

4	3	4	3	2

= 288 registrations

Q21. (i) $\binom{n}{2} = 10$

$$\Rightarrow \frac{n(n-1)}{2.1} = \frac{10}{1} \Rightarrow n^2 - n = 20$$
$$\Rightarrow n^2 - n - 20 = 0$$
$$\Rightarrow (n-5)(n+4) = 0$$
$$\Rightarrow n = 5 \text{ or } n = -4$$
since $n \in N$, $\Rightarrow n = 5$

(ii) $\binom{n}{2} = 45$

$$\Rightarrow \frac{n(n-1)}{2.1} = \frac{45}{1} \Rightarrow n^2 - n = 90$$
$$\Rightarrow n^2 - n - 90 = 0$$
$$\Rightarrow (n-10)(n+9) = 0$$
$$\Rightarrow n = 10 \text{ or } n = -9$$
since $n \in N$, $\Rightarrow n = 10$

(iii) $\binom{n+1}{2} = 28$

$$\Rightarrow \frac{(n+1)(n)}{2.1} = \frac{28}{1} \Rightarrow n^2 + n = 56$$
$$\Rightarrow n^2 + n - 56 = 0$$
$$\Rightarrow (n+8)(n-7) = 0$$
$$\Rightarrow n = -8 \text{ or } n = 7$$
since $n \in N$, $\Rightarrow n = 7$

Exercise 1.3

Q1. (i) Impossible
(ii) Very likely
(iii) Very unlikely
(iv) Very unlikely
(v) Even chance
(vi) Certain
(vii) Unlikely

Q2. (i) 6
(ii) 4
(iii) 0
(iv) 2

Q3. (i) 6
(ii) 8
(iii) 2

Q4. (i) $\frac{1}{6}$
(ii) $\frac{2}{6} = \frac{1}{3}$
(iii) $\frac{3}{6} = \frac{1}{2}$
(iv) $\frac{3}{6} = \frac{1}{2}$
(v) $\frac{2}{6} = \frac{1}{3}$
(vi) $\frac{3}{6} = \frac{1}{2}$

Q5. (i) $\frac{4}{52} = \frac{1}{13}$
(ii) $\frac{13}{52} = \frac{1}{4}$
(iii) $\frac{12}{52} = \frac{3}{13}$
(iv) $\frac{2}{52} = \frac{1}{26}$
(v) $\frac{20}{52} = \frac{5}{13}$

Q6. (i) $\frac{9}{17}$

(ii) $\frac{8}{17}$

(iii) $\frac{5}{17}$

(iv) $\frac{4}{17}$

Q7. (i) $\frac{1}{8}$

(ii) $\frac{2}{8}=\frac{1}{4}$

(iii) $\frac{3}{8}$

(iv) $\frac{4}{8}=\frac{1}{2}$

Q8. (i) $\frac{15}{30}=\frac{1}{2}$

(ii) $\frac{5}{30}=\frac{1}{6}$

(iii) $\frac{20}{30}=\frac{2}{3}$

(iv) $\frac{15}{30}=\frac{1}{2}$

Q9. (i) $\frac{3}{36}=\frac{1}{12}$

(ii) $\frac{9}{36}=\frac{1}{4}$

(iii) $\frac{6}{36}=\frac{1}{6}$

(iv) $\frac{12}{36}=\frac{1}{3}$

Q10. (i) $(3,3)$ gives a score $= 9 \Rightarrow P(9)=\frac{1}{36}$

(ii) $(1,4), (4,1), (2,2)$ give a score $= 4 \Rightarrow P(4)=\frac{3}{36}=\frac{1}{12}$

(iii) $(3,4)\ (4,3)\ (2,6)\ (6,2)$ give a score $= 12 \Rightarrow P(12)=\frac{4}{36}=\frac{1}{9}$

Q11. Box has 6 counters; 3 of these are green.

One green counter removed; hence, box has 2 green counters left.

$P(\text{green}) = \frac{2}{5}$

Q12. (i) $P(\text{purple}) = 1 - \frac{2}{5} = \frac{3}{5}$

(ii) 3

(iii) 3

Q13. Sample space S

$\#S = 12$

	1	2	3	4
5	6	7	8	9
7	8	9	10	11
8	9	10	11	12

(i) $\frac{1}{12}$

(ii) $\frac{2}{12} = \frac{1}{6}$

(iii) $\frac{6}{12} = \frac{1}{2}$

9 occurs most often $\Rightarrow P(9) = \frac{3}{12} = \frac{1}{4}$

Q14. $\#S = 36$

(i) Total $= 7$ occurs from $(3,4), (4,3), (2,5), (5,2), (6,1), (1,6)$

Total $= 11$ occurs from $(5,6)\ (6,5)$

$\Rightarrow \quad P(\text{wins}) = \frac{8}{36} = \frac{2}{9}$

(ii) Total $= 2$ occurs from $(1,1)$; Total $= 3$ occurs from $(1,2)\ (2,1)$

Total $= 12$ occurs from $(6,6)$

$\Rightarrow \quad P(\text{loses}) = \frac{4}{36} = \frac{1}{9}$

Q15. Sample space (S)

$\#S = 8$

HHH	HTT
HHT	THT
HTH	TTH
THH	TTT

(i) $P(\text{HHH}) = \frac{1}{8}$

(ii) $P(\text{HTH}) = \frac{1}{8}$

(iii) $P(\text{2H and 1T}) = \frac{3}{8}$

Q**16**. (i) $\frac{25}{50} = \frac{1}{2}$

(ii) $\frac{16}{50} = \frac{8}{25}$

(iii) $\frac{16}{50} = \frac{8}{25}$

25 males $\Rightarrow$ $P(\text{He wears glasses}) = \frac{16}{25}$

Q**17**. (i) $P(\text{Bus}) = \frac{60^\circ}{360^\circ} = \frac{1}{6}$

(ii) $\text{Walk} = 360^\circ - (90^\circ + 60^\circ) = 210^\circ$

$\Rightarrow$ $P(\text{Walk}) = \frac{210^\circ}{360^\circ} = \frac{7}{12}$

Exercise 1.4

Q**1**. (i) $P(2) = \frac{1}{6}$ $\Rightarrow$ Expected frequency $= \frac{1}{6} \times 900 = 150$

(ii) $P(6) = \frac{1}{6}$ $\Rightarrow$ Expected frequency $= \frac{1}{6} \times 900 = 150$

(iii) $P(2 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$ $\Rightarrow$ Expected frequency $= \frac{1}{3} \times 900 = 300$

Q**2**. (i) $P(\text{red}) = \frac{4}{8} = \frac{1}{2}$

(ii) (a) Expected frequency $= \frac{1}{2} \times 400 = 200$ times

(b) $P(\text{white}) = \frac{3}{8}$

$\Rightarrow$ Expected frequency $= \frac{3}{8} \times 400 = 150$ times

Q**3**. (i) Relative frequency (Heads) $= \frac{34}{100} = \frac{17}{50}$

(ii) No, probably not; as 34 is well below the expected value of 50.

Q**4**. (i) (a) Exp. $P(6) = \frac{60}{300} = \frac{1}{5}$

(b) Exp. $P(2) = \frac{40}{300} = \frac{2}{15}$

(ii) (a) $P(6) = \frac{1}{6}$

(b) $P(2) = \frac{1}{6}$

(iii) No; as 60 is well above the expected value of 50, and 40 is well below the expected value of 50.

Q5. (i) Estimate = relative frequency $= \frac{154}{300} = \frac{77}{150}$

(ii) No; as red is far higher than one would expect (54% higher).

Q6. $P(\text{red}) = \frac{5}{10} = \frac{1}{2}$

Expected frequency $= \frac{1}{2} \cdot 300 = 150$

Spinner is almost definitely not fair as red (120 times) should be much closer to 150 times.

Q7. (i) $x + 0.2 + 0.1 + 0.3 + 0.1 + 0.2 = 1$

$\Rightarrow \; x + 0.9 = 1$

$\Rightarrow \; x = 1 - 0.9 = 0.1$

(ii) $P(\text{number} > 3) = 0.3 + 0.1 + 0.2 = 0.6$

(iii) $P(6) = 0.2$

Expected frequency of 6 $= 0.2 \cdot 1000 = 200$ times

Q8. $P(\text{Gemma wins}) = \frac{21}{30} = \frac{7}{10}$

Q9. $P(6) = \frac{165}{1000} = \frac{33}{200}$

Use the largest number of trials.

Q10. (i) Bill's

(ii)

	Results		
Number of spins	0	1	2
Total = 580	187	267	126

Hence, spinner is biased.

(iii) $P(2) = \frac{126}{580} = \frac{63}{290}$

(iv) $P(0) = \frac{187}{580} = 0.322 \;\Rightarrow\;$ Expected frequency $= 0.322 \times 1000$

$= 322$

Q11. (i) $P(1) = \frac{2}{6} = \frac{1}{3}$

(ii) 1, 2, 2, 3, 3, 4

Exercise 1.5

Q**1**. (i) $\frac{8}{16}=\frac{1}{2}$

(ii) $\frac{4}{16}=\frac{1}{4}$

(iii) $\frac{8+4}{16}=\frac{12}{16}=\frac{3}{4}$

Q**2**. (i) $\frac{13}{52}=\frac{1}{4}$

(ii) $\frac{6}{52}=\frac{3}{26}$

(iii) $\frac{13+6}{52}=\frac{19}{52}$

Q**3**. (i) $\frac{10}{30}=\frac{1}{3}$

(ii) $\frac{6}{30}=\frac{1}{5}$

Not mutually exclusive as 15 and 30 are multiples of both 3 and 5

$\Rightarrow \quad P(\text{multiple of 3 or 5}) \;=\; \frac{10}{30}+\frac{6}{30}-\frac{2}{30}=\frac{14}{30}=\frac{7}{15}$

Q**4**. (i) $\frac{6}{12}=\frac{1}{2}$

(ii) $\frac{4}{12}=\frac{1}{3}$

(iii) $\frac{6}{12}+\frac{4}{12}-\frac{2}{12}=\frac{8}{12}=\frac{2}{3}$

Q**5**. (i) $\frac{13}{52}=\frac{1}{4}$

(ii) $\frac{4}{52}=\frac{1}{13}$

(iii) $\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$

(iv) $\frac{26}{52}=\frac{1}{2}$

(v) $\frac{4}{52}=\frac{1}{13}$

(vi) $\frac{26}{52}+\frac{4}{52}-\frac{2}{52}=\frac{28}{52}=\frac{7}{13}$

Q6. (i) $\frac{6}{36}=\frac{1}{6}$

(ii) Total of 8 $\Rightarrow$ $(2,6)\,(6,2)\,(3,5)\,(5,3)\,(4,4)$

$\Rightarrow$ $P(\text{total of } 8)=\frac{5}{36}$

(iii) $\frac{6}{36}+\frac{5}{36}-\frac{1}{36}=\frac{10}{36}=\frac{5}{18}$

Q7. $\#S=3+4+6+8+5+2=28$

(i) $\frac{14}{28}=\frac{1}{2}$

(ii) $\frac{21}{28}=\frac{3}{4}$

(iii) $\frac{8}{28}=\frac{2}{7}$

(iv) $\frac{14}{28}+\frac{14}{28}-\frac{6}{28}=\frac{22}{28}=\frac{11}{14}$

(v) $\frac{21}{28}=\frac{3}{4}$

(vi) $\frac{0}{28}=0$

Q8. $\#S=68+62+26+32+6+6=200$

(i) $\frac{32}{200}=\frac{4}{25}$

(ii) $\frac{100}{200}=\frac{1}{2}$

(iii) $\frac{12}{200}+\frac{100}{200}-\frac{6}{200}=\frac{106}{200}=\frac{53}{100}$

Q9. (i) $\frac{4}{16}=\frac{1}{4}$

(ii) $\frac{4}{16}=\frac{1}{4}$

(iii) $\frac{7}{16}$

(iv) $\frac{12}{16}=\frac{3}{4}$

(v) $\frac{4}{16}=\frac{1}{4}$

(vi) $\frac{12}{16}+\frac{4}{16}-\frac{2}{16}=\frac{14}{16}=\frac{7}{8}$

(vii) $\frac{4}{16}+\frac{8}{16}-\frac{2}{16}=\frac{10}{16}=\frac{5}{8}$

Q10. $n=$ number of green beads

$\Rightarrow \quad \#S=8+12+n=20+n$

$P(\text{green})=\frac{n}{20+n}=\frac{1}{5}$

$\Rightarrow \quad 5n=20+n$

$\Rightarrow \quad 4n=20$

$\Rightarrow \quad n=5$

Q11. 40 red, all even

30 blue, all odd

30 green — 20 even, 10 odd

(i) $P(\text{red})=\frac{40}{100}=\frac{2}{5}$

(ii) $P(\text{not blue})=\frac{70}{100}=\frac{7}{10}$

(iii) $P(\text{green or even})=\frac{30}{100}+\frac{60}{100}-\frac{20}{100}=\frac{70}{100}=\frac{7}{10}$

Q12. 100 people
- 40 male
  - 4 play tennis
  - 36 do not play tennis
- 60 female
  - 9 play tennis
  - 51 do not play tennis

(i) $\frac{4}{100}=\frac{1}{25}$

(ii) $\frac{4+9}{100}=\frac{13}{100}$

(iii) $\frac{60}{100}+\frac{13}{100}-\frac{9}{100}=\frac{64}{100}=\frac{16}{25}$

Q13. $A=\{20, 21, 22, 23\}$

$B=\{20, 25\}$

$C=\{23, 29\}$

$D=\{21, 24, 27\}$

(i)(a) No

(b) No

(c) No

(d) Yes

(e) Yes

(ii) $\dfrac{2}{10}+\dfrac{3}{10}=\dfrac{5}{10}=\dfrac{1}{2}$

(iii) No, as these 2 events are not mutually exclusive.

Q14. $\#S=6+2+9+3=20$

(i) $P(A)=\dfrac{8}{20}=\dfrac{2}{5}$

(ii) $P(B)=\dfrac{11}{20}$

(iii) $P(A\cup B)=\dfrac{6+2+9}{20}=\dfrac{17}{20}$

$P(A)+P(B)-P(A\cap B)=\dfrac{8}{20}+\dfrac{11}{20}-\dfrac{2}{20}=\dfrac{17}{20}$

Hence, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Q15. (i) $P(C)=0.4+0.2=0.6$

(ii) $P(D)=0.2+0.3=0.5$

(iii) $P(C\cup D)=0.4+0.2+0.3=0.9$

(iv) $P(C\cap D)=0.2$

$P(C)+P(D)-P(C\cap D)=0.6+0.5-0.2=0.9$

Hence, $P(C\cup D)=P(C)+P(D)-P(C\cap D)$

Q16. (i) $25+5+x+8=50$

$\Rightarrow \quad x+38=50$

$\Rightarrow \quad x=50-38=12$

(ii) $P(\text{French})=\dfrac{25+5}{50}=\dfrac{30}{50}=\dfrac{3}{5}$

(iii) $P(\text{French and Spanish})=\dfrac{5}{50}=\dfrac{1}{10}$

(iv) $P(\text{French or Spanish})=\dfrac{25+5+12}{50}=\dfrac{42}{50}=\dfrac{21}{25}$

(v) $P(\text{one language only})=\dfrac{25+12}{50}=\dfrac{37}{50}$

Q17. (i) $\#S= 5+3+11+1+2+8+7+3=40$

(ii) $\dfrac{1+2}{40}=\dfrac{3}{40}$

(iii) $\dfrac{1+2}{5+3+2+1}=\dfrac{3}{11}$

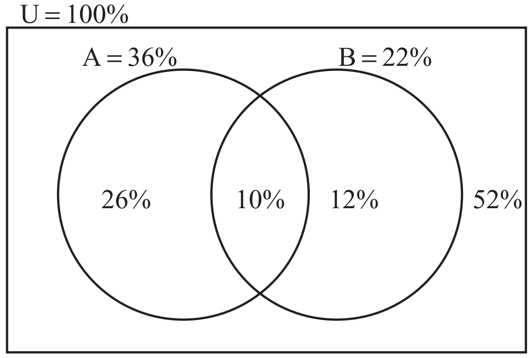
(iv) $\dfrac{3+2}{3+11+8+2}=\dfrac{5}{24}$

(v) $\dfrac{2}{1+2}=\dfrac{2}{3}$

Q18. (i) Venn Diagram

(ii) 52%

(iii) 26%



Q19.

$$P(A)+P(B)-P(A\cap B)=P(A\cup B)$$

$$\Rightarrow \quad \frac{2}{3}+P(B)-\frac{5}{12}=\frac{3}{4}$$

$$\Rightarrow \quad P(B)+\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow \quad P(B)=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$$

Q20.

$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$$

$$\Rightarrow \quad \frac{9}{10}=\frac{1}{2}+\frac{3}{5}-P(X\cap Y)$$

$$\Rightarrow \quad \frac{9}{10}=\frac{11}{10}-P(X\cap Y)$$

$$\Rightarrow \quad P(X\cap Y)=\frac{11}{10}-\frac{9}{10}=\frac{2}{10}=\frac{1}{5}$$

Q21.

$$P(C)+P(D)-P(C\cap D)=P(C\cup D)$$

$$\Rightarrow \quad 0.7+P(D)-0.3=0.9$$

$$\Rightarrow \quad P(D)+0.4=0.9$$

$$\Rightarrow \quad P(D)=0.9-0.4=0.5$$

Q22. (i)

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow \quad P(A\cup B)=0.8+0.5-0.3$$

$$=1$$

(ii) $P(A\cup B)=1$

$P(A)+P(B)-P(A\cap B)=0.8+0.5-0.3=1$

Hence, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Q23. (i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{8}{15}+\frac{2}{3}-\frac{1}{3}$$

$$=\frac{13}{15}$$

(ii) No, because $A\cap B\neq\phi$.

Q**24**. $P(A \cup B) = P(A) + P(B)$

$$= \frac{3}{7} + \frac{1}{5} = \frac{22}{35}$$

Exercise 1.6

Q**1**. (i) $P(R, R) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

(ii) $P(G, G) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$

(iii) $P(Y, Y) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

(iv) $P(R, G) = \frac{2}{5} \cdot \frac{1}{5} = \frac{2}{25}$

Q**2**. (i) $P(6, 6) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

(ii) $P(6, \text{Even}) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

(iii) $P(\text{Odd, multiple of 3}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

Q**3**. (i) $P(\text{Heads, 6}) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

(ii) $P(\text{Heads, Even}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Q**4**. (i) $P(\text{Black, Black}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(ii) $P(\text{King, King}) = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$

(iii) $P(\text{Black Ace, Diamond}) = \frac{1}{26} \cdot \frac{1}{4} = \frac{1}{104}$

Q**5**. (i) $P(\text{Red, Red}) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

(ii) $P(\text{Blue, Red}) = \frac{3}{5} \cdot \frac{2}{5} = \frac{6}{25}$

(iii) $P(\text{Red, Blue}) = \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{25}$

(iv) $P(\text{Blue, Blue}) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$

(v) $P(\text{same colour}) = \frac{4}{25} + \frac{9}{25} = \frac{13}{25}$

Q6. $P(\text{rain tomorrow, forget umbrella}) = \frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

Q7. (i) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(ii) $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

(iii) $\frac{1}{13} \cdot \frac{3}{13} = \frac{3}{169}$

(iv) $\frac{1}{13} \cdot \frac{1}{52} = \frac{1}{676}$

(v) $\frac{1}{52} \cdot \frac{1}{52} = \frac{1}{2704}$

Q8. $P(\text{rasp berry, rasp berry, rasp berry}) = \frac{4}{12} \cdot \frac{3}{12} \cdot \frac{2}{12} = \frac{24}{1728} = \frac{1}{72}$

Q9. $P(\text{hit gold area}) = 0.2 \Rightarrow P(\text{miss gold area}) = 0.8$

(i) $P(\text{hit, hit}) = (0.2)(0.2) = 0.04$

(ii) $P(\text{hit, miss}) + P(\text{miss, hit}) = (0.2)(0.8) + (0.8)(0.2) = 0.32$

Q10. $P(\text{Chris passes}) = 0.8 \Rightarrow P(\text{Chris fails}) = 0.2$

$P(\text{Georgie passes}) = 0.9 \Rightarrow P(\text{Georgie fails}) = 0.1$

$P(\text{Phil passes}) = 0.7 \Rightarrow P(\text{Phil fails}) = 0.3$

(i) $P(\text{all 3 pass}) = (0.8)(0.9)(0.7) = 0.504$

(ii) $P(\text{all 3 fail}) = (0.2)(0.1)(0.3) = 0.006$

(iii) $P(\text{at least one passes})$

$= 1 - P(\text{all 3 fail}) = 1 - 0.006 = 0.994$

Q11. $P(\text{Alan hits target}) = \frac{1}{2} \Rightarrow P(\text{Alan misses target}) = \frac{1}{2}$

$P(\text{Shane hits target}) = \frac{2}{3} \Rightarrow P(\text{Shane misses target}) = \frac{1}{3}$

(i) $P(\text{both men hit}) = \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

(ii) $P(\text{both men miss}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

(iii) $P(\text{only one hits}) = \frac{1}{2}\cdot\frac{1}{3}+\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{6}+\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$

Q12. $P(\text{stops at first}) = 0.6 \quad \Rightarrow \quad P(\text{doesn't stop at first}) = 0.4$

$P(\text{stops at second}) = 0.7 \quad \Rightarrow \quad P(\text{doesn't stop at second}) = 0.3$

$P(\text{stops at third}) = 0.8 \quad \Rightarrow \quad P(\text{doesn't stop at third}) = 0.2$

(i) $P(\text{stops at all three}) = (0.6)(0.7)(0.8) = 0.336$

(ii) $P(\text{he is late}) = P(\text{stop, stop, doesn't stop}) + P(\text{stop, doesn't stop, stop}) + P(\text{doesn't stop, stop, stop}) + P(\text{stop, stop, stop})$

$$= (0.6)(0.7)(0.2) + (0.6)(0.3)(0.8) + (0.4)(0.7)(0.8) + (0.6)(0.7)(0.8)$$
$$= 0.084 + 0.144 + 0.224 + 0.336$$
$$= 0.788$$

Q13. (i) $P(\text{no sixes}) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{125}{216}$

(ii) $P(\text{at least one six})$

$= 1 - P(\text{no sixes}) = 1 - \frac{125}{216} = \frac{91}{216}$

(iii) $P(\text{exactly one six}) = P(6, \text{other, other}) + P(\text{other}, 6, \text{other}) + P(\text{other, other}, 6)$

$$= \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} + \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$$
$$= \frac{25}{72}$$

$P(\text{same number}) = \left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\right) \times 6 = \frac{1}{36}$

Q14. (i) $P(\text{both on Monday}) = \frac{1}{7}\cdot\frac{1}{7} = \frac{1}{49}$

(ii) $P(\text{both on same day}) = 1\cdot\frac{1}{7} = \frac{1}{7}$

(iii) $P(\text{both on different days}) = 1\cdot\frac{6}{7} = \frac{6}{7}$

(iv) $P(\text{both on Monday}) + P(\text{Monday, other day}) + P(\text{other day, Monday})$

$= \frac{1}{7}\cdot\frac{1}{7} + \frac{1}{7}\cdot\frac{6}{7} + \frac{6}{7}\cdot\frac{1}{7} = \frac{13}{49}$

Q15. (i) $P(\text{none on a Sunday}) = \frac{6}{7}\cdot\frac{6}{7}\cdot\frac{6}{7} = \frac{216}{343}$

(ii) $P(\text{one on a Sunday})$

$= P(\text{Sunday, other, other}) + P(\text{other, Sunday, other}) + P(\text{other, other, Sunday})$

$= \frac{1}{7}\cdot\frac{6}{7}\cdot\frac{6}{7} + \frac{6}{7}\cdot\frac{1}{7}\cdot\frac{6}{7} + \frac{6}{7}\cdot\frac{6}{7}\cdot\frac{1}{7} = \frac{108}{343}$

(iii) $P(\text{at least one on a Sunday}) = 1 - P(\text{none on a Sunday})$

$$= 1 - \frac{216}{343} = \frac{127}{343}$$

Exercise 1.7

Q1. (i) $\#S = 26 \quad \Rightarrow \quad P(\text{Spade}) = \frac{13}{26} = \frac{1}{2}$

(ii) $\#S = 26 \quad \Rightarrow \quad P(\text{Queen}) = \frac{2}{26} = \frac{1}{13}$

(iii) $\#S = 12 \quad \Rightarrow \quad P(\text{King}) = \frac{4}{12} = \frac{1}{3}$

Q2. (i) $\#S = 90 \quad \Rightarrow \quad P(\text{Person can drive}) = \frac{70}{90} = \frac{7}{9}$

(ii) $\#S = 40 \quad \Rightarrow \quad P(\text{Man can drive}) = \frac{32}{40} = \frac{4}{5}$

(iii) $\#S = 50 \quad \Rightarrow \quad P(\text{Female can drive}) = \frac{38}{50} = \frac{19}{25}$

Q3. (i) $\frac{2}{12} = \frac{1}{6}$

(ii) $\frac{8}{12} = \frac{2}{3}$

Q4. (i) $\#S = 120 \quad \Rightarrow \quad P(\text{Ordinary}) = \frac{45}{120} = \frac{3}{8}$

(ii) $\#S = 55 \quad \Rightarrow \quad P(\text{Girl, Higher}) = \frac{35}{55} = \frac{7}{11}$

(iii) $\#S = 45 \quad \Rightarrow \quad P(\text{Boy, Ordinary}) = \frac{25}{45} = \frac{5}{9}$

Q5. (i) $P(\text{Red}) = \frac{5}{8}$

(ii) $P(\text{Red, Red}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$

(iii) $P(\text{Blue, Blue}) = \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$

(iv) $P(\text{both same colour}) = \frac{5}{14} + \frac{3}{28} = \frac{10}{28} + \frac{3}{28} = \frac{13}{28}$

Q**6**. (i) $P(\text{Red, Red}) = \frac{5}{11}\cdot\frac{4}{10} = \frac{2}{11}$

(ii) $P(\text{Red, Black}) = \frac{5}{11}\cdot\frac{6}{10} = \frac{3}{11}$

(iii) $P(\text{Black, Black}) = \frac{6}{11}\cdot\frac{5}{10} = \frac{3}{11}$

(iv) $P(\text{Same colour}) = \frac{2}{11}+\frac{3}{11} = \frac{5}{11}$

(v) $P(\text{Second is Red}) = P(\text{Red, Red}) + P(\text{Black, Red})$

$$= \frac{5}{11}\cdot\frac{4}{10}+\frac{6}{11}\cdot\frac{5}{10} = \frac{20}{110}+\frac{30}{110} = \frac{2}{11}+\frac{3}{11} = \frac{5}{11}$$

Q**7**. (i) $P(T,N) = \frac{1}{5}\cdot\frac{1}{4} = \frac{1}{20}$

(ii) $P(E,V) = \frac{2}{5}\cdot\frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

(iii) $P(\text{Second is } E) = P(E,E)+P(V,E)+P(N,E)+P(T,E)$

$$= \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{2}{4} + \frac{1}{5}\cdot\frac{2}{4} + \frac{1}{5}\cdot\frac{2}{4} = \frac{8}{20} = \frac{4}{10} = \frac{2}{5}$$

Q**8**. $P(\text{Same letters}) = P(I,I)+P(M,M)$

$$= \frac{2}{8}\cdot\frac{1}{7}+\frac{2}{8}\cdot\frac{1}{7} = \frac{2}{56}+\frac{2}{56} = \frac{4}{56} = \frac{1}{14}$$

Q**9**. (i) $\#S = 33 \;\Rightarrow\; P(\text{girl}) = \frac{20}{33}$

(ii) $\#S = 13 \;\Rightarrow\; P(\text{boy, left-handed}) = \frac{4}{13}$

(iii) $\#S = 13 \text{ boys} \;\Rightarrow\; P(\text{left-handed}) = \frac{4}{13}$

$\#S = 20 \text{ girls} \;\Rightarrow\; P(\text{left-handed}) = \frac{5}{20} = \frac{1}{4}$

$\Rightarrow\; P(\text{both left-handed}) = \frac{4}{13}\cdot\frac{1}{4} = \frac{4}{52} = \frac{1}{13}$

(iv) $\#S = 24 \;\Rightarrow\; P(\text{boy, right-handed}) = \frac{9}{24} = \frac{3}{8}$

Q**10**. (i) $(0.8)(0.6) = 0.48$

(ii) $P(\text{fails in at least one task}) = 1 - P(\text{succeeds in both tasks})$

$$= 1 - 0.48 = 0.52$$

Q**11**. $\left(\frac{3}{5}\cdot\frac{2}{4}\right)\times 2 = \frac{3}{5}\; \left[\text{OR } P(O,E)+P(E,O) \Rightarrow \frac{3}{5}\cdot\frac{2}{4}+\frac{2}{5}\cdot\frac{3}{4} = \frac{6}{20}+\frac{6}{20} = \frac{3}{5}\right]$

Q**12**. (i) $P(A) = 0.4 + 0.2 = 0.6$

(ii) $P(A \cap B) = 0.2$

(iii) $P(A \cup B) = 0.4 + 0.2 + 0.3 = 0.9$

(iv) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.5} = 0.4$

(v) $B(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.2}{0.6} = \dfrac{1}{3}$

Q**13**. (i) $P(A) = \dfrac{8+4}{30} = \dfrac{12}{30} = \dfrac{2}{5}$

(ii) $P(A \cap B) = \dfrac{4}{30} = \dfrac{2}{15}$

(iii) $P(A \cup B) = \dfrac{8+4+12}{30} = \dfrac{24}{30} = \dfrac{4}{5}$

(iv) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{4}{16} = \dfrac{1}{4}$

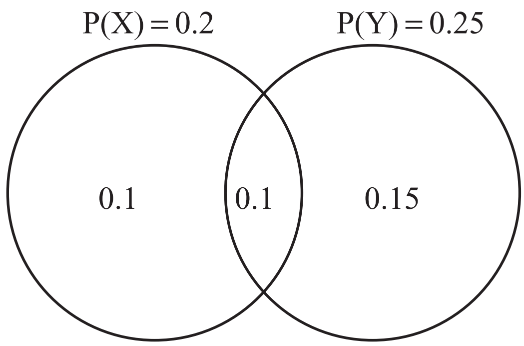
$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{4}{12} = \frac{1}{3} \neq \frac{1}{4}$$

Hence, $P(A \mid B) \neq P(B \mid A)$

Q**14**. (i) $P(X \cup Y) = 0.1 + 0.1 + 0.15$
$= 0.35$

(ii) $P(X \mid Y) = \dfrac{P(X \cap Y)}{P(Y)} = \dfrac{0.1}{0.25} = 0.4$

(iii) $P(Y \mid X) = \dfrac{P(Y \cap X)}{P(X)} = \dfrac{0.1}{0.2} = 0.5$



Q**15**. (i) $P(A) = 0.2 + 0.1 = 0.3$

(ii) $P(A \cup B) = 0.2 + 0.1 + 0.4 = 0.7$

(iii) $P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

(iv) $P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

(v) $P(A' \cap B) = 0.4$

(vi) $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$

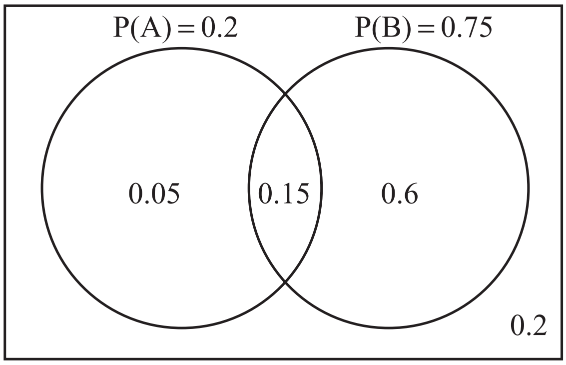
Q16. (i) Complete the Venn diagram.

(ii) 0.2

(iii) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.15}{0.75}=0.2$

(iv) $P(B\mid A)=\dfrac{P(B\cap A)}{P(A)}=\dfrac{0.15}{0.2}=0.75\neq 0.2$

Hence, $P(A\mid B)\neq P(B\mid A)$

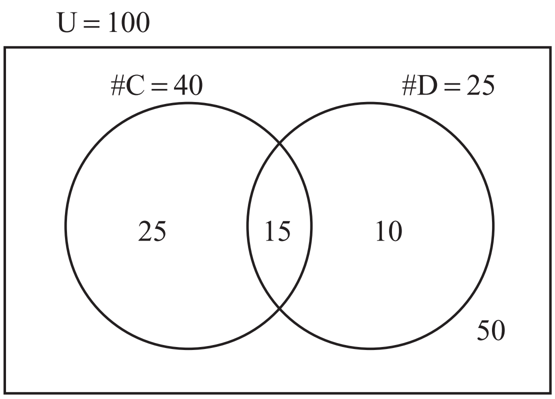


Q17. (i) $\dfrac{25+15+10}{100}=\dfrac{50}{100}=0.5$

(ii) $\dfrac{25+10}{100}=\dfrac{35}{100}=0.35$

(iii) $P(D\mid C)=\dfrac{P(D\cap C)}{P(C)}=\dfrac{15}{40}=0.375$

(iv) $P(C'\mid D)=\dfrac{P(C'\cap D)}{P(D)}=\dfrac{10}{25}=0.4$

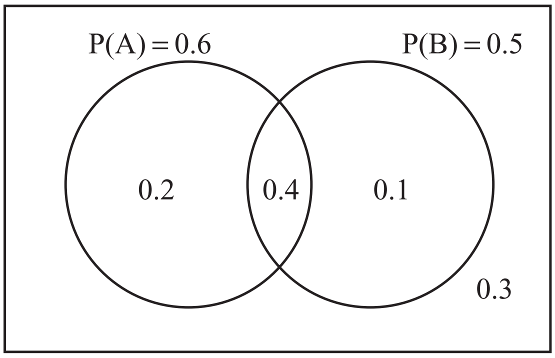


Q18. (i) $P(A\cup B)=0.2+0.4+0.1=0.7$

(ii) $P(B\mid A)=\dfrac{P(B\cap A)}{P(A)}=\dfrac{0.4}{0.6}=\dfrac{2}{3}$

(iii) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.4}{0.5}=0.8$

(iv) $P(B\cap A')=0.1$

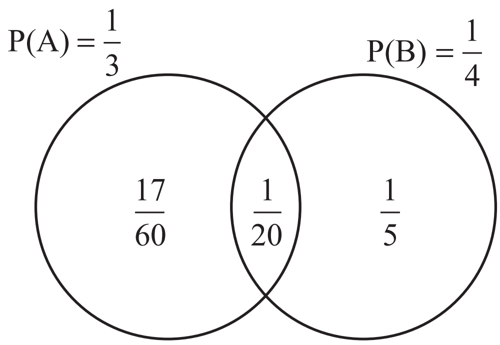


Q19. (i) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$

$\Rightarrow\ P(A\cap B)=P(A\mid B).P(B)$

$=\dfrac{1}{5}\cdot\dfrac{1}{4}=\dfrac{1}{20}$

(ii) $P(B\mid A)=\dfrac{P(B\cap A)}{P(A)}=\dfrac{\frac{1}{20}}{\frac{1}{3}}=\dfrac{3}{20}$



Q20. (i) $P(B)=0.18+0.17+0.02+0.05=0.42$

(ii) $P(A\cap C)=0.08+0.02=0.1$

(iii) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.2}{0.42}=\dfrac{10}{21}$

(iv) $P(C\mid B)=\dfrac{P(C\cap B)}{P(B)}=\dfrac{0.07}{0.42}=\dfrac{1}{6}$

(v) $P(A\cap C')=0.3+0.18=0.48$

(vi) $P[B|(A\cap C)]=\dfrac{P(B\cap A\cap C)}{P(A\cap C)}=\dfrac{0.02}{0.1}=0.2$

Test Yourself 1

A Questions

Q**1**. $\boxed{5}\boxed{4}\boxed{3}=60$ three-digit numbers

(i) $\boxed{1}\boxed{4}\boxed{3}=12$

(ii) $\boxed{3}\boxed{4}\boxed{3}=36$

Q**2**. (i) $\dbinom{11}{4}=330$ different groups

(ii) $\dbinom{\mathbf{5}}{2}\times\dbinom{\mathbf{6}}{2}=(10)(15)=150$

Q**3**. (i) $\dfrac{1}{36}$

(ii) $\left(\dfrac{1}{6}\right)\left(\dfrac{1}{6}\right)\times 6=\dfrac{6}{36}=\dfrac{1}{6}$

(iii) $\dfrac{1}{36}+\dfrac{6}{36}-\dfrac{1}{36}=\dfrac{6}{36}=\dfrac{1}{6}$

Q**4**. (i) $1-(0.35+0.1+0.25+0.15)=0.15$

(ii) 1

(iii) $0.25\cdot\ 200=50$ times

Q**5**. (i) $\boxed{6}\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1}=6!=720$ arrangements

(ii) $5!\ \cdot\ 2!=240$

Q**6**. (i) (a)

(ii) $P(\text{H or T})=P(\text{H}\cup\text{T})=P(\text{H})+P(\text{T})=\dfrac{10}{30}+\dfrac{12}{30}=\dfrac{22}{30}=\dfrac{11}{15}$

Q7. (i) $P(2)=\dfrac{2}{6}=\dfrac{1}{3}$

(ii) $P(2\text{ on first two throws})=\dfrac{1}{3}\cdot\dfrac{1}{3}=\dfrac{1}{9}$

(iii) $P(\text{first 2 on third throw})=\dfrac{2}{3}\cdot\dfrac{2}{3}\cdot\dfrac{1}{3}=\dfrac{4}{27}$

Q**8**. $P(\text{blue, red, red or green}) = \frac{6}{13} \cdot \frac{4}{12} \cdot \frac{6}{11} = \frac{144}{1716} = \frac{12}{143}$

Q**9**. Event $A = \{3,\ 6,\ 9,\ 12,\ 15,\ 18\}$

Event $B = \{4,\ 8,\ 12,\ 16,\ 20\}$

(i) $P(A) = \frac{6}{20} = \frac{3}{10} = 0.3$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}$$

(iii) $P(A \cap B)' = \frac{19}{20}$

Q**10**. (i) $\#S = 25 \Rightarrow P(E) = \frac{6}{25} + \frac{5}{25} = \frac{11}{25}$

(ii) $\#S = 13 \Rightarrow P(E) = \frac{7}{13}$

(iii) $P(E) = \frac{5}{25} \cdot \frac{4}{24} = \frac{20}{600} = \frac{1}{30}$

$\left[\text{Note: Here E = Event}\right]$

B Questions

Q**1**. (i) $P(\text{6 on first throw}) = \frac{1}{6}$

(ii) $P(\text{first 6 on second throw}) = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$

(iii) $P(\text{first 6 on either first or second throw}) = \frac{1}{6} + \frac{5}{36} = \frac{11}{36}$

Q**2**. (i) $\binom{8}{4} = 70$ choices

(ii) $\binom{7}{3} = 35$

(iii) A and 6 others, select $3 = \binom{6}{3} = 20$

B and 6 others, select $3 = \binom{6}{3} = \underline{20}$

Total = 40

Q3. (i) $\boxed{7}\boxed{6}\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1} = 7! = 5040$ arrangements

(ii) $\boxed{1}\boxed{1}\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1} = 120$

(iii) $\boxed{4}\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1}\boxed{3} = 1440$

Q4. (i) Score of 6 $\Rightarrow$ $P(2,2,2) = \frac{2}{3}\cdot\frac{1}{2}\cdot\frac{2}{3} = \frac{2}{9}$

(ii) Score of 9 $\Rightarrow$ $P(3,3,3) = \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{3} = \frac{1}{18}$

(iii) Score of 7 $\Rightarrow$ $P(2,2,3) + P(2,3,2) + P(3,2,2)$

$$= \frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}$$

$$= \frac{2}{18} + \frac{4}{18} + \frac{2}{18} = \frac{8}{18} = \frac{4}{9}$$

Q5. (i) Events L and M cannot happen at the same time.

(ii) (a) $\binom{22}{4} = 7315$ possible selections

(b) 22 students $\Rightarrow$ select Janelle and 3 others $= \binom{21}{3} = 1330$

(c) $P(\text{Janelle included}) = \frac{1330}{7315} = \frac{2}{11}$

Q6. (i) $P(\text{first 10c, second 5c}) = \frac{4}{6}\cdot\frac{2}{5} = \frac{8}{30} = \frac{4}{15}$

(ii) $P(\text{sum} = 15\text{c}) = P(\text{first 10c, second 5c}) + P(\text{first 5c, second 10c})$

$$= \frac{4}{6}\cdot\frac{2}{5} + \frac{2}{6}\cdot\frac{4}{5} = \frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}$$

(iii) $P(\text{sum} = 20\text{c}) = P(\text{first 10c, second 10c}) = \frac{4}{6}\cdot\frac{3}{5} = \frac{12}{30} = \frac{2}{5}$

Q7. (i) $0.2 + 0.3 + x + 0.1 = 1$

$$\Rightarrow x = 1 - 0.6 = 0.4$$

(ii) $P(A) = 0.2 + 0.3 = 0.5$

(iii) $P(A \cup B) = 0.2 + 0.3 + 0.4 = 0.9$

(iv) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$

(v) $P(A \mid B) = \frac{3}{7}$ and $\frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$

Hence, $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Q**8**. (i) $P(A) = \frac{20}{35} = \frac{4}{7}$

(ii) $P(B) = \frac{26}{35}$

(iii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{16}{26} = \frac{8}{13}$

(iv) $P(A \cap B) = \frac{16}{35}$

(v) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{7} + \frac{26}{35} - \frac{16}{35} = \frac{30}{35} = \frac{6}{7}$

$P(B).P(A|B) = \frac{26}{35} \cdot \frac{8}{13} = \frac{16}{35} = P(A \cap B)$

Events are not mutually exclusive, hence we cannot apply $P(A \cup B) = P(A) + P(B)$.

Q**9**. (i) $P(\text{green, green}) = \frac{x}{x+6} \cdot \frac{x-1}{x+5}$

(ii) $\frac{x}{x+6} \cdot \frac{x-1}{x+5} = \frac{4}{13}$

$\Rightarrow 4x^2 + 44x + 120 = 13x^2 - 13x$

$\Rightarrow 9x^2 - 57x - 120 = 0$

$\Rightarrow 3x^2 - 19x - 40 = 0$

$\Rightarrow (3x+5)(x-8) = 0$

$\Rightarrow x = -\frac{5}{3}$ or $x = 8$

$\Rightarrow$ valid answer: $x = 8$

$\Rightarrow$ number of discs $= 4 + 2 + 8 = 14$

(iii) $P(\text{not green, not green}) = \frac{6}{14} \cdot \frac{5}{13} = \frac{15}{91}$

Q**10**. (i) $P(\text{shaded square first throw}) = \frac{2}{6} = \frac{1}{3}$

(ii) $\frac{2}{6} = \frac{1}{3}$

(iii) (a) $\{3,4,5\}$ or $\{5,4,3\}$ or $\{2,6,4\}$ etc (i.e. see below)

(b) $\{3,5,4\}$ or $\{1,6,5\}$ or $\{2,5,5\}$ or $\{3,6,3\}$ or $\{5,2,5\}$ or $\{5,3,4\}$ or $\{5,6,1\}$

$\Rightarrow$ Total $= 10$ ways

(iv) (a) 3 throws

(b) $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{216}$

C Questions

Q1. (i) $P(\text{red, red, red}) + P(\text{green, green, green})$

$$= \frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7} + \frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7} = \frac{5}{42} + \frac{1}{21} = \frac{7}{42} = \frac{1}{6}$$

(ii) $P(\text{at least one is red}) = 1 - P(\text{none red})$

$$= 1 - \frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7} = 1 - \frac{1}{21} = \frac{20}{21}$$

(iii) $P(\text{at most one is green}) = P(G,R,R) + P(R,G,R) + P(R,R,G) + P(R,R,R)$

$$= \frac{4}{9}\cdot\frac{5}{8}\cdot\frac{4}{7} + \frac{5}{9}\cdot\frac{4}{8}\cdot\frac{4}{7} + \frac{5}{9}\cdot\frac{4}{8}\cdot\frac{4}{7} + \frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7}$$

$$= \frac{25}{42}$$

Q2. (i) $\frac{8}{100}\cdot\frac{1}{10} = 0.8\%$

(ii) $\frac{8}{100}\cdot\frac{9}{10} = 7.2\%$

(iii) $\frac{92}{100}\cdot\frac{1}{10} = 9.2\%$

Q3. (i) Equally likely outcomes.

(ii) Probability of second event is dependent on the outcome of first.

$$P(A\,|\,B) = \frac{P(A\cap B)}{P(B)}$$

(iii) (a) $P(C\,|\,D) = \dfrac{P(C\cap D)}{P(D)} \Rightarrow \dfrac{P(C\cap D)}{\frac{1}{3}} = \dfrac{1}{5} \Rightarrow P(C\cap D) = \dfrac{1}{15}$

(b) $P(C\cup D) = P(C) + P(D) - P(C\cap D)$

$$= \frac{8}{15} + \frac{1}{3} - \frac{1}{15} = \frac{12}{15} = \frac{4}{5}$$

(c) $P[(C\cup D)'] = 1 - P(C\cup D) = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

Q4. (i) $1 - \frac{1}{5} = \frac{4}{5}$

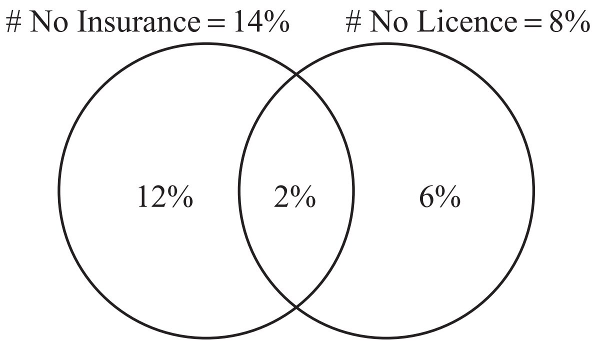
(ii) $P(\text{blue eyes and left-handed}) = \frac{2}{5}\cdot\frac{1}{5} = \frac{2}{25}$

2 people chosen at random $= \binom{2}{1} = 2$

$P(\text{blue eyes and not left-handed}) = \frac{2}{5}\cdot\frac{4}{5} = \frac{8}{25}$

Hence, $P(E) = 2\cdot\frac{2}{25}\cdot\frac{8}{25} = \frac{32}{625}$

Q5. (i) Venn diagram



(ii) $P(\text{drive illegally})$

$$= 12\% + 2\% + 6\% = 20\% = \frac{1}{5}$$

(iii) $300 \cdot 12\% = \text{Roughly } 36$

Q6. (i) $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{625}{20736} = 0.03014 = 0.030$

(ii) $\left(\frac{7}{12}\right)^4 + \left(\frac{5}{12}\right)^4 = 0.11578 + 0.03014 = 0.14592 = 0.146$

(iii) 0

Q7.

$$\begin{aligned} P(\text{correct answer}) &= \frac{5}{8} + \frac{1}{5} \cdot \frac{3}{8} \\ &= \frac{28}{40} \\ &= \frac{7}{10} \end{aligned}$$

Q8. (i)

$$\begin{aligned} P(B) = 0.4 &\Rightarrow & 0.1 + 0.05 + 0.05 + x &= 0.4 \\ &\Rightarrow & 0.2 + x &= 0.4 \\ &\Rightarrow & x &= 0.2 \\ P(C) = 0.35 &\Rightarrow & 0.05 + 0.05 + 0.05 + y &= 0.35 \\ &\Rightarrow & 0.15 + y &= 0.35 \\ &\Rightarrow & y &= 0.2 \end{aligned}$$

$$0.3 + 0.1 + 0.2 + 0.05 + 0.05 + 0.05 + 0.2 + z = 1$$

$$\begin{aligned} &\Rightarrow & 0.95 + z &= 1 \\ &\Rightarrow & z &= 0.05 \end{aligned}$$

(ii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.4} = \frac{3}{8}$

(iii) $P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{0.1}{0.35} = \frac{2}{7}$

(iv) $P[(A \cup B)'] = 0.2 + 0.05 = 0.25$

(v) $P(A \cup B \cup C) = 0.95$

$P(A \mid B) = \frac{3}{8}$ and $\frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.4} = \frac{3}{8}$

Q9. (i) $P(\text{at least one } 6) = 1 - P(\text{no } 6)$

$$= 1 - \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{25}{36} = \frac{11}{36} \quad [= P(A)]$$

(ii) $P(\text{sum is 8}) \Rightarrow \text{possibilities} = \{(2,6)(6,2)(3,5)(5,3)(4,4)\} \quad [= P(E)]$

$$P(E) = \frac{5}{36}$$

(iii) $P(A \cap E) = \frac{2}{36} = \frac{1}{18}$

(iv) $P(A \cup E) = \frac{11}{36} + \frac{5}{36} - \frac{1}{18} = \frac{14}{36} = \frac{7}{18}$

(v) $P(A | E) = \frac{P(A \cap E)}{P(E)} = \frac{\frac{1}{18}}{\frac{5}{36}} = \frac{2}{5}$

Q**10**. (i) $P(E) = 0.6 + (0.4)(0.6) + (0.4)(0.4)(0.6) = 0.936$

(ii) $P(\text{not successful at } 1.70\,\text{m}) = 1 - P(\text{successful at } 1.70\,\text{m})$

$$= 1 - [(0.2) + (0.8)(0.2) + (0.8)(0.8)(0.2)]$$

$$= 1 - 0.488 = 0.512$$

(iii) $1 - 0.936 = 0.064$

(iv) $(0.936).(0.512) = 0.479232 = 0.479$

Chapter 2: Statistics 1

Exercise 2.1

Q1. (i) Numerical
(ii) Categorical
(iii) Numerical
(iv) Categorical

Q2. (i) Discrete
(ii) Discrete
(iii) Continuous
(iv) Discrete
(v) Continuous
(vi) Discrete
(vii) Discrete
(viii) Discrete

Q3. (i) Categorical
(ii) Numerical
(iii) Numerical
Part (ii) is discrete

Q4. Race time is continuous
Number on bib is discrete

Q5. (i) No
(ii) Yes
(iii) Yes
(iv) No

Q6. (i) Contains two pieces of information
(ii) Number of eggs
(iii) Amount of flour

Q7. (i) Categorical
(ii) Numerical
(iii) Numerical
(iv) Categorical
Part (iii) is discrete
Part (ii) is bivariate continuous numerical

Q**8**. (i) True
(ii) True
(iii) False
(iv) False
(v) True
(vi) True
(vii) True
(viii) True

Q**9**. Small, medium, large;
1-bedroom house, 2-bedroom house, 3-bedroom house;
Poor, fair, good, very good

Q**10**. (i) Primary
(ii) Secondary
(iii) Primary
(iv) Secondary

Q**11**. (i) Secondary
(ii) Roy's data; It is more recent.

Q**12**. (i) Number of bedrooms in family home and the number of children in the family
(ii) An athlete's height and his distance in a long-jump competition.

Exercise 2.2

Q**1**. (i) Too personal (it identifies respondent)
(ii) Too vague/subjective

Q**2**. (i) Too personal
(ii) Too leading
(iii) (a) Overlapping (b) "Roughly how many times per annum do you visit your doctor?"

Q**3**. **Q A**: Judgmental and subjective
Q B: Leading and biased

Q**4**. Not suitable; too vague, not specific enough

Q**5**. Where did you go on holidays last year?

☐ Ireland
☐ Europe, excluding Ireland
☐ Rest of the world

What type of accommodation did you use?

☐ Self-catering
☐ Guesthouses / Hotels
☐ Camping

Q**6**. B and D are biased:
B gives an opinion: D is a leading question.

Q**7**. Do you have a part-time job?
Are you male or female?

Q**8**. Explanatory variable: Length of legs.
Response variable: Time recorded in sprint race.

Q**9**. (i) Explanatory variable: Number of operating theatres.
(ii) Response variable: Number of operations per day.

Q**10**. (i) Group B
(ii) Explanatory variable: The new drug.
(iii) Response variable: Blood pressure.
(iv) (a) a designed experiment = carry out some controlled activity and record the results.

Exercise 2.3

Q**1**. Census — all members of the population surveyed.
Sample — only part of the population surveyed.

Q**2**. Any sample of size n which has an equal chance of being selected.

Q**3**. (i) Likely biased
(ii) Random
(iii) Random
(iv) Random
(v) Random

Q**4**. Selecting a sample in the easiest way
(i) Convenience sample.
(ii) (a) High level of bias likely.
(b) Unrepresentative of the population.

Q5. (i) Convenience sampling
(ii) Systematic sampling
(iii) Stratified sampling

Q6. (i) Very small sample; not random and therefore not representative
(ii) Each member of the local population should have an equal chance of being asked. The sample should not be too small. The sample should be stratified to ensure all age and class groups are represented.

Q7. (i) Convenience sampling.
(ii) Her street may not be representative of the whole population.
(iii) Systematic random sampling from a directory or cluster sampling of travel agents' clients, ie. pick one travel agent at random and survey them about all their clients.

Q8. (i) Assign a number to each student and then use a random number generator to pick n numbers.
(ii) (a) $\frac{230}{1000} \cdot 100 = 23$ students
(b) $\frac{80}{1000} \cdot 100 = 8$ boys

Q9. (i) Quota sampling.
(ii) Advantage: Convenient as no sampling frame required.
Disadvantage: Left to the discretion of the interviewer so possible bias.

Q10. (i) Cost and time, without a great loss in accuracy.
(ii) Sampling frame: a list of all the items that could be included in the survey.

Q11. (i) Junior Cycle : $\frac{460}{880} \cdot 100 = 52.27 = 52$ pupils
Senior Cycle : $\frac{420}{880} \cdot 100 = 47.72 = 48$ pupils
(ii) Stratified sampling is better if there are different identifiable groups with different views in the population.

Q12. (i) Cluster sampling
(ii) Convenience sampling
(iii) Systematic sampling

Exercise 2.4

Q1. (a) $2,2,5,5,7,8,8,8,11$

$\Rightarrow$ (i) Mode $=8$ (ii) Median $=7$

(b) $3,3,5,7,7,7,8,8,9,11,12$

$\Rightarrow$ (i) Mode $=7$ (ii) Median $=7$

Q2. $31,34,36,37,41,41,42,42,42,43,45$

(i) Median speed $=41$ km/hr

(ii) Mean speed $=\dfrac{31+34+36+37+41+41+42+42+42+43+45}{11}$

$=\dfrac{434}{11}=39.45$ km/hr

Q3. $7,11,12,14,14,14,18,22,22,36$

(i) Mode $=14$ points

(ii) Median $=\dfrac{14+14}{2}=14$ points

(iii) Mean $=\dfrac{7+11+12+14+14+14+18+22+22+36}{10}$

$=\dfrac{170}{10}=17$ points

Q4. The four numbers are $21,25,16$ and x.

$\Rightarrow \dfrac{21+25+16+x}{4}=19$

$\Rightarrow \quad 62+x=76$

$\Rightarrow \quad x=76-62=14,$ the fourth number.

Q5. Results for six tests were: $8,4,5,3,x$ and y.

Modal mark $=4 \Rightarrow x=4$

Mean $=5 \Rightarrow \dfrac{8+4+5+3+4+y}{6}=5$

$\Rightarrow \quad 24+y=30$

$\Rightarrow \quad y=30-24=6$

Q6. Numbers: $9,11,11,15,17,18,100$

(i) Mean $=\dfrac{9+11+11+15+17+18+100}{7}$

$=\dfrac{181}{7}=25.877$

(ii) Median $=15$

$\Rightarrow$ Median is the best

Q7. Numbers: $103, 35, x, x, x$.

$$\text{Mean} = 39 \Rightarrow \frac{103+35+x+x+x}{5} = 39$$

$$\Rightarrow 138+3x = 195$$

(i) Total of the five numbers $= 195$

(ii) $3x = 195 - 138$

$\Rightarrow 3x = 57$

$\Rightarrow x = \frac{57}{3} = 19$

Q8. Mean for 12 children = 76%

$\Rightarrow$ Total for 12 children = $76\% \times 12 = 912\%$

Mean for 8 children = 84%

$\Rightarrow$ Total for 8 children = $84\% \times 8 = 672\%$

$\Rightarrow$ Total for 20 children = $912\% + 672\% = 1584\%$

$\Rightarrow$ Overall mean = $\frac{1584\%}{20} = 79.2\%$

Q9. Median, since 50% of the marks will be above the median mark.

Q10. (i) Mean for 20 boys $= 17.4$

$\Rightarrow$ Total for 20 boys $= 17.4 \times 20$

$= 348$ marks

Mean for 10 girls $= 13.8$

$\Rightarrow$ Total for 10 girls $= 13.8 \times 10$

$= 138$ marks

Total for 30 students $= 348 + 138$

$= 486$

$\Rightarrow$ Mean for whole class $= \frac{486}{30}$

$= 16.2$

(ii) Median for $12, 18, 20, 25$ and x $= 20$

$$\text{Mean} = \frac{12+18+20+25+x}{5} = 22 \quad \text{[i.e. 20 (the median)+2]}$$

$$\Rightarrow \quad 75 + x = 110$$

$$\Rightarrow \quad x = 110 - 75 = 35$$

Q11.

$x =$ Marks	3	4	5	6	7	8	9	
$f =$ No. of students	3	2	6	10	0	3	1	$= 25$
$f.x=$	9	8	30	60	0	24	9	$= 140$

(i) Number of students $= 3+2+6+10+0+3+1 = 25$

(ii) Mode $= 6$ marks

(iii) $\text{Mean} = \dfrac{\sum fx}{\sum f} = \dfrac{140}{25} = 5.6$ marks

(iv) $10+0+3+1 = 14$ students

(v) 25 students $\Rightarrow$ median is the mark of 13^{th} student $= 6$ marks

Q12.

$x =$ No. in family	2	3	4	5	6	7	8	
$f =$ frequency	2	4	6	5	2	0	1	$= 20$
$f.x=$	4	12	24	25	12	0	8	$= 85$

(i) Mode $= 4$ people

(ii) $\text{Median} = \dfrac{4+4}{2} = 4$ people

(iii) $\text{Mean} = \dfrac{\sum fx}{\sum f} = \dfrac{85}{20} = 4.25$

Q13.

Age	10–20	20–30	30–40	40–50	
$f =$ No. of people	4	15	11	10	$= 40$
$x =$ mid-interval	15	25	35	45	
$f.x$	60	375	385	450	$= 1270$

(i) $\text{Mean} = \dfrac{\sum fx}{\sum f} = \dfrac{1270}{40} = 31.75 = 32$ years

(ii) $(30-40)$ years

Q14. (i) $\text{Mean} = \dfrac{\sum x}{N} = \dfrac{256.2}{6} = 42.7$

(ii) Mean will increase

Q15. (i) (a) Mode $= B$

(b) Median $= C$

(ii) Categorical data is not numerical

Q16.

Rainfall (mm)	0	1	2	3	3	26	3	2	3	0
Sunshine (hours)	70	15	10	15	18	0	15	21	21	80
Rainfall (mm)	0	0	1	2	2	3	3	3	3	26
Sunshine (hours)	0	10	15	15	15	18	21	21	70	80

(i) $\text{Mean} = \dfrac{0+0+1+2+2+3+3+3+3+26}{10} = \dfrac{43}{10} = 4.3$ mm of rainfall

(ii) $\text{Mean} = \dfrac{0+10+15+15+15+18+21+21+70+80}{10}$

$= \dfrac{265}{10} = 26.5$ hours of sunshine

(iii) Rainfall mode $= 3$ mm

Sunshine mode $= 15$ hours

(iv) Rainfall median $= \dfrac{2+3}{2} = 2.5$ mm

Sunshine median $= \dfrac{15+18}{2} = 16.5$ hours

(v) Median rainfall and mean sunshine

(least rainfall and highest sunshine).

Q17. Dice was thrown 50 times, mean score $= 3.42$.

$\Rightarrow$ Total scores $= 50 \times 3.42 = 171$

outcomes	1	2
frequency	9	12

outcomes	1	2
frequency	12	9

scores $= 9 + 24 = 33$

scores $= 12 + 18 = 30$

Hence, there is an increase of 3 (or a decrease of 3) in the total scores when the frequencies had been swapped.

$\text{Mean} = \dfrac{171+3}{50} = \dfrac{174}{50} = 3.48$

or $\text{Mean} = \dfrac{171-3}{50} = \dfrac{168}{50} = 3.36$

Exercise 2.5

Q1. (i) $\text{Range} = 10 - 2 = 8$

(ii) $\text{Range} = 73 - 16 = 57$

Q2. Marks in order: 4, 10, 27, 27, 29, 34, 34, 34, 37

(i) $\text{Range} = 37 - 4 = 33$

(ii) Median = 29

(iii) (a) Lower quartile = 27

(b) Upper quartile = 34

(c) Interquartile range $= 34 - 27 = 7$

Q3. Times in order: 6, 7, 8, 9, 9, 9, 11, 12, 15, 16, 19

(i) $\text{Range} = 19 - 6 = 13$

(ii) Lower quartile = 8

(iii) Upper quartile = 15

(iv) Interquartile range $= 15 - 8 = 7$

Q4. Marks in order: 12, 13, 14, 14, 14, 14, 14, 15, 15, 16, 16, 17

(i) $\text{Range} = 17 - 12 = 5$ marks

(ii) $\text{Mean} = \dfrac{12 + 13 + 5(14) + 2(15) + 2(16) + 17}{12} = \dfrac{174}{12} = 14.5$ marks

(iii) On average, the girls didn't do as well as the boys. The girls' marks were more dispersed.

Q5. Scores in order: 41, 50, 50, 51, 53, 59, 64, 65, 66

(i) $\text{Range} = 66 - 41 = 25$

(ii) Lower quartile = 50

(iii) Upper quartile = 65

(iv) Interquartile range $= 65 - 50 = 15$

Q6. Results in order: 2.2, 2.2, 2.3, 2.3, 2.5, 2.7, 3.1, 3.2, 3.6, 3.7, 3.7, 3.8, 3.8, 3.8, 3.8, 3.9, 3.9, 3.9, 4.0, 4.0, 4.0, 4.0, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5.1, 5.5

(i) Lower Quartile, $Q_1 = 3.2$
Upper Quartile, $Q_3 = 4.0$

Interquartile range $= 4.0 - 3.2 = 0.8$

(ii) $(1.5) \times (0.8) = 1.2$

$\Rightarrow$ Outlier = 5.5 as it is more than $1\frac{1}{2}$ times the interquartile range above Q_3.

Q7. (i) $\text{Mean} = \mu = \dfrac{\sum x}{n} = \dfrac{1+3+7+9+10}{5} = \dfrac{30}{5} = 6$

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{n}} = \sqrt{\frac{(1-6)^2+(3-6)^2+(7-6)^2+(9-6)^2+(10-6)^2}{5}}$$

$$= \sqrt{\frac{(-5)^2+(-3)^2+(1)^2+(3)^2+(4)^2}{5}}$$

$$= \sqrt{\frac{25+9+1+9+16}{5}}$$

$$= \sqrt{\frac{60}{5}} = \sqrt{12} = 3.464 = 3.5$$

(ii) $\text{Mean} = \mu = \dfrac{8+12+15+9}{4} = \dfrac{44}{4} = 11$

$$\sigma = \sqrt{\frac{(8-11)^2+(12-11)^2+(15-11)^2+(9-11)^2}{4}}$$

$$= \sqrt{\frac{(-3)^2+(1)^2+(4)^2+(-2)^2}{4}}$$

$$= \sqrt{\frac{9+1+16+4}{4}} = \sqrt{\frac{30}{4}} = \sqrt{7.5} = 2.73 = 2.7$$

(iii) $\text{Mean} = \mu = \dfrac{1+3+4+6+10+12}{6} = \dfrac{36}{6} = 6$

$$\sigma = \sqrt{\frac{(1-6)^2+(3-6)^2+(4-6)^2+(6-6)^2+(10-6)^2+(12-6)^2}{6}}$$

$$= \sqrt{\frac{(-5)^2+(-3)^2+(-2)^2+(0)^2+(4)^2+(6)^2}{6}}$$

$$= \sqrt{\frac{25+9+4+0+16+36}{6}} = \sqrt{\frac{90}{6}} = \sqrt{15} = 3.87 = 3.9$$

Q8. Mean $= \mu = \dfrac{2+3+4+5+6}{5} = \dfrac{20}{5} = 4$

$$\sigma = \sqrt{\frac{(2-4)^2+(3-4)^2+(4-4)^2+(5-4)^2+(6-4)^2}{5}}$$

$$= \sqrt{\frac{(-2)^2+(-1)^2+(0)^2+(1)^2+(2)^2}{5}}$$

$$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2} = 1.414$$

Mean $= \mu = \dfrac{12+13+14+15+16}{5} = \dfrac{70}{5} = 14$

$$\sigma = \sqrt{\frac{(12-14)^2+(13-14)^2+(14-14)^2+(15-14)^2+(16-14)^2}{5}}$$

$$= \sqrt{\frac{(-2)^2+(-1)^2+(0)^2+(1)^2+(2)^2}{5}}$$

$$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2} = 1.414$$

(i) New set is $x+10$.

(ii) Both the same.

(iii) If all the numbers are increased by the same amount, the standard deviation does not change.

Q9. Mean $= \mu = \dfrac{2+3+4+5+6+8+8}{7} = \dfrac{36}{7}$

$$\sigma = \sqrt{\frac{(2-\frac{36}{7})^2+(3-\frac{36}{7})^2+(4-\frac{36}{7})^2+(5-\frac{36}{7})^2+(6-\frac{36}{7})^2+(8-\frac{36}{7})^2+(8-\frac{36}{7})^2}{7}}$$

$$= \sqrt{\frac{(\frac{-22}{7})^2+(\frac{-15}{7})^2+(\frac{-8}{7})^2+(\frac{-1}{7})^2+(\frac{6}{7})^2+(\frac{20}{7})^2+(\frac{20}{7})^2}{7}}$$

$$= \sqrt{\frac{\frac{484}{49}+\frac{225}{49}+\frac{64}{49}+\frac{1}{49}+\frac{36}{49}+\frac{400}{49}+\frac{400}{49}}{7}}$$

$$= \sqrt{\frac{1610}{343}} = \sqrt{4.69388} = 2.166 = 2.17$$

Q10. (i) Route 1 mean $= \dfrac{15+15+11+17+14+12}{6} = \dfrac{84}{6} = 14$

Route 2 mean $= \dfrac{11+14+17+15+16+11}{6} = \dfrac{84}{6} = 14$

(ii) Route 1 $\Rightarrow$ $\sigma = \sqrt{\dfrac{(15-14)^2+(15-14)^2+(11-14)^2+(17-14)^2+(14-14)^2+(12-14)^2}{6}}$

$$= \sqrt{\frac{(1)^2+(1)^2+(-3)^2+(3)^2+(0)^2+(-2)^2}{6}}$$

$$= \sqrt{\frac{1+1+9+9+0+4}{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$

Route 2 $\Rightarrow$ $\sigma = \sqrt{\dfrac{(11-14)^2+(14-14)^2+(17-14)^2+(15-14)^2+(16-14)^2+(11-14)^2}{6}}$

$$= \sqrt{\frac{(-3)^2+(0)^2+(3)^2+(1)^2+(2)^2+(-3)^2}{6}}$$

$$= \sqrt{\frac{9+0+9+1+4+9}{6}}$$

$$= \sqrt{\frac{32}{6}} = \sqrt{5\frac{1}{3}} = 2.309 = 2.3$$

(iii) Route 1, as times are less dispersed.

Q11.

Variable $= x$	0	2	3	4	
frequency $= f$	4	3	2	3	$= 12$
$f.x$	0	6	6	12	$= 24$

$$\text{Mean} = \mu = \frac{\sum fx}{\sum f} = \frac{24}{12} = 2$$

x	f	$(x-\mu)$	$(x-\mu)^2$	$f(x-\mu)^2$
0	4	–2	4	16
2	3	0	0	0
3	2	1	1	2
4	3	2	4	12

$\sum f = 12$ $\qquad$ $\sum f(x-\mu)^2 = 30$

$$\sigma = \sqrt{\frac{\sum f(x-\mu)^2}{\sum f}} = \sqrt{\frac{30}{12}} = \sqrt{2.5} = 1.58 = 1.6$$

Q**12**. Mean $= \dfrac{(1\times1)+(4\times2)+(9\times3)+(6\times4)}{1+4+9+6}$

$$\Rightarrow \mu = \frac{1+8+27+24}{20} = \frac{60}{20} = 3$$

x	f	$x-\mu$	$(x-\mu)^2$	$f(x-\mu)^2$
1	1	–2	4	4
2	4	–1	1	4
3	9	0	0	0
4	6	1	1	6

$\downarrow$ $\Sigma f = 20$ $\qquad$ $\downarrow$ $\Sigma f(x-\mu)^2 = 14$

$$\sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}} = \sqrt{\frac{14}{20}} = 0.83666 = 0.84$$

Q**13**.

Class	Mid-interval $= x$	f	fx	$(x-\mu)$	$(x-\mu)^2$	$f(x-\mu)^2$
1–3	2	4	8	–3	9	36
3–5	4	3	12	–1	1	3
5–7	6	9	54	1	1	9
7–9	8	2	16	3	9	18
		18				66

$$\text{Mean} = \mu = \frac{\Sigma fx}{\Sigma f} = \frac{90}{18} = 5$$

$$\sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}} = \sqrt{\frac{66}{18}} = 1.91 = 1.9$$

Q14.

Class	Mid-interval $= x$	f	fx	$x-\mu$	$(x-\mu)^2$	$f(x-\mu)^2$
0–4	2	2	4	–9	81	162
4–8	6	3	18	–5	25	75
8–12	10	9	90	–1	1	9
12–16	14	7	98	3	9	63
16–20	18	3	54	7	49	147
		24	264			456

$$\text{Mean} = \frac{264}{24} = 11$$

$$\sigma = \sqrt{\frac{456}{24}} = \sqrt{19} = 4.358 = 4.36$$

Q15. (i) $\text{Mean } (\bar{x}) = \dfrac{18+26+22+34+25}{5} = \dfrac{125}{5} = 25 \text{ letters}$

(ii) $$\sigma = \sqrt{\frac{(18-25)^2+(26-25)^2+(22-25)^2+(34-25)^2+(25-25)^2}{5}}$$

$$= \sqrt{\frac{(-7)^2+(1)^2+(-3)^2+(9)^2+(0)^2}{5}}$$

$$= \sqrt{\frac{49+1+9+81+0}{5}} = \sqrt{\frac{140}{5}} = \sqrt{28} = 5.29 = 5.3$$

(iii) $\bar{x}+\sigma = 25+5.3 = 30.3$

$\bar{x}-\sigma = 25-5.3 = 19.7$

(iv) 3 days

Q16. (i) Mean $(\bar{x}) = \dfrac{1+9+a+3a-2}{4}$

$$= \frac{4a+8}{4} = a+2$$

(ii) $\sigma = \sqrt{20} \;\Rightarrow\; \sigma = \sqrt{\dfrac{[1-(a+2)]^2+[9-(a+2)]^2+[a-(a+2)]^2+[3a-2-(a+2)]^2}{4}}$

$$= \sqrt{\frac{(-a-1)^2+(7-a)^2+(-2)^2+(2a-4)^2}{4}}$$

$$= \sqrt{\frac{a^2+2a+1+49-14a+a^2+4+4a^2-16a+16}{4}}$$

$$= \sqrt{\frac{6a^2-28a+70}{4}} = \sqrt{20}$$

$$\Rightarrow \frac{3a^2-14a+35}{2} = 20$$

$$\Rightarrow 3a^2-14a+35 = 40$$

$$\Rightarrow 3a^2-14a-5 = 0$$

$$\Rightarrow (3a+1)(a-5) = 0$$

$$\Rightarrow a = -\frac{1}{3}, \quad a = 5$$

$$\Rightarrow a = 5 \quad \text{as} \quad a \in Z$$

Q17. (i) 80%

(ii) 20%

Q18. (i) No, as it does not tell you what percentage did worse than Elaine.

(ii) $P_{40} = \dfrac{40}{100} \times \dfrac{800}{1} = 320$

$\Rightarrow$ $800 - 320 = 480$ people did better than Tanya.

Q19. (i) $P_{25} = \dfrac{52+55}{2} = 53.5$

(ii) $P_{75} = \dfrac{72+77}{2} = 74.5$

(iii) $P_{40} = \dfrac{63+65}{2} = 64$

$\Rightarrow$ $P_{75} - P_{40} = 74.5 - 64 = 10.5$

(iv) $P_{80} = \dfrac{77+79}{2} = 78$

$\Rightarrow$ 4 people have scores $\geq P_{80}$

(v) $\dfrac{9}{20} \times \dfrac{100}{1} = 45$ $\Rightarrow$ Eoins mark is at the 45^{th} percentile

Q20. (i) $\dfrac{70}{100} \times 36 = 25.2 \quad \Rightarrow \quad \text{Next whole number} = 26$

$\Rightarrow \quad P_{70} = 26^{\text{th}}$ number in the set = € 55

(ii) $\dfrac{40}{100} \times 36 = 14.4 \quad \Rightarrow \quad \text{Next whole number} = 15$

$\Rightarrow \quad P_{40} = 15^{\text{th}}$ number in the set = € 32

(iii) 14

(iv) $\dfrac{80}{100} \times 36 = 28.8 \quad \Rightarrow \quad \text{Next whole number} = 29$

$\Rightarrow \quad P_{80} = 29^{\text{th}}$ number in the set = € 59 and 7 are more expensive.

(v) Price = €40 $\Rightarrow$ 19 t-shirts are lower than this

$\Rightarrow \quad \dfrac{19}{36} \times \dfrac{100}{1} = 52.77 \quad \Rightarrow \quad 53^{\text{rd}}$ to 56^{th} percentile

Q21. Mean $= \dfrac{a+b+8+5+7}{5} = 6$

$$\Rightarrow \quad a+b+20 = 30$$
$$\Rightarrow \quad a+b = 10$$
$$\Rightarrow \quad a = 10-b$$

Find σ for $10-b, b, 8, 5, 7$.

$$\Rightarrow \quad \sigma = \sqrt{\frac{(10-b-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(7-6)^2}{5}}$$
$$\Rightarrow \quad \sqrt{\frac{(4-b)^2+(b-6)^2+(2)^2+(-1)^2+(1)^2}{5}} = \sqrt{2}$$
$$\Rightarrow \quad \sqrt{\frac{16-8b+b^2+b^2-12b+36+4+1+1}{5}} = \sqrt{2}$$
$$\Rightarrow \quad \sqrt{\frac{2b^2-20b+58}{5}} = \sqrt{2}$$
$$\Rightarrow \quad \frac{2b^2-20b+58}{5} = 2$$
$$\Rightarrow \quad 2b^2-20b+58 = 10$$
$$\Rightarrow \quad 2b^2-20b+48 = 0$$
$$\Rightarrow \quad b^2-10b+24 = 0$$
$$\Rightarrow \quad (b-4)(b-6) = 0$$
$$\Rightarrow \quad b = 4, \ b = 6$$
$$\Rightarrow \quad a = 10-4 = 6, \quad a = 10-6 = 4$$

Since $a > b$, hence $a = 6, \ b = 4$.

Exercise 2.6

Q1. (i) 4 people

(ii) 27 years

(iii) 8 people

(iv) Median age is the age of the 13^{th} person = 36 years

Q2. (i)

stem	leaf							
0	0	1	2	4	6	6	7	8
1	2	4	4	5	7	8	8	9
2	1	1	3	5	6			
3	1	1	2					

Key: 2|3 = 23 CDs

(ii) 8 pupils

(iii) Median $= \frac{15+17}{2} = 16$ CDs

Q3.

stem	leaf								
1	5								
2	4	4	5	6	7	8	8	9	
3	0	1	2	3	5	5	5	6	7
4	2	2	3						
5	6	6	8						

Key: 3|2 means 3.2 seconds

(i) 6 calls

(ii) $5.8 - 1.5 = 4.3$ seconds

(iii) Median $= \frac{3.2+3.3}{2} = 3.25$ seconds

(iv) Mode = 3.5 seconds

Q4. (i) Range $= 84 - 22 = 62$ marks

(ii) Q_1 ? $\Rightarrow \frac{1}{4}(19) = 4.75 \Rightarrow Q_1$ is the 5^{th} value = 47 marks

(iii) Q_3 ? $\Rightarrow \frac{3}{4}(19) = 14.25 \Rightarrow Q_3$ is the 15^{th} value = 67 marks

(iv) $67 - 47 = 20$ marks

Q5. (i) Median $= \frac{38+44}{2} = \frac{82}{2} = 41$ laptops

(ii) $Q_1 ? \Rightarrow \frac{1}{4}(26) = 6.5 \Rightarrow Q_1$ is the 7^{th} value $= 32$ laptops

(iii) $Q_3 ? \Rightarrow \frac{3}{4}(26) = 19.5 \Rightarrow Q_3$ is the 20^{th} value $= 47$ laptops

(iv) Interquartile range $= 47 - 32 = 15$ laptops

(v) Mode $= 47$ laptops

Q6. (i) 19 students took both Science and French

(ii) (a) Science range $= 91 - 25 = 66$ marks

(b) French range $= 85 - 36 = 49$ marks

(iii) Median for Science = 55 marks

(iv) $Q_1 ? \Rightarrow \frac{1}{4}(19) = 4.75 \Rightarrow Q_1$ is the 5^{th} value $= 48$ marks

$Q_3 ? \Rightarrow \frac{3}{4}(19) = 14.25 \Rightarrow Q_3$ is the 15^{th} value $= 74$ marks

$\Rightarrow$ Interquartile range of the french marks $= 74 - 48 = 26$ marks

Q7. (i) Median = 76 bpm

Range $= 92 - 65 = 27$ bpm

(ii) Median $= \frac{68+68}{2} = 68$ bpm

Range $= 88 - 50 = 38$ bpm

(iii) Those who did not smoke; significantly lower median.

Q8. (i)

French		English
2 1	3	8
7 6 5 4 4	4	3 4 4
8 8 7 3 3 0	5	2 6 8
9 6 1 1	6	3 5 5 8 9
8 5	7	1 2 2 7 9 9
1	8	4 5

Key: 7|5 = 57 marks Key: 6|9 = 69 marks

(ii) Median for French $= \frac{53+57}{2} = 55$ marks

(iii) Median for English $= \frac{65+68}{2} = 66.5$ marks

(iv) English; higher median.

Q**9**. (i) Range $= 33 - 2 = 31$ minutes

(ii) Median for Matrix $1 = \dfrac{17+18}{2} = 17.5$ minutes

(iii) 15 minutes (i.e. the digit 5 is missing)

(iv) Prob (person waited > 10 mins) $= \dfrac{15}{20} = 0.75$

(v) Median for Matrix $1 = 17.5$ minutes

Median for Matrix $2 = \dfrac{17+18}{2} = 17.5$ minutes

$\Rightarrow$ Both have median 17.5, similar ranges (one minute in the difference); hence no significant difference.

Q**10**.

Men		Women
2 1 0	4	0 1
2 2 2	5	1
5 5 4	6	2 3
1	7	5
	8	7 8
Key: 5|6 = 65 mins	9	3 5

Key: 8|7 = 87 mins

(i) Modal time for men = 52 minutes

(ii) (a) Median for men $= \dfrac{52+52}{2} = 52$ minutes

(b) Median for women $= \dfrac{63+75}{2} = 69$ minutes

(iii) (a) Range for men $= 71 - 40 = 31$ minutes

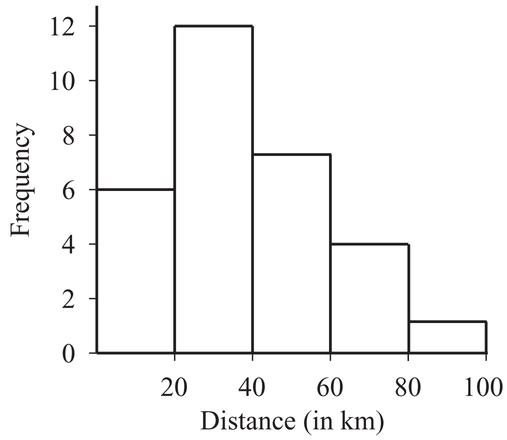
(b) Range for women $= 95 - 40 = 55$ minutes

(iv) Women in the survey have a higher median and a wider range.

The far wider range is significant here as both the men's and women's shortest time spent watching t.v. was the same (i.e. 40 mins). Accordingly, the women dominated the longer tv-watching times in this survey (87, 95 mins etc), especially with there being no outliers.

Exercise 2.7

Q1. (i)



(ii) 12 motorists

(iii) (20–40) km

(iv) Percentage $= \frac{12}{30} \cdot \frac{100}{1} = 40\%$

Q2. (i) 10 people

(ii) Modal class = (40–50) years

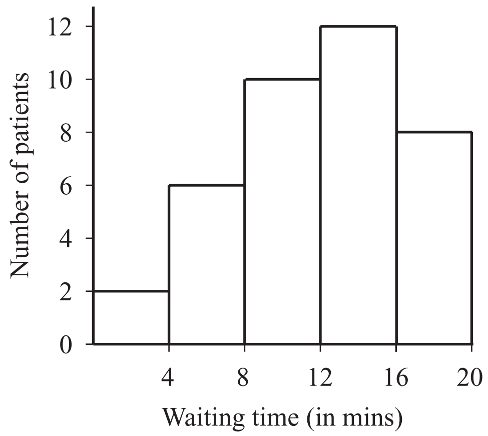
(iii) How many < 30 years? $= 2 + 4 + 6 = 12$ people

(iv) Total $= 2 + 4 + 6 + 10 + 17 + 12 + 6 + 3 = 60$ people

(v) (50–60) years

(vi) Median lies in the (40–50) years interval.

Q3. (i)



(ii) Number of patients $= 2 + 6 + 10 + 12 + 8 = 38$ patients

(iii) Modal class = (12–16) mins

(iv) Median lies in the (12–16) mins interval

(v) Greatest number > 10 minutes $= 10 + 12 + 8 = 30$ patients

(vi) Least number > 14 minutes $= 8$ patients

Q4. (i) Number of pupils ≥ 15 secs $= 10 + 9 = 19$ pupils

(ii) Total $= 8 + 12 + 15 + 10 + 9 = 54$ pupils

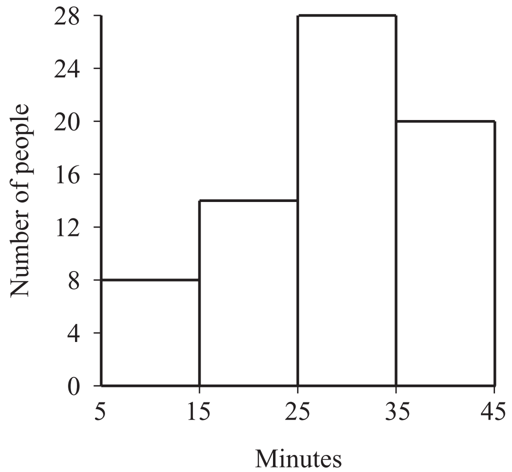
(iii) Modal class = (10–15) secs

(iv) Median lies in the (10–15) secs interval

(v) Greatest number < 8 secs $= 8 + 12 = 20$ pupils

(vi) Least number < 12 secs $= 8 + 12 = 20$ pupils

Q5. (i)



(ii) Modal class = (25–35) mins

(iii) Median lies in the (25–35) mins interval

(iv) $(15 - 25)$ mins because $\frac{14}{70} \times \frac{100}{1} = 20\%$

(v) Greatest number > 30 minutes $= 28 + 20 = 48$ people

(vi) $$\text{Mean} = \frac{(10)(8) + (20)(14) + (30)(28) + (40)(20)}{8 + 14 + 28 + 20}$$

$$= \frac{80 + 280 + 840 + 800}{70}$$

$$= \frac{2000}{70} = 28.57 = 29 \text{ minutes}$$

Exercise 2.8

Q1. Symmetrical;

(i) Normal distribution

(ii) Peoples' heights

Q2. Positively skewed; age at which people start third-level education.

Q3. (i) c

(ii) a

(iii) b

(iv) b

(v) c

Q4. Negatively skewed

(i) Mean

(ii) Mode

Q5. More of the data is closer to the mean in distribution A

Q6. (i) B

(ii) B

Q7. (i) A

(ii) Equal

Q8. (i) B

(ii) A

Q9. (i) A

(ii) B

Q10. (i)

A	B	C	D
✗	✗	✓	✗
✓	✗	✗	✗
✗	✓	✗	✓
✓	✗	✗	✗
✓	✓	✓	✗

(ii) D has the largest standard deviation as more of the data is located further from the mean.

Test Yourself 2

A Questions

Q**1**. (i) Primary

(ii) Secondary

(iii) Primary

(iv) Secondary

(v) Secondary

Q**2**. Times arranged in order: 6, 7, 8, 9, 9, 9, 11, 12, 15, 16, 19

(i) $\text{Range} = 19 - 6 = 13$ minutes

(ii) $Q_1 ? \;\Rightarrow\; \frac{1}{4}(11) = 2.75 \;\Rightarrow\; Q_1$ is the 3rd value $= 8$ minutes

(iii) $Q_3 ? \;\Rightarrow\; \frac{3}{4}(11) = 8.25 \;\Rightarrow\; Q_3$ is the 9th value $= 15$ minutes

(iv) Interquartile range $= 15 - 8 = 7$ minutes

Q**3**.

$$\text{Mean} = \frac{3+6+7+x+14}{5} = 8$$

$$\Rightarrow \quad 30 + x = 40$$

$$\Rightarrow \quad x = 40 - 30 = 10$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{(3-8)^2+(6-8)^2+(7-8)^2+(10-8)^2+(14-8)^2}{5}}$$

$$= \sqrt{\frac{(-5)^2+(-2)^2+(-1)^2+(2)^2+(6)^2}{5}}$$

$$= \sqrt{\frac{25+4+1+4+36}{5}} = \sqrt{\frac{70}{5}} = \sqrt{14} = 3.74 = 3.7$$

Q**4**. (i) Census surveys the entire population; sample surveys only part of the population.

(ii) $P_{72} \Rightarrow 28\%$ higher than his mark

$$= \frac{28}{100} \times 90 = 25.2$$

$= 25$ students

Q**5**. (i) 32 students

(ii) € 48

(iii) Median = amount spent by the 8th female = €25

(iv) Median = amount spent by the 9th male = €29

(v) Males; higher median.

Q6. (i) (b); because it has the greater spread.

(ii) Mean ( ) $= \dfrac{(2\cdot 0)+(5\cdot 1)+(6\cdot 2)+(5\cdot 3)+(2\cdot 4)}{2+5+6+5+2}$

$$= \frac{0+5+12+15+8}{20}$$

$$= \frac{40}{20} = 2$$

x	f	$x-\mu$	$(x-\mu)^2$	$f(x-\mu)^2$
0	2	–2	4	8
1	5	–1	1	5
2	6	0	0	0
3	5	1	1	5
4	2	2	4	8
	20			26

$$\sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}} = \sqrt{\frac{26}{20}} = \sqrt{1.3} = 1.140$$

Q7. (i) Stratified, then simple random sampling.

(ii) $\dfrac{100}{5} = 20$ students

(iii) Give each student a number and then select 10, using random number key on calculator.

Q8. (i) Yes; it may not be representative as there is no random element to the survey.

(ii) Use stratified sampling based on gender, age, marital status, income level, etc. and then use simple random sampling.

Q9. Stratified sampling is used when the population can be split into seperate groups or strata that are quite different from each other. The number selected from each group is proportional to the size of the group. Seperate random samples are then taken from each group.

In cluster sampling, the population is divided into groups or clusters. Then, some of these clusters are randomly selected and all items from these clusters are chosen. A large number of small clusters is best as this minimises the chances of the sample being unrepresentative. Cluster sampling is very popular with scientists.

Q**10**.

stem	leaf
3	1 2 3 4 5 6 8
4	0 1 2 6 6 7 7 7
5	9 Key: 4|2 = 4.2 mins

(ii) Mode = 4.7 minutes

(iii) Median = $\dfrac{4.0+4.1}{2}$ = 4.05 minutes

$Q_1\,?\ \Rightarrow\ \dfrac{1}{4}(16)=4\quad\Rightarrow\ Q_1$ is the 4^{th} value = 3.4 minutes

$Q_3\,?\ \Rightarrow\ \dfrac{3}{4}(16)=12\quad\Rightarrow\ Q_3$ is the 12^{th} value = 4.6 minutes

$\Rightarrow$ Interquartile range $= 4.6-3.4=1.2$ minutes

B Questions

Q**1**. David; as the standard deviation of his marks is smaller.

Q**2**. Marks in order: 37, 38, 42, 46, 46, 46, 48, 54, 55, 57, 59, 63, 64, 65, 66, 68, 68, 68, 71, 73, 74, 76, 78, 82.

(i) $\dfrac{40}{100}\times 24 = 9.6$

$\Rightarrow\quad P_{40} = 10^{\text{th}}$ number in the set = 57% (i.e. 57 marks out of 100)

(ii) Score = 71 marks $\Rightarrow$ 18 students are lower than this.

$\Rightarrow\ \dfrac{18}{24}\times\dfrac{100}{1} = 75$

$\Rightarrow$ Gillian's score is P_{75}, the 75^{th} percentile.

Q**3**. (i) A, D

(ii) C, A

(iii) B

(iv) A

(v) A

Q4.

Class	Mid-interval $= x$	f	fx	$x-\mu$	$(x-\mu)^2$	$f(x-\mu)^2$
1–3	2	4	8	−2	4	16
3–5	4	3	12	0	0	0
5–7	6	0	0	2	4	0
7–9	8	2	16	4	16	32
		9	36			48

$$\text{Mean } (\mu) = \frac{36}{9} = 4$$

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{48}{9}} = \sqrt{5\frac{1}{3}} = 2.309 = 2.3$$

Q5. (i) Negatively skewed as most of the data occurs at the higher values.

(ii) A = mode, B = median, C = mean.

(iii) Age when people retire

Q6.

$$300 + 500 + 400 = 1200 \text{ cans}$$

$$\text{Large} \Rightarrow \frac{300}{1200} \times 60 = 15 \text{ large cans}$$

$$\text{Medium} \Rightarrow \frac{500}{1200} \times 60 = 25 \text{ medium cans}$$

$$\text{Small} \Rightarrow \frac{400}{1200} \times 60 = 20 \text{ small cans}$$

Q7. (i)

$$\text{Mean } (\ ) = \frac{(26 \cdot 0) + (90 \cdot 1) + (57 \cdot 2) + (19 \cdot 3) + (5 \cdot 4) + (3 \cdot 5) + (200 \cdot 6)}{26 + 90 + 57 + 19 + 5 + 3 + 200}$$

$$= \frac{0 + 90 + 114 + 57 + 20 + 15 + 1200}{400}$$

$$= \frac{1496}{400} = 3.74$$

(ii)

x	f	$(x-\mu)$	$(x-\mu)^2$	$f(x-\mu)^2$
0	26	−3.74	13.9876	363.6776
1	90	−2.74	7.5076	675.684
2	57	−1.74	3.0276	172.5732
3	19	−0.74	0.5476	10.4044
4	5	0.26	0.0676	0.338
5	3	1.26	1.5876	4.7628
6	200	2.26	5.1076	1021.52
	400			2248.96

$$\sigma = \sqrt{\frac{2248.96}{400}} = \sqrt{5.6224} = 2.371 = 2.37$$

(iii) In the earlier study of the same junction, there were less crashes, on average, each day (mean 0.54 lower). The far lower standard deviation also tells us that there used to be far fewer days where there were a high number (i.e. 5 or 6) of road accidents at the junction.

Q8. (i) Explanatory: Fertilizer used

Response: Wheat yield

(ii) Explanatory: Suitable habitat

Response: Number of species

(iii) Explanatory: Amount of water

Response: Time taken to cool

(iv) Explanatory: Size of engine

Response: Petrol consumption

Q9. A: Systematic

B: Convenience

C: Simple random

D: Stratified

E: Quota

Q10. Runs in order: 0, 0, 13, 28, 35, 40, 47, 51, 63, 77, a

(i) Median = 40

$Q_1 ? \Rightarrow \frac{1}{4}(11) = 2.75 \quad \Rightarrow$ Lower Q. is the 3rd number $= 13$

$Q_3 ? \Rightarrow \frac{3}{4}(11) = 8.25 \quad \Rightarrow$ Upper Q. is the 9th number $= 63$

$\Rightarrow$ Interquartile range $= 63 - 13 = 50$

(ii) Mode = 0 $\Rightarrow$ not an appropriate average because zero would not be a typical value. (In fact, it is the lowest value.)

Range is between 0 and $a > 100$ so it would be distorted by the two zeros and the one very high value.

C Questions

Q1. (i) (a) Median is between 190^{th} and 191^{st} matches

$$= \frac{2+2}{2} = 2 \text{ goals}$$

$$Q_1 ? \Rightarrow \frac{1}{4}(380) = 95 \Rightarrow 95^{\text{th}} \text{ match had 1 goal scored in it}$$

$$Q_3 ? \Rightarrow \frac{3}{4}(380) = 285 \Rightarrow 285^{\text{th}} \text{ match had 4 goals scored in it}$$

$\Rightarrow$ Interquartile range $= 4 - 1 = 3$ goals.

(b)

Goals $= x$	Matches $= f$	$f.x$	$x-\mu$	$(x-\mu)^2$	$f(x-\mu)^2$
0	30	0	–2.56	6.5536	196.608
1	79	79	–1.56	2.4336	192.2544
2	99	198	–0.56	0.3136	31.0464
3	68	204	0.44	0.1936	13.1648
4	60	240	1.44	2.0736	124.416
5	24	120	2.44	5.9536	142.8864
6	11	66	3.44	11.8336	130.1696
7	6	42	4.44	19.7136	118.2816
8	2	16	5.44	29.5936	59.1872
9	1	9	6.44	41.4736	41.4736
	380	974			1049.488

$$\text{Mean } (\mu) = \frac{974}{380} = 2.563 = 2.56$$

$$\sigma = \sqrt{\frac{1049.488}{380}} = \sqrt{2.76181} = 1.661 = 1.66$$

(ii) The mean is slightly higher in the 2008/09 season and the standard deviation is also marginally higher. The wider spread in the 2008/09 season suggests a few more open, high-scoring games. However, the median number of goals per game is the same for both seasons. Overall, there is little significant difference between the two seasons.

Q2. (i) Mode = 22

$$Q_1 = X \Rightarrow \frac{1}{4}(21) = 5.25 \Rightarrow Q_1 \text{ is the } 6^{\text{th}} \text{ number} = 11 = X$$

(ii) Median $= Y \Rightarrow 11^{\text{th}}$ number for Jack $= 27 = Y$

$$Q_3 = Z \Rightarrow \frac{3}{4}(21) = 15.75 \Rightarrow Q_3 \text{ is the } 16^{\text{th}} \text{ number} = 22 = Z$$

(iii) Strand road; median is more than double that of market street.

Q3. (i) Driver: Positively skewed as a lot of the data is clustered to the left, especially in the (20–30) year age-group.

Passenger: From the ages (0–40) years, it is a symmetrical distribution with a mean of approximately 20 years. The values then fall away as you move away from the centre.

(ii) (a) Driver: 20 years old

(b) Passenger: 18 years old

(iii) A uniform distribution suggesting casualities equally likely at all ages, with a moderate peak from (15–25) years.

(iv) The (17–30) years age-group. Most of the casualities among both drivers and passengers occur in this group.

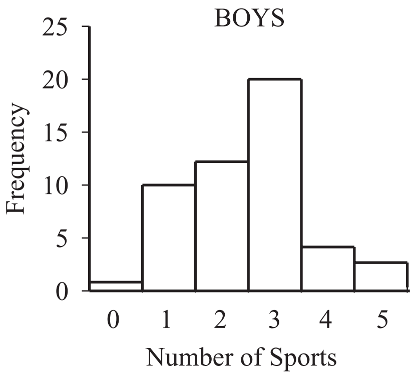
Q4. Boys

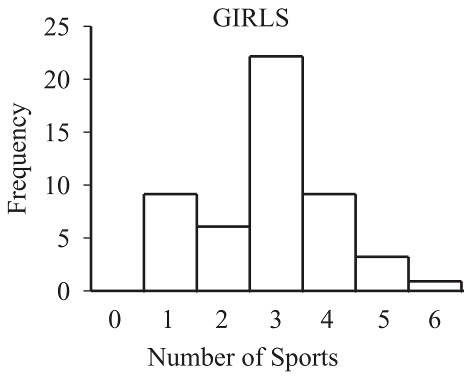
(i)

Number of sports	0	1	2	3	4	5
Frequency	1	10	12	20	4	3

Girls

Number of sports	1	2	3	4	5	6
Frequency	9	6	22	9	3	1





(ii) **Similarity**: Both have the same mode (3).

Difference: Girls' distribution resembles a normal distribution. For the boys, most of the data is concentrated at the lower values (1–3).

(iii) Though the medians are the same, the girls' distribution has a greater spread. The samples' findings are sufficiently different to suggest that this could not happen by chance, i.e. that there is evidence that there are differences between the two populations.

(iv) They could include more boys and girls who are not in G.A.A. clubs. Include both urban and rural children from different parts of the country so the sample would be less biased. Also, be more precise about what "playing sports" means.

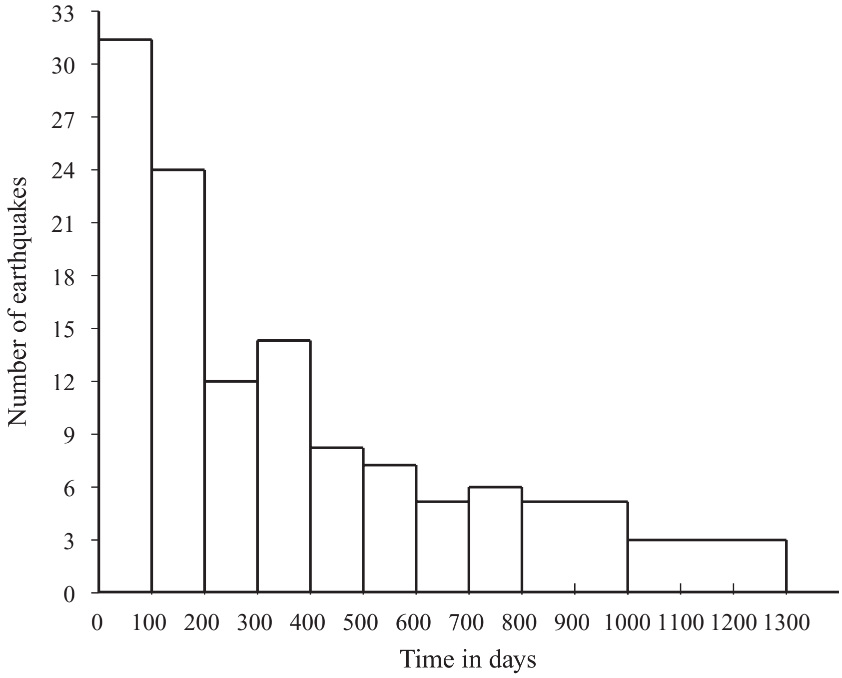
Q5. (i) A – run; B – cycle; C – swim

(ii) 25 minutes

(iii) Standard deviation for the swim times lies between 4.553 and 3.409 $\Rightarrow$ Approx: 3 minutes

(iv) It would be very unusual for two (or more) athletes to have the same time as it is continuous numerical data and times were to the nearest 1000th of a second.

Q6. (i)



(ii) The distribution has a positive skew (tail to the right).

$$\text{Median} = 58^{\text{th}} \text{ earthquake} = 31 + 24 + \frac{1}{3}(12)$$

$$\Rightarrow \frac{1}{4} \text{ into the } 3^{\text{rd}} \text{ interval } [200 - 300] = 225 \text{ days}$$

(iii) It is not a normal distribution and so z-scores are not appropriate. The distribution has a positive skew and hence, it is not a normal distribution.

(iv) $\frac{24}{115} = 0.2087 = 0.21.$ This is the relative frequency of the next earthquake occuring between 100 and 200 days later.

(v) They could have looked at the number of earthquakes each year, or some other interval of time (eg. distribution of earthquakes per decade, per year, etc)

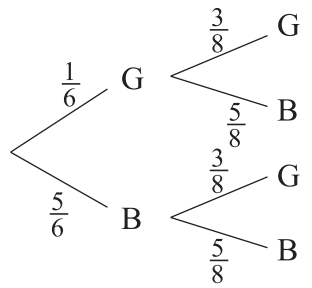
They could have redefined serious earthquakes as earthquakes greater than a certain magnitude; earthquakes in less-populated areas are not included.

The data set could have been broadened to include less serious earthquakes. This could result in a different pattern.

Chapter 3: Probability 2

Exercise 3.1

Q1. (i) 1st Spinner 2nd Spinner



$GG = \frac{1}{6} \times \frac{3}{8}$

GB

BG

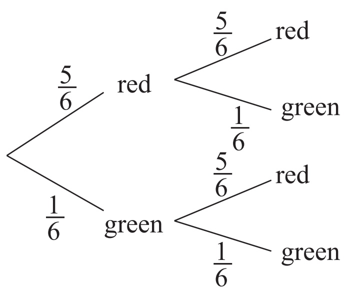
$BB = \frac{5}{6} \times \frac{5}{8}$

(ii) P (the two spinners show the same colour)

$= \left(\frac{1}{6} \times \frac{3}{8}\right) + \left(\frac{5}{6} \times \frac{5}{8}\right)$

$= \frac{28}{48} = \frac{7}{12}$

Q2. (i) 1st Roll 2nd Roll



$RR = \frac{5}{6} \times \frac{5}{6}$

RG

GR

$GG = \frac{1}{6} \times \frac{1}{6}$

(ii) $P(RR) = \frac{25}{36}$

$P(GG) = \frac{1}{36}$

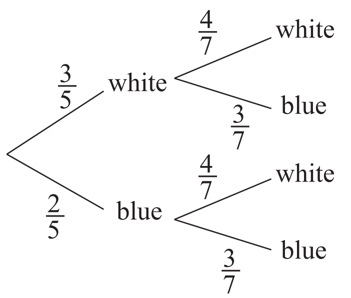
P(same colour) $= P$(both red) **or** P(both green)

$= \frac{25}{36} + \frac{1}{36}$

$= \frac{26}{36} = \frac{13}{18}$

(iii) $P(\text{G and R}) = \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{36}$

Q3. **Bag A** **Bag B**



$$WW = \frac{3}{5} \times \frac{4}{7}$$

$$WB = \frac{3}{5} \times \frac{3}{7}$$

$$BW = \frac{2}{5} \times \frac{4}{7}$$

$$BB = \frac{2}{5} \times \frac{3}{7}$$

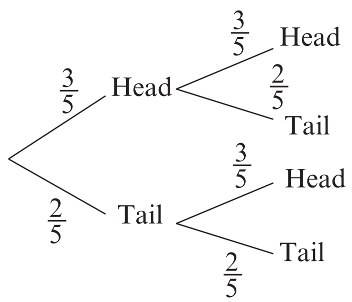
(i) $P(\text{both counters white}) = \frac{3}{5} \cdot \frac{4}{7} = \frac{12}{35}$

(ii) $P(\text{both blue}) = \frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35}$

(iii) $P(\text{both white})$ or $P(\text{both blue})$

$$= \frac{12}{35} + \frac{6}{35} = \frac{18}{35}$$

Q4. (i) **1st Throw** **2nd Throw**



$$HH = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

$$HT = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$$

$$TH = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$

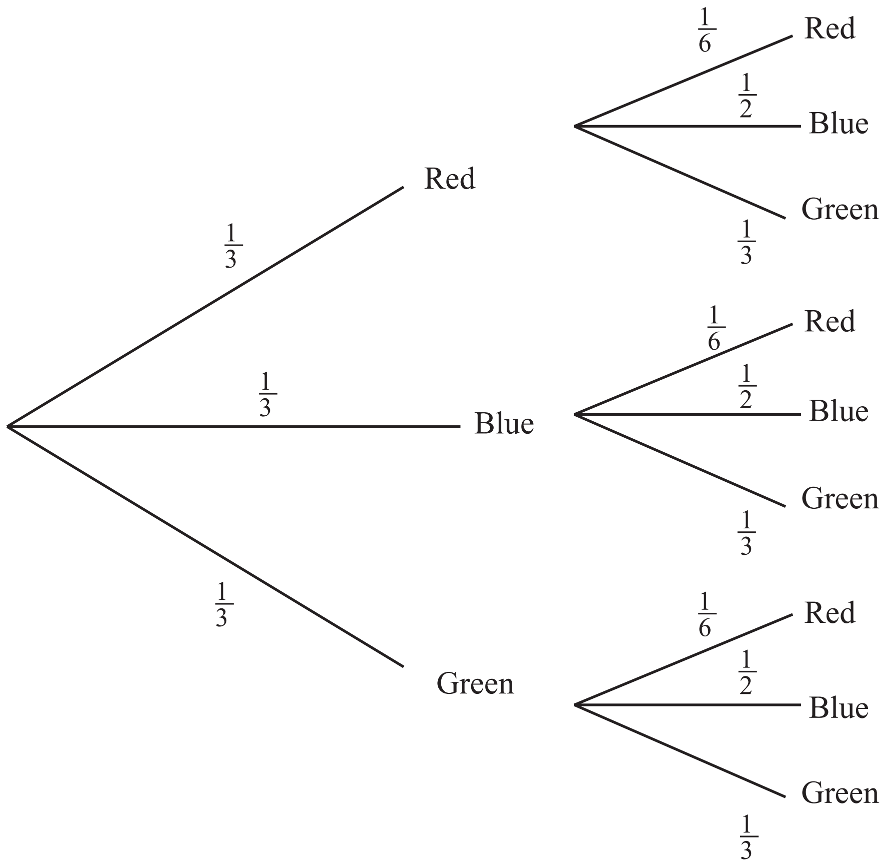
$$TT = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

(ii) $P(\text{two heads}) = P(\text{H,H}) = \frac{9}{25}$

(iii) $P(\text{H,T})$ or $P(\text{T,H}) = \frac{6}{25} + \frac{6}{25}$

$$= \frac{12}{25}$$

Q5. (i) **CUBE A** **CUBE B**



RR $= \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

RB $= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

RG $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

BR $= \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

BB $= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

BG $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

GR $= \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

GB $= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

GG $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

(ii) $P(\text{RR})$ or $P(\text{BB})$ or $P(\text{GG})$

$= \frac{1}{18} + \frac{1}{6} + \frac{1}{9}$

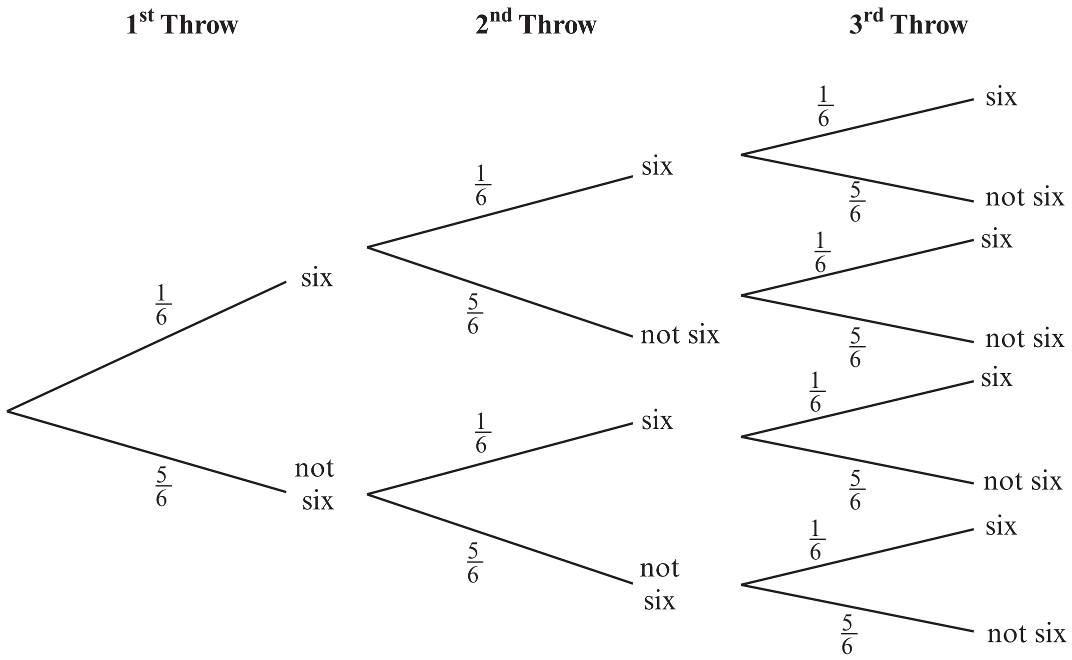
$= \frac{6}{18} = \frac{1}{3}$

(iii) $P(\text{BG})$ or $P(\text{GB})$

$= \frac{1}{9} + \frac{1}{6}$

$= \frac{5}{18}$

Q6. (i)



$=\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}$

$=\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}$

$=\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}$

$=\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}$

(ii) $P(\text{two sixes})$ or $P(\text{three sixes})$

$$= \left(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\right)+\left(\frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}\right)$$

$$+\left(\frac{5}{6}\times\frac{1}{6}\times\frac{1}{6}\right) = \frac{2}{27}$$

Q7. (i) $P(1^{st}\text{ Black})$ and $P(2^{nd}\text{ Black})$ **or**

$P(1^{st}\text{ White})$ and $P(2^{nd}\text{ White})$

$\therefore\ P(1^{st}\text{ Black}) = \frac{1}{3} \qquad P(2^{nd}\text{ Black}) = \frac{1}{5}$

$\therefore\ P(1^{st}\text{ White}) = \frac{2}{3} \qquad P(2^{nd}\text{ White}) = \frac{3}{5}$

$\therefore\ P(\text{same colour}) = \left(\frac{1}{3}\times\frac{1}{5}\right)+\left(\frac{2}{3}\times\frac{3}{5}\right)$

$\therefore \qquad = \frac{1}{15}+\frac{6}{15}$

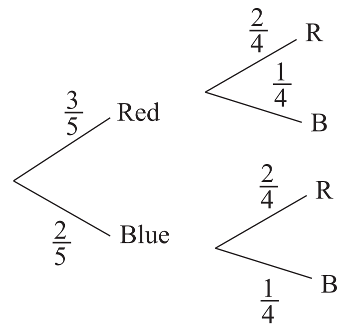
$= \frac{7}{15}$

(ii) $P(\text{different colours}) = 1 - P(\text{same colour})$

$= 1 - \frac{7}{15}$

$= \frac{8}{15}$

Q8. (i) 1^{st} Removal 2^{nd} Removal



$$RR = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$

$$RB = \frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$$

$$BR = \frac{2}{5} \times \frac{2}{4} = \frac{4}{20}$$

$$BB = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$$

(ii) $P(\text{both cubes same colour})$

$= P(\text{RR}) \quad \text{OR} \quad P(\text{BB})$

$$= \left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{1}{4}\right)$$

$$= \frac{6}{20} + \frac{2}{20}$$

$$= \frac{8}{20} = \frac{2}{5}$$

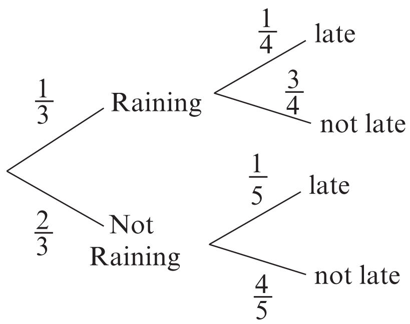
(iii) $P(\text{cubes are different colours})$

$= 1 - P(\text{both same colour})$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

Q9. (i) *Weather* *Simon*



$$\text{R/Late} = \frac{1}{3} \times \frac{1}{4}$$

$$\text{R/not late} = \frac{1}{3} \times \frac{3}{4}$$

$$\text{not Rain/Late} = \frac{2}{3} \times \frac{1}{5}$$

$$\text{not Rain/not Late} = \frac{2}{3} \times \frac{4}{5}$$

(ii) $P(\text{simon late}) = P(\text{raining and late})$

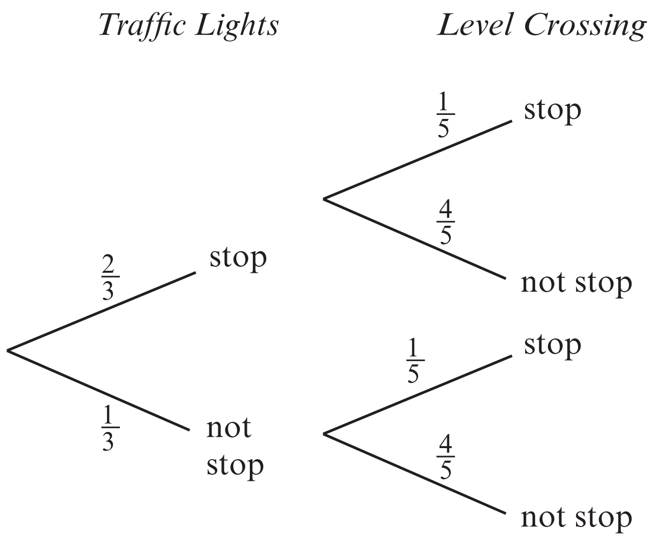
or $P(\text{not raining and late})$

$$\therefore \left(\frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{2}{3} \times \frac{1}{5}\right)$$

$$\therefore \frac{1}{12} + \frac{2}{15}$$

$$= \frac{13}{60}$$

Q10. (i)



$$SS = \frac{2}{3} \times \frac{1}{5}$$

$$SN = \frac{2}{3} \times \frac{4}{5}$$

$$NS = \frac{1}{3} \times \frac{1}{5}$$

$$NN = \frac{1}{3} \times \frac{4}{5}$$

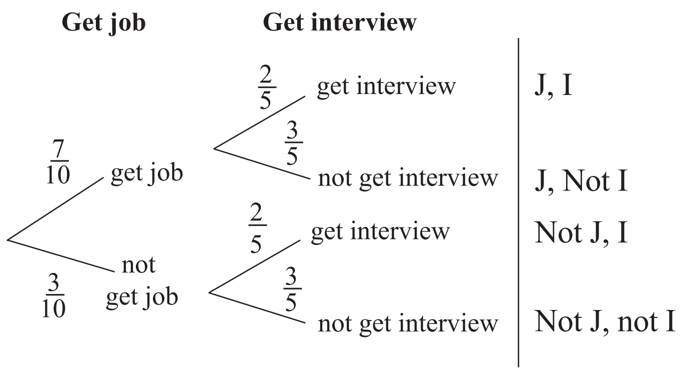
(ii) P(not have to stop at lights or crossing)

$= P$(not stop lights) **and** P(not stop crossing)

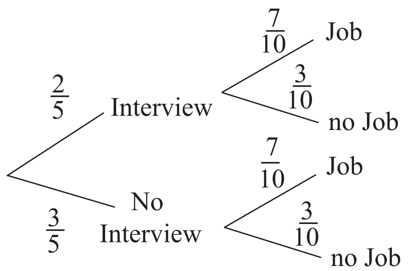
$= \frac{1}{3} \cdot \frac{4}{5}$

$= \frac{4}{15}$

Q11.



or



(i) $P(\text{interview with no job}) = \dfrac{2}{5}\times\dfrac{3}{10} = \dfrac{6}{50} = \dfrac{3}{25}$

P(interview with no job) and P (no interview, no job)

$$\left(\frac{2}{5}\times\frac{3}{10}\right)+\left(\frac{3}{5}\times\frac{3}{10}\right)$$

$$=\frac{6}{50}+\frac{9}{50}$$

$$=\frac{15}{50} = 0\cdot 3$$

Or probability karen does

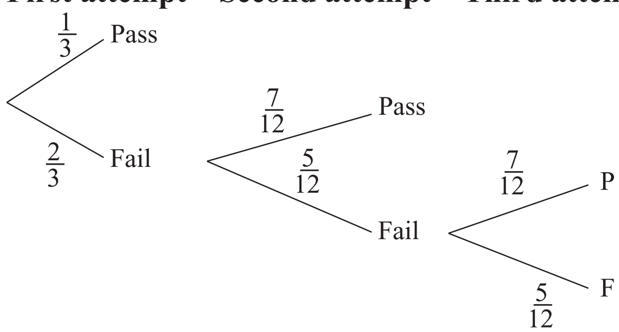
not get the job $= 30\%$

(ii) $P(\text{karen not get job}) = 1 - P(\text{karen get interview and get job})$

$$=1-\left(\frac{7}{10}\times\frac{2}{5}\right)$$

$$=1-\frac{14}{50} = \frac{36}{50} = \frac{18}{25}$$

Q12. **First attempt** **Second attempt** **Third attempt**



$$\text{P} = \frac{1}{3}$$

$$\text{FP} = \frac{2}{3}\cdot\frac{7}{12}$$

$$\text{FFP} = \frac{2}{3}\cdot\frac{5}{12}\cdot\frac{7}{12}$$

$$\text{FFF} = \frac{2}{3}\cdot\frac{5}{12}\cdot\frac{5}{12}$$

$P(\text{pass at } 3^{\text{rd}} \text{ attempt}) = \text{FFP}$

$$=\frac{2}{3}\cdot\frac{5}{12}\cdot\frac{7}{12} = \frac{70}{432} = \frac{35}{216}$$

Exercise 3.2

Q1.

Outcome (x)	Probability (P)	$x \times P$
10	$\frac{1}{4}$	$2\frac{1}{2}$
12	$\frac{1}{2}$	6
6	$\frac{1}{4}$	$1\frac{1}{2}$

$$\therefore \sum x.P(x) = 2.5 + 6 + 1.5$$
$$= 10$$

Q2.

Outcome (x)	2	6	8	9	12
Probability (P)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$x.P(x)$	$\frac{2}{3}$	1	$\frac{8}{6}$	$\frac{9}{6}$	2

$$\therefore \sum x.P(x) = \frac{2}{3} + 1 + 1\frac{1}{3} + 1\frac{1}{2} + 2$$
$$= 6\frac{1}{2} \ = 6.5$$

Q3.

Outcome (x)	2	10	15	20
Probability (P)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$
$x.P(x)$	$\frac{1}{4}$	$3\frac{3}{4}$	$3\frac{3}{4}$	5

$$\sum x.P(x) = \frac{1}{4} + 3\frac{3}{4} + 3\frac{3}{4} + 5$$
$$= €\,12.75$$

Q4.

$$\sum x.P(x) = 0.1 + 0.1 + 0.75 + 0.6 + 1.25 + 1.2$$
$$= 4$$

Q5. Expected value of x

i.e. $\sum x.P(x) = -0.6 - \cancel{0.1} + 0 + 0.4 + \cancel{0.1}$

$= -0.6 + 0.4$

$= -0.2$

Q6.

Outcome (x)	0	1	2	3	4	5
Probability (P)	0·21	0.37	0.25	0.13	0.03	0.01
$x.P(x)$	0	0.37	0.50	0.39	0.12	0.05

$\sum x.P(x) = 0.37 + 0.5 + 0.39 + 0.12 + 0.05$

$= 1.43$

Q7.

Outcome (x)	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
Probability (P)	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{0}{8}$

$\sum x.P(x) = \frac{3}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$

$= \frac{12}{8} = 1.5$

Q8.

Outcome (x)	5	10	20
Probability (P)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$
$x.P(x)$	$\frac{5}{3}$	$\frac{10}{6}$	10

Costs € 10 to play the game.

$\therefore$ Since $\sum x.P(x) = \frac{5}{3} + \frac{10}{6} + 10 = €\,13\frac{1}{3}$,

you expect to win $13\frac{1}{3} - 10$

$= €\,3\frac{1}{3}$

The game is not fair as
mathematical expectation $\neq 0$.

Q9.

Outcome (x)	1	2	3	4	5	6
Probability (P)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$x.P(x)$	$+\frac{10}{6}$	$+\frac{10}{6}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{5}{6}$	$-\frac{5}{6}$

$$\sum x.P(x) = \frac{20}{6} - \frac{20}{6} = 0$$

Yes, the game is fair since
the expected amount is 0 (zero).

Q10. (i) $\sum x.P(x) = 3.52 + 4.76 + 4.62 + 4.8 + 4.8$

$= €\,22.50$

(ii) Grandad will have a **loss**, since his bet on the 5 horses was € 25.

Q11.

$$P(\text{dying}) = \frac{1}{1{,}000} = 0.001$$

$$P(\text{disability}) = \frac{3}{1{,}000} = 0.003$$

$$\begin{aligned}\sum x.P(x) &= 50{,}000(0.001) + 20{,}000(0.003)\\ &= 50 + 60\\ &= €\,110\end{aligned}$$

$$\text{Profit} = €\,300 - €\,110 = €\,190$$

Q12. (i)

$$\begin{aligned}y &= 1 - (0.1 + 0.3 + 0.2 + 0.1)\\ &= 1 - 0.7\\ &= 0.3\end{aligned}$$

(ii)

$$\begin{aligned}\sum x.P(x) &= 1(0.1) + 2(0.3) + 3(0.3) + 4(0.2) + 5(0.1)\\ &= 0.1 + 0.6 + 0.9 + 0.8 + 0.5\\ &= 2.9\end{aligned}$$

Q13.

Outcome (x)	1	2	3	4	5	6
Probability (P)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$x.P(x)$	$-\frac{15}{6}$	$\frac{20}{6}$	0	0	0	$\frac{20}{6}$

$\sum x.P(x) = \frac{25}{6} = €\,4.17$

Costs € 5 to play, $\therefore$ lose $5 - 4.17 = 0.83$

In 20 games $\therefore$ $20 \times 0.83 = €\,16.67$

Q14. (i) $\mathrm{E}(x) = 3$

$\therefore \quad 0.1 + 2p + 0.9 + 4q + 1 = 3$

$\therefore \quad 2p + 4q = 3 - 2$

$\therefore \quad 2p + 4q = 1 \quad \cdots\cdots \quad (1)$

Since $P(x) \quad = 1$,

$\therefore \quad 0.1 + p + 0.3 + q + 0.2 = 1$

$\therefore \quad p + q = -0.6 + 1$

$\quad = 0.4 \quad \cdots\cdots \quad (2)$

(ii) Solve

$$\begin{array}{ll} 2p + 4q = 1 & \cdots\cdots \quad (1) \\ \underline{p + q \;\; = 0.4} & \cdots\cdots \quad (2) \end{array}$$

$$\begin{array}{ll} \cancel{2p} + 4q = 1 & \cdots\cdots \quad (1) \\ \underline{-\cancel{2p} - 2q = -0.8} & \cdots\cdots \quad (2) \times -2 \\ 2q = 0.2 & \\ q = 0.1 & \end{array}$$

$p = 0.4 - q$

$\quad = 0.4 - 0.1$

$\quad = 0.3$

$\therefore \quad p = 0.3 \;, \qquad q = 0.1$

Q15. (i) $P(\text{rural claim}) = \frac{210}{4600} = 0.0456$

(ii) Expected value of cost

$= 0.0456 \cdot \; €\,1705$

$= 77.836$

$= €\,77.84$

(iii) No. of households $= 6250$; Premium $= €\,580$

$$\begin{array}{lll} 6250 \cdot \; 580 = & €\,3{,}625{,}000 & \text{payments} \\ 480 \cdot \; 2840 = & \underline{€\,1{,}363{,}200} & \text{claims} \\ & €\,2{,}261{,}800 & \text{profit} \end{array}$$

Profit per household

$= \frac{2{,}261{,}800}{6{,}250}$

$= 361.888$

$= €\,361.89$

(iv) $P(\text{rural claim}) = 0.05$

$\therefore\ 1550 \times 0.05 = €\,77.5$

Profit $= €\,350$

$\therefore$ annual premium

$= €\,350 + €\,77.5$

$= €\,427.50$

Q16. **Section 1**

$P(\text{A}),\ P(\text{B}),\ P(\text{C}),\ P(\text{D})$

$= \frac{1}{4} \quad = \frac{1}{4} \quad = \frac{1}{4} \quad = \frac{1}{4}$

$\therefore$ 20 questions; expected number of correct answers

$= 20 \times \frac{1}{4}$

$= 5$

Section 2

$P(\text{T}) = \frac{1}{2} \quad P(\text{F}) = \frac{1}{2}$

$\therefore$ with 10 questions, expected number of correct answers

$= 10 \times \frac{1}{2}$

$= 5$

Section 3

$P(\text{A}) = \frac{1}{3}, \quad P(\text{B}) = \frac{1}{3}, \quad P(\text{C}) = \frac{1}{3}$

$\therefore$ 10 questions give $\quad 10 \times \frac{1}{3}$

Expected no. of correct answers $= 3\frac{1}{3}$

$\therefore$ Total correct answers expected

$= 5 + 5 + 3\frac{1}{3}$

$= 13\frac{1}{3}$

.

Q17. **Table 1**

Deck of cards = 52 cards

$P(\text{pick one card}) = \frac{1}{52}$

Outcome (x)	Heart	Other suit
Probability (P)	$\frac{13}{52}$	$\frac{39}{52}$
$x\,.\,P(x)$	$\frac{13}{52}\times 30$	$\frac{39}{52}\times -5$

$\therefore$ Expected payout $=(0.25\times 30)+(0.75\times(-5))$

$$= €\,7.5 - €\,3.75$$
$$= €\,3.75$$

Costs € 10 to play the table

$\therefore$ € $10 - 3.75$

= expected loss of € 6.25

Table 2

Throw 2 dice

Outcome (x)	Dice total 10	Dice total 11	Dice total 12
Probability (P)	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$P(\text{sum } 10, \text{sum } 11, \text{sum } 12) = \frac{6}{36} = \frac{1}{6}$$

$\therefore$ $P(\text{any other sum total}) \quad = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ Expected value

$$= \left(\frac{1}{6}\times 50\right) + \frac{5}{6}(-2)$$
$$= \frac{50}{6} - \frac{10}{6} = \frac{40}{6} = €\,6\frac{2}{3}$$

Costs € 10 to play the table

$\therefore$ € $10 - 6\frac{2}{3}$

= € 3.33 expected loss

Hence, to get the better
expected return, play the dice table
since with cards we lose 6.25 and with dice we lose 3.33.
The difference between the two
expected returns is:

$$€\,6.25 - €\,3.33$$
$$= €\,2.92$$

Exercise 3.3

Q1. (i) There is a fixed number of independent trials, with two outcomes that have constant probabilities.

(ii) $p = \frac{1}{2}, \quad q = \frac{1}{2}, \quad n = 8$

Q2. (i) $\binom{5}{1}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = 5 \cdot \frac{1}{16} \cdot \frac{1}{2} = \frac{5}{32}$

(ii) $\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32}$

$= \frac{5}{16}$

Q3. (i) $P(\text{success}) = \frac{1}{6} \qquad P(\text{failure}) = \frac{5}{6}$

$\therefore\ P(\text{a three}) = \frac{1}{6} \qquad P(\text{not a three}) = \frac{5}{6}$

$\binom{5}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 = \frac{3{,}125}{7{,}776}$

(ii) $\binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 = \frac{3{,}125}{7{,}776}$

(iii) $\binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 10 \cdot \frac{1}{36} \cdot \frac{125}{216} = \frac{625}{3{,}888}$

Q4. $P(\text{success}) = \frac{1}{3} \qquad P(\text{failure}) = \frac{2}{3}$

$\binom{7}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4 = 35 \cdot \frac{1}{27} \cdot \frac{16}{81}$

$= \frac{560}{2{,}187}$

Q5. $P(\text{boy}) = \frac{1}{2}, \qquad P(\text{girl}) = \frac{1}{2}$

$P(3\text{ boys}) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32}$

$\therefore \frac{10}{32} = \frac{5}{16}$

$P(2\text{ girls}) = \binom{5}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = 10 \cdot \frac{1}{4} \cdot \frac{1}{8} = \frac{10}{32}$

$\therefore \frac{10}{32} = \frac{5}{16}$

Q6. (i) $P(\text{success}) = 0.7 \qquad P(\text{failure}) = 0.3$

$\left[\text{Here "success" equals walking to school and not walking equals "failure".}\right]$

$P(\text{walks to school once})$

$$= \binom{5}{1}(0.7)^1(0.3)^4 = 5(0.7)(0.0081)$$

$$= 0.028$$

(ii) $P(\text{walks to school 3 times})$

$$= \binom{5}{3}(0.7)^3(0.3)^2 = 10(0.343)(0.09)$$

$$= 0.3087$$

$$= 0.31$$

Q7. $P(\text{success}) = P(\text{vote X}) = \dfrac{3}{5}$

$P(\text{failure}) = P(\text{not vote for X}) = \dfrac{2}{5}$

$P(\text{3 people vote for party X})$

$$= \binom{8}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^5 = 56 \cdot \frac{27}{125} \cdot \frac{32}{305} = \frac{48{,}384}{390{,}625}$$

$$= 0.1238$$

$$= 0.124$$

Q8. $P(\text{success}) = \dfrac{1}{3} \qquad P(\text{failure}) = \dfrac{2}{3}$

$\qquad = p \qquad\qquad\qquad = q$

$P(\text{3 students completing 4 yrs})$

$$= \binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = 4\left(\frac{1}{27}\right)\left(\frac{2}{3}\right)$$

$$= \frac{8}{81}$$

$P(\text{4 students completing 4 yrs})$

$$= \binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = 1\left(\frac{1}{81}\right)(1)$$

$$= \frac{1}{81}$$

$\therefore \;\; P(\text{3 students at least completing 4 yrs study})$

$$= \frac{8}{81} + \frac{1}{81} = \frac{9}{81} = \frac{1}{9}$$

Q9. (i) $20\%\ \text{ defective} = \frac{20}{100} = \frac{1}{5}$

$P(\text{defective}) = \frac{1}{5}$

$P(\text{not defective}) = \frac{4}{5}$

$P(\text{two bolts defective}) = \binom{4}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = 6\left(\frac{1}{25}\right)\left(\frac{16}{25}\right)$

$= \frac{96}{625}$

(ii) $P(\text{not more than 2 defective})$

$= P(\text{none defective})$ **or** $P(\text{one defective})$

or $P(\text{two defective})$

$P(1\ \text{defective}) = \binom{4}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3 = 4\left(\frac{1}{5}\right)\left(\frac{64}{125}\right)$

$= \frac{256}{625}$

$P(0\ \text{defective}) = \binom{4}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$

$\therefore P(\text{not more than 2 defective}) = \frac{256}{625} + \frac{256}{625} + \frac{96}{625}$

$= \frac{608}{625}$

Q10. $P(\text{success}) = \frac{2}{5} = p$

$P(\text{failure}) = \frac{3}{5} = q$

(i) $P(\text{none travel by bus}) = \binom{4}{0}\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^4$

$= 1\left(\frac{81}{625}\right) = \frac{81}{625}$

(ii) $P(\text{three travel by bus}) = \binom{4}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$

$= 4\left(\frac{8}{125}\right)\left(\frac{3}{5}\right)$

$= \frac{96}{625}$

(iii) $P(\text{at least one of the children travel by bus})$

$$= 1 - P(\text{none travel by bus})$$

$$\therefore \quad 1 - \frac{81}{625}$$

$$= \frac{544}{625}$$

Q11. $P(\text{sink a putt}) = \frac{7}{10} \qquad = p$

$P(\text{not sink a putt}) = \frac{3}{10} \qquad = q$

(i) $n = 3$

$P(\text{sink 2 putts in 3 attempts})$

$$= \binom{3}{2}\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^1 = 3\left(\frac{49}{100}\right)\left(\frac{3}{10}\right) = \frac{441}{1{,}000}$$

(ii) $P(\text{miss 3 putts in 4 attempts}) \qquad n = 4$

$$= \binom{4}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^1 = 4\left(\frac{343}{1{,}000}\right)\left(\frac{3}{10}\right) = \frac{1029}{2{,}500}$$

Q12. $P(A \text{ will win race}) = \frac{2}{5} = p$

$P(A \text{ not win race}) = \frac{3}{5} = q$

(i) $n = 5$

$$\binom{5}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1 = 10\left(\frac{8}{125}\right)\left(\frac{9}{25}\right) = \frac{144}{625}$$

$= P(\text{winning exactly 3 races})$

(ii) $P(A \text{ win } 1^{\text{st}}, 3^{\text{rd}}, 5^{\text{th}} \text{ races}) \qquad n = 5$

$$P(A \text{ win } 1^{\text{st}} \text{ race}) = \binom{5}{1}\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^4 = \cancel{5}\left(\frac{2}{\cancel{5}}\right)\left(\frac{81}{625}\right)$$

$$= \frac{162}{625}$$

$$P(A \text{ win } 3^{\text{rd}} \text{ race}) = \binom{5}{3} = \frac{144}{625}$$

$$P(A \text{ win } 5^{\text{th}} \text{ race}) = \binom{5}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^0 = \frac{32}{3125}$$

$$P(A \text{ lose } 2^{\text{nd}} \text{ race}) = \binom{5}{2}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^3 = 10\left(\frac{9}{25}\right)\left(\frac{8}{125}\right)$$

$$= \frac{720}{3125}$$

$$P(A \text{ lose } 4^{\text{th}} \text{ race}) = \binom{5}{4}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^1 = 5\left(\frac{81}{3125}\right)\left(\frac{2}{5}\right)$$

$$= \frac{162}{3125}$$

$\therefore P(\text{win } 1^{\text{st}}, 3^{\text{rd}}, 5^{\text{th}} \text{ and lose } 2^{\text{nd}} \text{ \& } 4^{\text{th}} \text{ races})$

$$= \frac{144}{625} + \frac{162}{625} + \frac{32}{3125} - \left(\frac{720}{3125} + \frac{162}{3125}\right)$$

$$- \frac{882}{3125}$$

Q13. $P(\text{boy}) = \frac{1}{2} \qquad P(\text{girl}) = \frac{1}{2} \qquad n = 4$

(i) $P(2 \text{ boys}) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

In 2,000 families, those with 4 children (2 boys) are expected to number:

$$2{,}000 \times \frac{3}{8} = 750 \text{ families}$$

(ii) $P(\text{no girls})$ i.e. 4 boys 0 girls

$$\binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = 1\left(\frac{1}{16}\right)$$

With 2,000 families expect:

$$\frac{1}{16} \times 2000 = 125 \text{ families}$$

(iii) $P(\text{at least one boy}) = 1 - P(\text{no boy})$

$$\therefore\ 1-\binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = 1-1\left(\frac{1}{16}\right)$$

$$= \frac{15}{16}$$

In 2,000 families: $\therefore\ \dfrac{15}{16}\times 2{,}000$

$= 1{,}875$ families

Q14. $P(\text{answer correct}) \quad = \dfrac{1}{3}$

$P(\text{answer incorrect}) = \dfrac{2}{3}$

(i) • Suitable because there is a fixed number of independent trials

• There are two outcomes (correct or incorrect)

• Outcomes have constant probabilities

(ii) $P(\text{all 4 answers correct}) = \binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$

$$= 1\left(\frac{1}{81}\right)1 \ = \frac{1}{81}$$

(iii) $P(\text{one answer correct}) = \binom{4}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$

$$= 4\cdot\frac{1}{3}\cdot\frac{8}{27} \ = \frac{32}{81}$$

Probability that Ray gets the first answer correct $= \dfrac{1}{3}$ since in the test there are 3 alternative answers of which exactly one is correct, and he is guessing.

Q15. When a coin is tossed there are only two outcomes:

(1) Getting Head (2) Getting Tail

$P(\text{success}) = P(\text{head}) = p$

$P(\text{failure}) = P(\text{tail}) \quad = q$

Q16. (i) $P(\text{getting a 5 on a throw}) = \frac{1}{6} = p$

$P(\text{not getting a 5 on a throw}) = \frac{5}{6} = q$

$n = 10$

$P(\text{two 5's}) = \binom{10}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8$

$= 45\left(\frac{1}{36}\right)\left(\frac{5}{6}\right)^8$

$= 0.29071$

(ii) $P(\text{getting 3}^{\text{rd}} \text{ five on 11}^{\text{th}} \text{ throw})$

$= P(\text{getting 2 fives in 10 throws}) \cdot P(5)$

$= 0.29071 \cdot \frac{1}{6}$

$= 0.048451$

$= 0.04845$

Q17. (i) $n = 52$ cards

$P(\text{card is picture card}) = \frac{12}{52} = \frac{3}{13}$

(ii) $P(\text{card not picture card}) = \frac{10}{13}$

$P(3^{\text{rd}} \text{ picture card on } 13^{\text{th}} \text{ attempt})$

i.e. 2 picture cards in 12 selections

$\therefore \binom{12}{2}\left(\frac{3}{13}\right)^2\left(\frac{10}{13}\right)^{10} = 66 \times \frac{9}{169} \times \left(\frac{10}{13}\right)^{10}$

$= 0.2548$

$P(\text{picture card on } 13^{\text{th}} \text{ selection}) = \frac{3}{13}$

Thus, $P(3^{\text{rd}} \text{ picture card on } 13^{\text{th}} \text{ selection})$

$= 0.2548 \times \frac{3}{13}$

$= 0.0588$

Q18. Probability (spinner stops on red) = 0.3

P(spinner stops on another colour) = 0.7

$\therefore\ p = 0.3 \qquad q = 0.7$

For 4^{th} red on 10^{th} spin,

$\therefore$ there must be 3 red on first 9 spins.

$$\therefore\ \binom{9}{3}(0.3)^3(0.7)^6 = 84(0.027)(0.117649)$$

$$= 0.2668$$

$P(\text{red on } 10^{\text{th}} \text{ spin}) = 0.3$

$$\therefore\ P(4^{\text{th}} \text{ red on } 10^{\text{th}} \text{ spin}) = 0.2668 \times 0.3$$

$$= 0.08$$

Q19. (i)

$$P(\text{red counter}) = 40\% = \frac{2}{5}$$

$$P(\text{yellow counter}) = 60\% = \frac{3}{5}$$

$$n = 8$$

$$P(3 \text{ red counters}) = \binom{8}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^5$$

$$= 56 \cdot \frac{8}{125} \cdot \frac{243}{3125}$$

$$= 0.27869$$

(ii)

$$P(\text{red counter on ninth draw}) = \frac{2}{5}$$

$\therefore P(4^{\text{th}} \text{ red counter on } 9^{\text{th}} \text{ draw})$

$$= 0.27869 \times \frac{2}{5} = 0.11148$$

Q20. (i) $P(\text{correct answer}) = \dfrac{1}{4} \qquad P(\text{incorrect answer}) = \dfrac{3}{4}$

$$p = \frac{1}{4} \qquad q = \frac{3}{4} \qquad n = 10$$

$$P(\text{no correct answer out of } 10) = \binom{10}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$$

$$= (1)(1)(0.75)^{10} = 0.0563$$

(ii) $P(7 \text{ correct answers}) = \binom{10}{7}\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3$

$\therefore\ 120 \times \frac{1}{16,384} \times \frac{27}{64}$

$= 0.003089$

$= 0.00309$

$P(2 \text{ correct answers in } 9 \text{ questions}) = \binom{9}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^7$

$= 36 \times \frac{1}{16} \times \frac{2,187}{16,384}$

$= 0.3003387$

$P(\text{correct answer on } 10^{\text{th}} \text{ question}) = \frac{1}{4}$

$\therefore P(3^{\text{rd}} \text{ correct answer on } 10^{\text{th}} \text{ question})$

$= 0.3003387 \times \frac{1}{4}$

$= 0.07508$

Exercise 3.4

Q1. (i) $P(A)=\frac{12}{30}=\frac{2}{5}$

(ii) $P(B)=\frac{10}{30}=\frac{1}{3}$

(iii) $P(A\cap B)=P(A).P(B)$

$=\frac{2}{5}\times\frac{1}{3}$

$=\frac{2}{15}$

From diagram, $P(A\cap B)=\frac{4}{30}=\frac{2}{15}$

$\therefore$ since $P(A\cap B)=P(A).P(B)=\frac{2}{15}$

$\therefore$ A and B are independent events

Q2. (i) $P(A)=\frac{1}{3}$

(ii) $P(B)=\frac{1}{4}$

From diagram, $P(A\cap B)=\frac{1}{12}$

$P(A\cap B)=P(A).P(B)$

$=\frac{1}{3}\cdot\frac{1}{4}$

$=\frac{1}{12}$

$\therefore P(A\cap B)=P(A).P(B)=\frac{1}{12}$

$\therefore$ A and B are independent

Q3. $P(A)=0.8$ $\quad$ $P(B)=0.6$

$P(A\cap B)=P(A).P(B)$

$0.48=0.8\times 0.6$ (given)

$=0.48$

$\therefore$ Yes, A and B are independent

since $P(A).P(B)=P(A\cap B)$

Q4. $P(A)=0.4$ $\quad$ $P(B)=0.25$

$P(A\cap B)=P(A).P(B)$

$=0.4\times 0.25$

$=0.1$

Q5. $P(A) = 0.4$ $\quad$ $P(A \cup B) = 0.7$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.7 = 0.4 + P(B) - [P(A).P(B)]$$
$$0.7 - 0.4 = P(B) - 0.4P(B)$$
$$0.3 = 0.6P(B)$$
$$\therefore P(B) = \frac{0.3}{0.6}$$
$$= 0.5$$

Q6. (i) $P(A) = 0.45$ $\quad$ $P(B) = 0.35$

$$P(A \cup B) = 0.7$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.7 = 0.45 + 0.35 - P(A \cap B)$$
$$0.7 - 0.45 - 0.35 = -P(A \cap B)$$
$$0.7 - 0.8 = -P(A \cap B)$$
$$\therefore \ -0.1 = -P(A \cap B)$$
$$\therefore P(A \cap B) = 0.1$$

(ii) $P(A \cap B) = P(A).P(B)$

$$= 0.45 \times 0.35$$
$$= 0.1575$$
$$\therefore \ P(A \cap B) \neq P(A).P(B)$$

$\Rightarrow$ events are not independent

(iii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.35}$

$$= \frac{2}{7}$$

Q7. $P(A) = 0.8$ $\quad$ $P(B) = 0.7$

$$P(A \mid B) = 0.8$$

(i) To find

$P(A \cap B)$:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
$$\therefore \ P(A \cap B) = P(A \mid B) \times P(B)$$
$$= 0.8 \times 0.7$$
$$= 0.56$$

(ii) $P(A \cap B) = P(A) \times P(B)$

$$= 0.8 \times 0.7$$
$$= 0.56$$

A and B are independent events

since $P(A \cap B) = P(A) \times P(B) = 0.56$

Q8. $P(A)=\frac{2}{5}$ $\qquad$ $P(B)=\frac{1}{6}$

$P(A\cup B)=\frac{13}{30}$

(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\frac{13}{30}=\frac{2}{5}+\frac{1}{6}-P(A\cap B)$$

$$\therefore\ \frac{13}{30}-\frac{2}{5}-\frac{1}{6}=-P(A\cap B)$$

$$\frac{2}{15}=P(A\cap B)$$

$$\therefore\ P(A\cap B)=\frac{2}{15}$$

(ii) $P(A\cap B)=P(A).P(B)$

$$=\frac{2}{5}\times\frac{1}{6}$$

$$=\frac{2}{30}\quad=\frac{1}{15}$$

Since $P(A\cap B)=\frac{2}{15}$ and $P(A)\times P(B)=\frac{1}{15}$

they are not equal.

$\therefore$ Events A and B are not independent.

Q9. Given $P(C\mid D)=\frac{2}{3}$ and

$$P(C\cap D)=\frac{1}{3}$$

(i) $P(C\mid D)=\dfrac{P(C\cap D)}{P(D)}$

$$\therefore\quad \frac{2}{3}=\frac{\frac{1}{3}}{P(D)}$$

$$\therefore\ \frac{2}{3}P(D)=\frac{1}{3}$$

$$\therefore\quad P(D)=\frac{1}{3}\div\frac{2}{3}$$

$$=\frac{1}{2}$$

(ii) Since events are independent

$P(C\cap D)=P(C)\times P(D)$

$$\therefore\quad \frac{1}{3}=P(C)\times\frac{1}{2}$$

$$\therefore\ P(C)=\frac{1}{3}\div\frac{1}{2}$$

$$=\frac{2}{3}$$

Q10. Given $P(B) = 0.7, \quad P(C) = 0.6, \quad P(C \mid B) = 0.7$

To find $P(B \cap C)$:

$$P(C \mid B) = \frac{P(C \cap B)}{P(B)}$$

$$\therefore \quad 0.7 = \frac{P(C \cap B)}{0.7}$$

$$\therefore \quad P(C \cap B) = 0.7 \times 0.7$$
$$= 0.49$$

Also, $P(C \cap B) = P(C) \times P(B)$
$$= 0.6 \times 0.7$$
$$= 0.42$$

B and C are not independent
since $0.49 \neq 0.42$

Q11. Given $P(A) = 0.2 \qquad P(B) = 0.15$

(i) To find $P(A \cap B)$, we use
$P(A \cap B) = P(A) \times P(B)$ since events are independent.

$$\therefore \quad P(A \cap B) = 0.2 \times 0.15$$
$$= 0.03$$

(ii) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.03}{0.15}$$
$$= 0.2$$

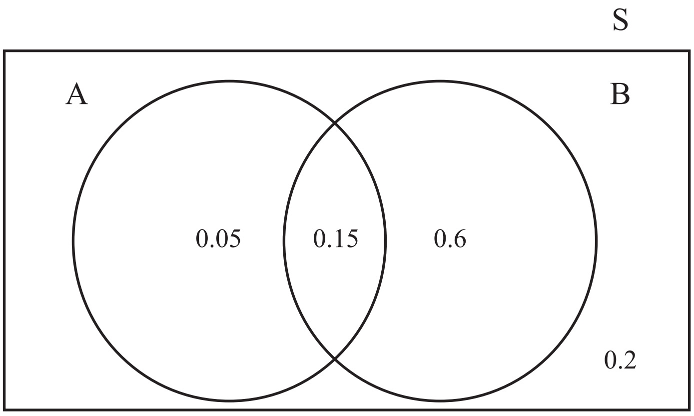
(iii) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.2 + 0.15 - 0.03$$
$$= 0.32$$

Q12. Given:

$P(A) = 0.2 \qquad P(A \cap B) = 0.15$

$P(A' \cap B) = 0.6$

(i)



(ii) $P(\text{neither } A \text{ nor } B) = 1 - P(A \cup B)$

$$= 1 - 0.8$$

$$= 0.2$$

(iii) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.15}{0.75}$$

$$= 0.2$$

(iv) $$P(A \cap B) = P(A) \times P(B)$$

$$= 0.2 \times 0.75$$

$$= 0.15$$

Yes, A and B are independent as
$P(A \cap B) = P(A) \times P(B) = 0.15.$

Q13. Given $P(A) = \frac{8}{15}$ $\quad P(B) = \frac{1}{3}$ $\quad P(A \mid B) = \frac{1}{5}$

(i) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \quad \frac{1}{5} = \frac{P(A \cap B)}{\frac{1}{3}}$$

$$\therefore P(A \cap B) = \frac{1}{5} \times \frac{1}{3}$$

$$= \frac{1}{15}$$

$\therefore P(\text{both events occur}) = \frac{1}{15}$

(ii) $P(\text{only } A \text{ or } B \text{ occurs})$ i.e. $P(A) + P(B)$

$$= \frac{8}{15} + \frac{1}{3}$$

$$= \frac{13}{15}$$

Q14. (i) A and B are independent events whereby the outcome of A does not affect the outcome of B; e.g. B is the event obtaining a head when a coin is tossed.

(ii) If $P(C \text{ or } D) = P(C) + P(D)$,

then we can say that C and D are mutually exclusive events; value of $P(C \text{ and } D) = 0$.

Q15. Given $P(A \mid B) = 0.4$

$P(B \mid A) = 0.25$

$P(A \cap B) = 0.12$

(i) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \quad 0.4 = \frac{0.12}{P(B)}$$

$$\therefore \quad 0.4P(B) = 0.12$$

$$\therefore \quad P(B) = \frac{0.12}{0.4}$$

$$\therefore \quad P(B) = 0.3$$

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

$$\therefore \quad 0.25 = \frac{0.12}{P(A)}$$

$$\therefore \quad P(A) \times 0.25 = 0.12$$

$$\therefore \quad P(A) = \frac{0.12}{0.25}$$

$$\therefore \quad P(A) = 0.48$$

(ii) A and B are not independent

since $P(A \cap B) \neq P(A) \times P(B)$

as $0.144 \neq 0.12$.

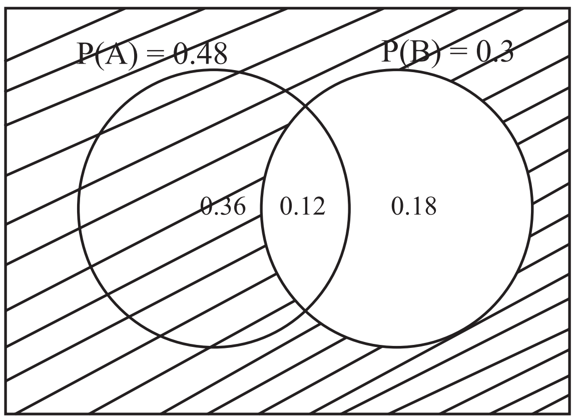
(iii) $P(A \cap B')$

$P(A) = 0.48$ $\qquad P(A \cap B) = 0.12$

$$\therefore \quad P(A \cap B') = P(A) - P(A \cap B)$$

$$= 0.48 - 0.12$$

$$= 0.36$$



B' = Shaded

Q16. Given $P(E)=\frac{2}{5}$, $P(F)=\frac{1}{6}$, $P(E\cup F)=\frac{13}{30}$

$$P(E\cap F)=P(E)\times P(F)$$

$$=\frac{2}{5}\times\frac{1}{6}=\frac{2}{30}=\frac{1}{15}$$

Also, $P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$$\therefore\ \frac{13}{30}=\frac{2}{5}+\frac{1}{6}-P(E\cap F)$$

$$\therefore -\frac{4}{30}=-P(E\cap F)$$

$$\therefore P(E\cap F)=\frac{2}{15}$$

Hence, since $P(E\cap F)\neq P(E)\times P(F)$

$\left(\text{as } \frac{2}{15}\neq\frac{1}{15}\right)$ events E and F are *not* independent.

$P(E\cap F)\neq 0$, so it can be concluded that E and F are *not* mutually exclusive.

Exercise 3.5

Q1. (i) 4 cards can be selected from a pack of 52 in

$$\binom{52}{4} \text{ ways} = 270,725$$

2 queens can be selected in $\binom{4}{2}$ ways

$$\therefore P(\text{exactly 2 queens}) = \frac{6}{270,725}$$

(ii) 4 spades can be selected in $\binom{13}{4}$ ways

$$\therefore P(4 \text{ spades}) = \frac{\binom{13}{4}}{\binom{52}{4}} = \frac{715}{270,725} = \frac{11}{4,165}$$

or 0.00264

(iii) 4 red cards can be selected in $\binom{26}{4}$ ways

$$\therefore P(4 \text{ red cards}) = \frac{\binom{26}{4}}{\binom{52}{4}} = \frac{14,950}{270,725} = \frac{46}{833}$$

(iv) 4 cards of the same suit can be

4 spades *or* 4 clubs *or* 4 hearts *or* 4 diamonds

$\therefore$ P(4 cards of the same suit)

$= 4 \times P(4 \text{ spades})$

$= 4 \times \frac{11}{4,165} = \frac{44}{4,165}$

Q2. (i) A team of 4 can be chosen

in $\binom{11}{4}$ ways $= 330$

$$\text{Selecting 2 men \& 2 women on team} = \binom{6}{2} \times \binom{5}{2} \text{ ways}$$

$$= 15 \times 10$$

$$= 150$$

$$\therefore P(\text{team of 2 men and 2 women}) = \frac{150}{330} = \frac{5}{11}$$

(ii) 1 man and 3 women can be selected

in $\binom{6}{1} \times \binom{5}{3}$ ways $= 60$

$\therefore \quad P(\text{team of 1 man and 3 women}) = \dfrac{60}{330} = \dfrac{2}{11}$

(iii) A team of all women can be selected

in $\binom{5}{4}$ ways $= 5$

$\therefore \; P(\text{team of all women}) = \dfrac{5}{330} = \dfrac{1}{66}$

Q3. Four discs are chosen from 16 in

$\binom{16}{4}$ ways $= 1820$

(i) $P(\text{four discs are blue}) = \dfrac{\binom{5}{4}}{\binom{16}{4}} = \dfrac{5}{1820}$

$= \dfrac{1}{364}$

(ii) 4 discs same colour means:

4 blue, 4 red

$\therefore P(\text{4 discs blue})$ *or* $P(\text{4 discs red})$

$$= \frac{1}{364} + \frac{\binom{6}{4}}{1820}$$

$$= \frac{1}{364} + \frac{15}{1820} \quad = \frac{1}{364} + \frac{3}{364}$$

$$= \frac{4}{364} \quad = \frac{1}{91}$$

$\therefore P(\text{4 discs of same colour}) = \dfrac{1}{91}$

(iii) $P(\text{4 discs of different colours})$

means $P(\text{red disc})$ *and* $P(\text{blue disc})$

and $P(\text{yellow disc})$ *and* $P(\text{green disc})$

$$\therefore \quad \frac{\binom{6}{1} \times \binom{5}{1} \times \binom{3}{1} \times \binom{2}{1}}{1820} = \frac{180}{1820} = \frac{9}{91}$$

$\therefore \; P(\text{4 discs of different colours}) = \dfrac{9}{91}$

(iv) $P(2 \text{ blue and } 2 \text{ not blue})$

$$= \frac{\binom{5}{2} \times \binom{11}{2}}{1820} = \frac{550}{1820} = \frac{55}{182}$$

$\therefore P(2 \text{ blue discs and } 2 \text{ not blue}) = \dfrac{55}{182}$

Q4. (i) Disc numbers are $2, 3, \ldots 10$

Prime numbers are 2, 3, 5, 7

$P(1^{\text{st}} \text{ number prime}) = \dfrac{4}{9}$

$P(2^{\text{nd}} \text{ number prime}) = \dfrac{4}{9}$

$\therefore P(\text{both discs show prime numbers})$

$$= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

(ii) 3 discs can be picked in $\binom{9}{3}$ ways

$$= 84$$

Odd-numbered discs are 3, 5, 7, 9

Even-numbered discs are 2, 4, 6, 8, 10

$$P(\text{picking 3 odd-numbered discs}) = \frac{\binom{4}{3}}{\binom{9}{3}} = \frac{4}{84}$$

$$P(\text{picking 3 even-numbered discs}) = \frac{\binom{5}{3}}{\binom{9}{3}} = \frac{10}{84}$$

$\therefore P(3 \text{ odd- or } 3 \text{ even-numbered discs})$

$$= \frac{4}{84} + \frac{10}{84} = \frac{14}{84} = \frac{1}{6}$$

Q5. 3 cards drawn from $9 = \binom{9}{3} = 84$

Drawing the card numbered 8 means there are only 8 numbers to draw 3 numbers from.

$$\therefore \binom{8}{3} = 56$$

(i) $P(\text{card number 8 not drawn}) = \dfrac{56}{84} = \dfrac{2}{3}$

(ii) Odd-numbered cards are 1, 3, 5, 7, 9

$$P(\text{all 3 cards have odd numbers}) = \frac{\binom{5}{3}}{\binom{9}{3}}$$

$$= \frac{10}{84} = \frac{5}{42}$$

Q6. Sample space $= \binom{24}{3} = 2024$

(i) $P(\text{3 boys celebrating birthday}) = \dfrac{\binom{14}{3}}{2024} = \dfrac{364}{2024}$

$P(\text{3 girls celebrating birthday}) = \dfrac{\binom{10}{3}}{2024} = \dfrac{120}{2024}$

$\therefore P(\text{students are 3 boys or 3 girls})$

$$= \frac{364}{2024} + \frac{120}{2024} = \frac{484}{2024} = \frac{11}{46}$$

(ii) P(a person has a birthday on a particular day in the week)

$$= \frac{1}{7}$$

P(a person does *not* have a birthday on a particular day in the week)

$$= \frac{6}{7}$$

Total probability of a birthday is $\frac{7}{7}$ (i.e. a certainty)

P(one of the 3 has a birthday on a particular day of the week)

$$= \frac{7}{7} \quad \text{i.e. } 1$$

P(the next of the 3 has a birthday on a different day from the first)

$$= \frac{6}{7}$$

P(the third has a birthday on a different day from the two above)

$$= \frac{5}{7}$$

Hence,

P(their birthdays fall on different days of the week)

$$= 1 \cdot \frac{6}{7} \cdot \frac{5}{7} = \frac{30}{49}$$

Q7. (i) $\binom{10}{7}\text{ways} = 120$

(ii) Include Q_1, Q_2 : $\therefore \binom{8}{5}\text{ ways} = 56$

(iii) $P(\text{choosing both } Q_1 \text{ and } Q_2) = \frac{56}{120} = \frac{7}{15}$

(iv) $P(\text{choosing at least one of } Q_1 \text{ or } Q_2)$:

We can use $1 - P(\text{neither } Q_1 \text{ nor } Q_2 \text{ chosen})$

Excluding Q_1 and Q_2 requires choice of selecting from 8 questions

$\therefore$ selection is $\binom{8}{7} = 8$

$\therefore P(\text{neither } Q_1 \text{ nor } Q_2 \text{ chosen}) = \frac{8}{120}$

$\therefore \quad 1 - P(\text{neither } Q_1 \text{ nor } Q_2)$

$$= 1 - \frac{8}{120}$$

$$= 1 - \frac{1}{15} = \frac{14}{15}$$

Q8. 2 pupils to be chosen as prefects can be done

in $\binom{16}{2}\text{ ways} = 120$

(i) $P(\text{one girl and one boy})$:

one girl can be selected in $\binom{10}{1}$ ways

one boy can be selected in $\binom{6}{1}$ ways

$\therefore P(\text{one boy and one girl})$

$$= \frac{\binom{10}{1} \times \binom{6}{1}}{\binom{16}{2}} = \frac{10 \times 6}{120} = \frac{60}{120}$$

$$= \frac{1}{2}$$

(ii) To select left-handed girl is $\binom{3}{1}$

To select left-handed boy is $\binom{1}{1}$

$\therefore P$(one girl left-handed and one boy left-handed)

$$= \frac{\binom{3}{1} \times \binom{1}{1}}{\binom{16}{2}} = \frac{3 \times 1}{120} = \frac{3}{120}$$

$$= \frac{1}{40}$$

(iii) P(two left-handed pupils)

$$= \frac{\binom{4}{2}}{\binom{16}{2}} = \frac{6}{120} = \frac{1}{20}$$

(iv) P(at least one pupil who is left-handed)

= P(one pupil left-handed) **and** P(two pupils left-handed)

P(one pupil left-handed and one not left-handed)

$$= \frac{\binom{4}{1} \times \binom{12}{1}}{\binom{16}{2}} = \frac{4 \times 12}{120} = \frac{48}{120}$$

P(two left-handed pupils) $= \dfrac{1}{20}$ [see part (iii)]

$\therefore P$(at least one pupil left-handed)

$$= \frac{48}{120} + \frac{1}{20} = \frac{48}{120} + \frac{6}{120}$$

$$= \frac{54}{120} = \frac{9}{20}$$

Q9. Given 1 fair dice and 2 biased dice. Bias assigns 6 as twice as likely as any other score.

$\therefore$ scoring on bias dice $= P(6) = \dfrac{2}{7}$ and $P(\text{not } 6) = \dfrac{5}{7}$

$P(\text{rolling exactly two sixes}) =$

$P(6 \text{ on } 1^{\text{st}}, 6 \text{ on second, not } 6)$ *or*

$P(6 \text{ on } 1^{\text{st}}, \text{not } 6 \text{ on second}, 6)$ *or*

$P(\text{not six on } 1^{\text{st}}, 6 \text{ on second}, 6)$

$$\therefore \left(\frac{1}{6}\times\frac{2}{7}\times\frac{5}{7}\right)+\left(\frac{1}{6}\times\frac{5}{7}\times\frac{2}{7}\right)+\left(\frac{5}{6}\times\frac{2}{7}\times\frac{2}{7}\right)$$

$$= \frac{10}{294} + \frac{10}{294} + \frac{20}{294}$$

$$= \frac{40}{294} = \frac{20}{147}$$

Q10. Of the 8 letters, there are 2 A's, 3 P's and C, E, L.

(i) $P(\text{letters } P, E, A \text{ drawn in that order})$

$$= \frac{1}{\binom{8}{3}} = \frac{1}{56}$$

(ii) $P(\text{letters } P, E, A \text{ are drawn in any order})$

$$= \frac{\binom{3}{1}\times\binom{2}{1}\times\binom{1}{1}}{\binom{8}{3}} = \frac{3\times 2\times 1}{56}$$

$$= \frac{6}{56} = \frac{3}{28}$$

(iii) $P(\text{Excluding letters } E \text{ and } P)$

$$= \frac{\binom{4}{3}}{\binom{8}{3}} = \frac{4}{56} = \frac{1}{14}$$

(iv) Consonants $= C, L, P$

Vowels $= A, E$

$P(\text{three letters all vowels})$

$$= \frac{\binom{5}{3}}{\binom{8}{3}} = \frac{10}{56}$$

$$P(\text{3 letters all consonants}) = \frac{\binom{3}{3}}{\binom{8}{3}} = \frac{1}{56}$$

$\therefore P$(3 letters are all consonants *or* all vowels)

$$= \frac{10}{56} + \frac{1}{56}$$

$$= \frac{11}{56}$$

Exercise 3.6

Q1. (i) $P(z \le 1.2) = 0.8849$

(ii) $P(z \ge 1) = 1 - P(z \le 1)$

$= 1 - 0.8413$

$= 0.1587$

(iii) $P(z \le -1.92)$

$= 1 - P(z \ge 1.92)$

(because the curve is symmetrical, w**e** find the area to the left of 1.92)

$\therefore P(z \le -1.92) = 1 - P(z \ge 1.92)$

$= 1 - 0.9726$

$= 0.0274$

(iv) $P(-1.8 \le z \le 1.8)$

Area to the left of $1.8 = 0.9641$

Area to the right of $-1.8 = 1 - P(z \le 1.8)$

$= 1 - 0.9641$

$= 0.0359$

$\therefore$ Area shaded portion is $0.9641 - 0.0359$

$= 0.9282$

Q2. $P(z \le 1.42) = 0.9222$

Q3. $P(z \le 0.89) = 0.8133$

Q4. $P(z \le 2.04) = 0.9793$

Q5. $P(z \ge 2) = 0.9722$

Q6. $P(z \ge 1.25) = 0.8944$

Q7. $P(z \ge 0.75) = 0.7723$

Q8. $P(z \le -2.3)$

Use the fact the curve is symmetrical

$\therefore P(z \le -2.3) = 1 - P(z \ge 2.3)$

$= 1 - 0.9893$

$= 0.0107$

Q9. $P(z \le -1.3) = 1 - P(z \ge 1.3)$

$= 1 - 0.9032$

$= 0.0968$

Q10. $P(z \le -2.13) = \text{left tail}$

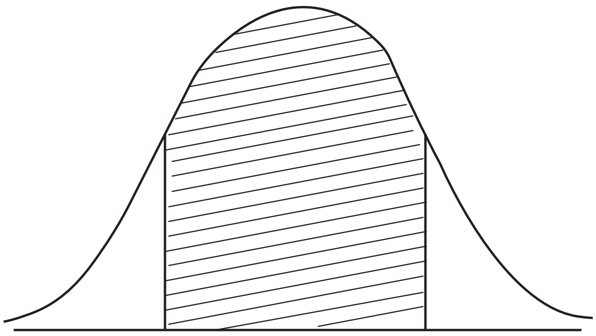
$\therefore P(z \le -2.13) = 1 - P(z \ge 2.13)$

$= 1 - 0.9834$

$= 0.0166$

Q11. $P(z \le 0.56) = 0.7123$

Q12. $P(-1 \le z \le 1)$



(i) Area to left of $1 = 0.8413$

(ii) Area to right of $-1 = 1 - 0.8413 = 0.1587$

Then subtract (ii) from (i)

Shaded area $= 0.8413 - 0.1587$

$= 0.6833$

Q13. $P(-1.5 \le z \le 1.5)$

Area to left of $1.5 = 0.9332$

Area to right of $-1.5 = 1 - P(z \le 1.5)$

$= 1 - 0.9332$

$= 0.0668$

$\therefore$ shaded portion $= 0.9332 - 0.0668$

$= 0.8664$

Q14. $P(0.8 \le z \le 2.2)$

Area to left of $2.2 = 0.9861$

Area to right of $0.8 = 0.7881$

$\therefore$ Area (ii) – Area (i) $= 0.9861 - 0.7881$

$= 0.1980$

Q15. $P(-1.8 \le z \le 2.3)$

Area to left of $2.3 = 0.9893$

Area to right of $-1.8 = 1 - P(z \le 1.8)$

$= 1 - 0.9641$

$= 0.0359$

$\therefore$ Area (ii) – Area (i) $= 0.9893 - 0.0359$

$= 0.9534$

Q16. $P(-0.83 \leq z \leq 1.4)$

$$\text{Area to left of } 1.4 = 0.9192$$

$$\text{Area to right of } -0.83 = 1 - P(z \leq 0.83)$$

$$= 1 - 0.7967$$

$$\therefore \quad = 0.2033$$

$$\therefore \quad 0.9192 - 0.2033 = 0.7159$$

Q17. $P(z \leq z_1) = 0.8686$

$$\therefore z_1 = 1.12$$

Q18. $P(z \leq z_1) = 0.6331$

$$\therefore z_1 = 0.34$$

Q19. $P(-z_1 \leq z \leq z_1) = 0.6368$

$$z_1 = 0.91$$

Q20. $P(-z_1 \leq z \leq z_1) = 0.8438$

$$\therefore z_1 = 1.42$$

Q21. $\mu = 50 \qquad \sigma = 10$

(i) $P(z \leq 60)$

$$z\text{-score} = \frac{60-50}{10} = \frac{10}{10} = 1$$

$$\therefore P(z \leq 1) = 0.8413$$

(ii) $P(x \leq 55)$

$$z\text{-score} = \frac{55-50}{10} = \frac{5}{10} = 0.5$$

$$\therefore P(z \leq 0.5) = 0.6915$$

(iii) $P(x \geq 45)$

$$z\text{-score} = \frac{45-50}{10} = \frac{-5}{10} = -\frac{1}{2}$$

$$\therefore P(x \geq -0.5) = 0.6915$$

Q22. $z\text{-score} = \frac{60-55}{25} = \frac{-6}{25}$

$$\therefore P(z \geq -0.24)$$

$$= 0.5948$$

(ii) $P(x \le 312)$

$$z\text{-score} = \frac{312-300}{25} = \frac{12}{25}$$

$$= 0.48$$

$\therefore P(z \le 0.48) = 0.6844$

Q23. (i) $\mu = 250, \qquad \sigma = 40$

$P(z \ge 300)$

$$z\text{-score} = \frac{300-250}{40} = \frac{50}{40}$$

$\therefore P(z \ge 1.25)$

$$= 1 - 0.8944$$

$$= 0.1056$$

(ii) $P(x \le 175)$

$$z\text{-score} = \frac{175-250}{40} = \frac{75}{40}$$

$\therefore P(z \le 1.875)$

$$= 1 - 0.9699$$

$$= 0.0301$$

Q24. (i) $\mu = 50 \qquad \sigma = 8$

$P(52 \le x \le 55)$

$$z\text{-score} = \frac{52-50}{8} = \frac{2}{8} = 0.25$$

$$z\text{-score} = \frac{55-50}{8} = \frac{5}{8} = 0.625$$

$\therefore P(0.25 \le z \le 0.625)$

$$= 0.7357 - 0.5987$$

$$= 0.1370$$

(ii) $P(48 \le x \le 54)$

$$z\text{-score} = \frac{48-50}{8} = \frac{-2}{8} = -0.25$$

$$z\text{-score} = \frac{54-50}{8} = \frac{4}{8} = 0.5$$

$\therefore P(-0.25 \le z \le 0.5)$

$$P(z \le 0.5) = 0.6915$$

$$P(-0.25 \le z) = 1 - P(z \le 0.25)$$

$$= 1 - 0.5987$$

$$= 0.4013$$

$\therefore P(-0.25 \le z \le 0.5) = 0.6915 - 0.4013$

$$= 0.2902$$

Q25. $\mu = 100, \qquad \sigma = 80$

(i) $P(85 \le x \le 112)$

$$z\text{-score} = \frac{85-100}{80} = \frac{-15}{80} = -0.1875$$

$$z\text{-score} = \frac{112-100}{80} = \frac{12}{80} = 0.15$$

$$\therefore P(-0.1875 \le z \le 0.15)$$
$$= P(z \le 0.15) = 0.5596$$
$$P(-0.1875 \le z) = 1 - P(z \le 0.1875)$$
$$= 1 - 0.5753$$
$$= 0.4247$$
$$\therefore P(85 \le x \le 112) = 0.5596 - 0.4247$$
$$= 0.1349$$

(ii) $P(105 \le x \le 115)$

$$z\text{-score} = \frac{105-100}{80} = \frac{5}{80} = 0.0625$$

$$z\text{-score} = \frac{115-100}{80} = \frac{15}{80} = 0.1875$$

$$\therefore P = P(0.0625 \le z \le 0.1875)$$
$$\therefore P(105 \le x \le 115) = P(0.0625 \le z \le 0.1875)$$
$$= 0.5753 - (0.5239)$$
$$= 0.0514$$

Q26. $\mu = 200 \qquad \sigma = 20$

(i) $P(190 \le x \le 210)$

$$z\text{-score} = \frac{190-200}{20} = \frac{-10}{20} = -0.5$$

$$z\text{-score} = \frac{210-200}{20} = \frac{10}{20} = 0.5$$

$$\therefore P(-0.5 \le z \le 0.5)$$
$$= 0.6915 - (1 - 0.6915)$$
$$= 0.6915 - 0.3085$$
$$= 0.3830$$

(ii) $P(185 \le x \le 205)$

$$z\text{-score} = \frac{185-200}{20} = \frac{15}{20} = -0.75$$

$$z\text{-score} = \frac{205-200}{20} = \frac{5}{20} = 0.25$$

$$\therefore \quad P = P(-0.75 \le z \le 0.25)$$
$$= 0.5987 - (1 - 0.7734)$$
$$= 0.5987 - 0.2266$$
$$= 0.3721$$

Q27. (i) $x = 240, \quad \mu = 210, \quad \sigma = 20$

$$z\text{-score} = \frac{240-210}{20} = \frac{30}{20} = 1.5$$

$$P(x > 240) = P(z > 1.5)$$
$$= 1 - 0.9332$$
$$= 0.0668$$

(ii) $P(\text{bulb last} \le 200 \text{ hrs})$

$$z\text{-score} = \frac{200-210}{20} = \frac{-10}{20} = -0.5$$

$$\therefore P(z \le -0.5)$$
$$= 1 - 0.6915$$
$$= 0.3085$$

Q28. (i) $\mu = 101 \text{ cm}, \quad \sigma = 5 \text{ cm}, \quad x = 103 \text{ cm}$

$P(\text{customer has chest measurement} < 103 \text{ cm})$

= writing expression in z-scores

$$z\text{-score} = \frac{103-101}{5} = \frac{2}{5} = 0.4$$

$$\therefore P = P(z < 0.4)$$
$$= 0.6554$$

$$\therefore P(\text{chest} < 103 \text{ cm}) = 0.6554$$

(ii) $P(\text{chest size} \ge 98 \text{ cm})$

$$= z\text{-score of } \frac{98-101}{5} = -\frac{3}{5}$$
$$= -0.6$$

$$\therefore P(z \ge -0.6)$$
$$= 0.7257$$

(iii) $P(\text{chest measurement between 95 cm and 100 cm})$

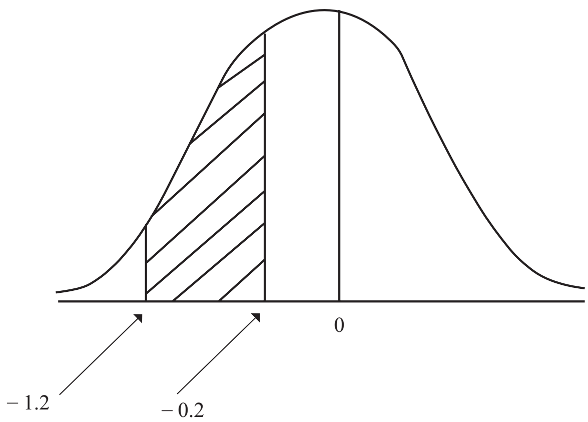
$$= z\text{-score of } \frac{95-101}{5} \text{ and } \frac{100-101}{5}$$

$$\therefore z = \frac{-6}{5} \text{ and } z = \frac{-1}{5}$$

$$= -1.2 \text{ and } z = -0.2$$

$$\therefore P(-1.2 \le z \le -0.2)$$

$$\therefore 0.8849 - 0.5793$$
$$= 0.3056$$



Q29. (i) $\mu = 12 \qquad \sigma = 2$

P(postman takes longer than 17 mins)

is changed to z-scores

$$z\text{-score} = \frac{17-12}{2} = \frac{5}{2} = 2\frac{1}{2}$$

$\therefore P(z > 2.5) = 1 - P(z < 2.5)$

$= 1 - 0.9938$

$= 0.0062$

(ii) P(taking less than 10 mins)

$$z\text{-score} = \frac{10-12}{2} = \frac{-2}{2} = -1$$

$\therefore P = P(z < -1) = 1 - 0.8413$

$= 0.1587$

(iii) P(taking between 9 and 13 mins)

1st get P(taking 9 mins) and

then get P(taking 13 mins)

$$z\text{-score} = \frac{9-12}{2} = \frac{-3}{2} = -1.5$$

$$z\text{-score} = \frac{13-12}{2} = 0.5$$

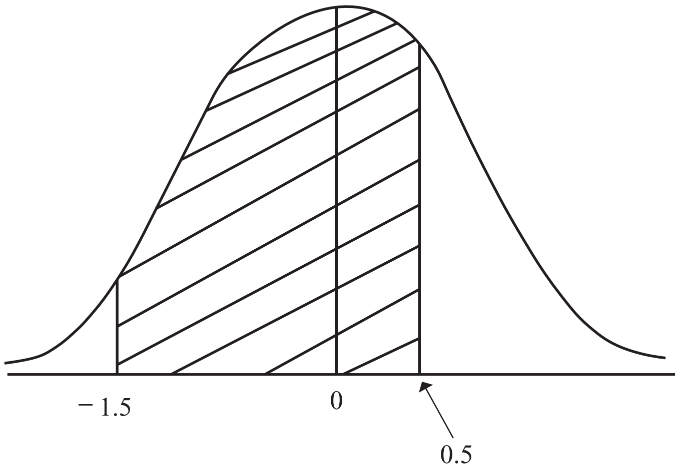
$\therefore P$(between 9 and 13 mins)

$= P(-1.5 \leq z \leq 0.5)$

$= 0.9332 - (1 - 0.6915)$

$= 0.9332 - 0.3085$

$= 0.6247$



Q30. $\mu = 53 \qquad \sigma = 15$

To find P(bill between €47 and €74):

$$z\text{-score} = \frac{47-53}{15} = \frac{-6}{15} = -0.4$$

$$z\text{-score} = \frac{74-53}{15} = \frac{21}{15} = 1.4$$

$\therefore P$(bill between €47 and €74)

$= P(-0.4 \leq z \leq 1.4)$

$= 0.6554 - (1 - 0.9192)$

$= 0.6554 - 0.0808$

$= 0.5746$

Q31. (i) $\mu = 165 \qquad \sigma = 3.5$ cm

P(a student is less than 160 cm high)

$$z\text{-score} = \frac{160-165}{3.5} = \frac{-5}{3.5} = -1.428$$

$$\therefore P = P(x < 160 \text{ cm})$$
$$= P(z < 1.428)$$
$$= 1 - 0.9236$$
$$= 0.0764$$

$\therefore P$(a student is less than 160 cm high)

$$= 0.0764$$

(ii) P(student with height between 168 cm and 174 cm)

$$z\text{-score} : \frac{168-165}{3.5} = \frac{3}{3.5} = 0.857$$

$$z\text{-score} : \frac{174-165}{3.5} = \frac{9}{3.5} = 2.571$$

$\therefore P$(a student with height between 168 cm and 174 cm)

$$= P(0.857 \le z \le 2.571)$$
$$= 1 - 0.8051 - 0.0051$$
$$= 0.1949 - 0.0051$$
$$= 0.1898 = 18.98\%$$

= approx. 19% of students from this group would statisfy the condition of having a height between 168 cm and 174 cm.

Q32. Given:

$x = 500, \qquad \mu = 151$ mm, $\qquad \sigma = 15$ mm

P(having leaves greater than 185 mm long)

$$z\text{-score} := \frac{185-151}{15} = \frac{34}{15} = 2.266$$

$$\therefore P(z > 2.266)$$
$$= 1 - 0.9881$$
$$= 0.0119$$

Of the 500 laurel leaves, then

$$500 \times 0.0119$$
$$= 5.95$$
$$= 6 \text{ leaves}$$

measure greater than 185 mm long.

(ii) z-score for a leaf 120 mm long

$$= \frac{120-151}{15} = -2.066$$

z-score for a leaf 155 mm long

$$= \frac{155-151}{15} = 0.26$$

$\therefore P$(leaves between 120 and 155 mm)

is $P(-2.06 \le z \le 0.26)$

$$= 0.9808 - 0.3936$$
$$= 0.5872.$$

Of the 500 leaves, then

500×0.5872 leaves have lengths

between 120 mm and 155 mm.

$$= 500 \times 0.5872$$
$$= 293.6 \text{ leaves}$$
$$= 294 \text{ leaves}$$

Q33. Given: $\mu = 300$ grams, $\sigma = 6$ grams

P(weight less than 295 grams) shows

$$z\text{-score} = \frac{295-300}{6} = \frac{-5}{6} = -0.833$$

$\therefore P(z < -0.833)$

$$= 1 - P(z > 0.833)$$
$$= 1 - 0.7967$$
$$= 0.2033$$

Out of 1,000 packages

then $1,000 \times 0.2033$

weigh less than 295 grams

(ii) To find the number of packages between 306 and 310 grams,

write the weights in z-scores.

z-score : $\dfrac{306-300}{6} = \dfrac{6}{6} = 0.1$

z-score : $\dfrac{310-300}{6} = \dfrac{10}{6} = 1.66$

$\therefore P$(a packet of weight between 306 and 310 grams)

$$= P(1 \le z \le 1.66)$$
$$= (1 - 0.8413) - (1 - 0.9527)$$
$$= 0.1587 - 0.0475$$
$$= 0.1112$$
$$\therefore 1000 \times 0.1112$$
$= 111$ packets weigh between 306 and 310 grams.

Q34. (i) $\mu = 60\%$ $\sigma = 10\%$

(a) P(mark less than 45%)

has z-score

$$= \frac{45-60}{10} = -\frac{15}{10} = -1.5$$

$P(z < -1.5)$

$= 1 - P(z > 1.5)$

$= 1 - 0.9332$

$= 0.0668$

(b) P(mark is between 50% and 75%)

has z-score

$$\frac{50-60}{10} = -\frac{10}{10} = -1$$

$$\frac{75-60}{10} = \frac{15}{10} = 1.5$$

$\therefore P(-1 \le z \le 1.5)$

$= 0.8413 - (1 - 0.9332)$

$= 0.8413 - 0.668$

$= 0.7745$

$\therefore P$(a randomly selected student scored between 50% and 75% in Geography)

$= 0.7745$ $(= 77.45\%)$

(ii) P(attaining more than 90%)

will give a special award.

Let x be the number of students attaining more than 90% so

$\therefore z$-score

$$= \frac{x-60}{10} = 0.9$$

From the tables, a z-score of 0.900 is given by 1.29,

i.e. 0.9015

$\therefore \frac{x-60}{10} = 1.29$

$\therefore x - 60 = 10(1.29)$

$\therefore x - 60 = 12.90$

$\therefore x = 72.9\%$

$= 73\%$

$\therefore$ the percentage mark students need in order to get a special award

is more than 73% in Geography.

Exercise 3.7

Q1. A possible generation can be carried out by generating random numbers 1–20 on a calculator.

A simulation like the one above indicates that you need to buy 34 packets of crisps to get the full set. Repeat the simulation as many times as you like. The more times you repeat the experiment, the more confidence you can have in your results.

Q2. 3 food options = meat, fish, vegetarian

Allocate numbers 1–8, allowing No. 1 and 2 be fish (told probability is 2/8)

Allocate No. 3 to vegetarian i.e. 1/8

Allocate numbers 4, 5, 6, 7 and 8 to meat i.e. meat = 5/8

Q3. A possible simulation would be to toss 4 coins where

H (head) stands for boy

T (tail) stands for girl

Outcomes of one such experiment

1.	HHTT	2B 2G
2.	HHHT	3B 1G
3.	TTTH	1B 3G
4.	HHTT	2B 2G
5.	HTTH	2B 2G
6.	HHTT	2B 2G
7.	HTTT	1B 3G
8.	HHHT	3B 1G
9.	HHTT	2B 2G
10.	TTTH	1B 3G
11.	HHTT	2B 2G
12.	TTHH	2B 2G
13.	TTHH	2B 2G
14.	TTTT	0B 4G
15.	HTTT	1B 3G
16.	HHTT	2B 2G

After 16 tosses:

(i) Probability that the girls outnumber the boys is $\frac{5}{16} = 0.3125$

(ii) Probability that all the 4 children are girls is $\frac{1}{16} = 0.0625$

Q4. You could generate random numbers; Allocate numbers 0 and 1 for cars turning right. Since 80% of cars turn left, allocate numbers 2, 3, 4, 5, 6, 7, 8, 9 for cars turning left. (The random numbers can be generated on a calculator or use a random number table.)

Q5. (i) $P(\text{win away}) = 0.4$

$P(\text{win at home}) = 0.7$

$\therefore$ In 12 home games $P(\text{winning})$

$= 12 \times 0.7$

$= 8.4$ games

$\therefore$ In 13 away games $P(\text{winning})$

$= 13 \times 0.4$

$= 5.2$ games

$\therefore$ The **Ringdogs** should win

$8.4 + 5.2 = 13.6$ games

$= 14$ games

(ii) The results of a simulation do approximately agree with the result above.

Q6. Possible simulations with discs, counters, calculators, computers, or even get your friends to buy the same breakfast cereal so they will have all 8 superhero figures.

Two possible simulations are presented by generating random number tables (numbers 1–8).

Simulation result:

1	4	3	7
5	6	8	1
6	7	2	5
8	2		

Based on this simulation, you would need to buy 14 packets.

Another simulation resulted in:

5	1	6	4	2
6	4	6	1	6
3	4	3	1	4
1	7	6	6	1
7	6	8		

In this case, 23 packets of *Chocopops* were purchased in order to collect the full set.

The more the experiment is repeated, the more confidence you have in the results.

Q7. $P(\text{at least one } 6) = 1 - P(\text{no six})$

$P(\text{no six in 4 rolls of a dice})$

$$= \binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4$$

$$= (1)(1)\frac{625}{1296}$$

$$= 0.48$$

Since $P(6) = \dfrac{1}{6}$ and $P(\text{not } 6) = \dfrac{5}{6}$

$\therefore P(\text{at least one } 6)$

$$= 1 - 0.48$$

$$= 0.52$$

Q8. The likely size of a family that contains (at least) one child of each gender is 3.

A simulation could assume an equal chance of being a boy or a girl. You could toss coins, or roll dice, to simulate the gender of the children.

Generally, the probability of boys and girls in families are approximately 1/2.

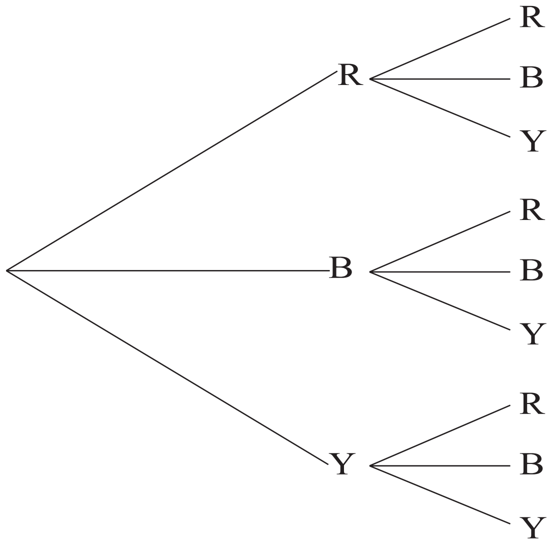
Test Yourself 3

A – Questions

Q1. $P(z \geq 0.93) = 1 - P(z \leq 0.93)$

$$= 1 - 0.8238$$
$$= 0.1762$$

Q2. (ii) event; selecting two counters from a bag of red, blue and yellow counters.



Q3. $P(\text{sink a 1m putt}) = 0.7$

$P(\text{not sink 1m putt}) = 0.3$

$\therefore\ P(\text{sink 3 in 4 attempts})$

$$= \binom{4}{3}(0.7)^3(0.3)^1$$
$$= 4 \times 0.343 \times 0.3$$
$$= 0.4116$$

Q4. (i) Children can be selected in $\binom{30}{5}$ ways

$$= 142{,}506$$

(ii) No. of selections with 2 boys and 3 girls

$$= \binom{10}{2} \times \binom{20}{3}$$
$$= 45 \times 1140$$
$$= 51{,}300$$

(iii) $P(\text{exactly 2 boys selected})$

$$= \frac{51,300}{142,506}$$
$$= \frac{950}{2639}$$
$$= 0.0359$$
$$= 0.36$$

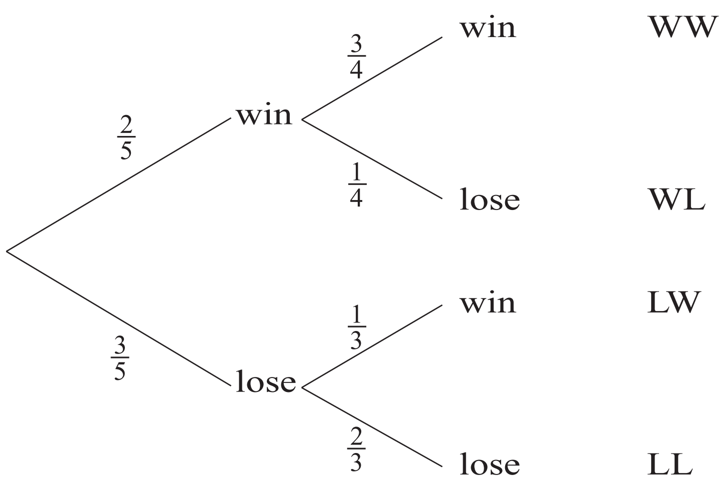
Q5. $P(-1 \le z \le 1.24)$

$$P(z \le 1.24) = 1 - 0.8925$$
$$= 0.1075$$
$$P(-1 \le z) = 0.8413$$
$$\therefore P(-1 \le z \le 1.24) = 0.8413 - 0.1075$$
$$= 0.7338$$

Q6.

$$P(\text{success} - \text{defective}) = \frac{1}{5}$$
$$P(\text{failure} - \text{not defective}) = \frac{4}{5}$$
$$P(\text{no item defective}) = \binom{4}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4$$
$$= (1)(1)\left(\frac{256}{625}\right)$$
$$= \frac{256}{625}$$

Q7. **1st Match** **2nd Match**



(i) $P(\text{loses both matches}) = \frac{3}{5} \cdot \frac{2}{3}$

$$= \frac{6}{15} = \frac{2}{5}$$

(ii) $P(\text{wins only one match}) = P(\text{wins } 1^{\text{st}}, \text{loses } 2^{\text{nd}})$ or $P(\text{loses } 1^{\text{st}}, \text{wins } 2^{\text{nd}})$

$$= \left(\frac{2}{5}\times\frac{1}{4}\right) + \left(\frac{3}{5}\times\frac{1}{3}\right)$$

$$= \frac{2}{20} + \frac{3}{15}$$

$$= \frac{3}{10}$$

Q8.

(i) $P(E) = 0.5$

(ii) $P(F) = 0.8$

(iii) $P(E \cup F) = 0.9$

If E and F are independent, then

from diagram, $P(E \cap F) = 0.4$

Also, $P(E \cap F) = P(E) \times P(F)$

$$= 0.5 \times 0.8$$

$$= 0.4$$

$\therefore$ Since $P(E \cap F) = P(E) \times P(F) = 0.4,$

events E and F are independent.

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{0.4}{0.8} = 0.5$$

$\therefore \quad P(E \mid F) = 0.5$

Q9. $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{not ace}) = \frac{12}{13}$

Drawing an ace wins € 10,

so $\therefore$ Net win $= 10 - 1$ (entry cost)

$= €\ 9$

Customer spends € 12 on the other turns of not getting an ace.

$\therefore$ Expected Profit $= \frac{12-9}{13} = \frac{3}{13}$

$= 0.23$ cents

Q10. The first number can be taken in 4 ways.

The second number can be taken in 3 ways.

$\therefore$ the two cards can be picked in 4×3 (i.e. 12) ways.

If 1 is picked, then 2, 3, 4 are higher $\Rightarrow$ 3 ways

If 2 is picked, then 3, 4 are higher $\Rightarrow$ 2 ways

If 3 is picked, then 4 only is higher $\Rightarrow$ 1 way

$\left[\text{Note: Obviously if 4 is picked then the } 2^{\text{nd}} \text{card cannot be higher, i.e. "0 ways"}\right]$

$$\therefore\ P(\text{2nd number is higher than first number}) = \frac{3+2+1}{12}$$

$$= \frac{6}{12} = \frac{1}{2}$$

Test Yourself 3

B – Questions

Q1. A tennis match has 2 or 3 sets.

$P(A \text{ wins a set}) = \frac{2}{3}$; $P(B \text{ wins a set}) = \frac{1}{3}$

To find $P(A$ wins the match in two or three sets) is made up of these three probabilities:

(i) $P(A \text{ wins}, A \text{ wins}) = \frac{2}{3}\times\frac{2}{3} = \frac{4}{9}$

or (ii) $P(A \text{ wins}, A \text{ loses}, A \text{ wins}) = \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3} = \frac{4}{27}$

or (iii) $P(A \text{ loses}, A \text{ wins}, A \text{ wins}) = \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{4}{27}$

$$\therefore \quad P(A \text{ wins the match}) = \frac{4}{9}+\frac{4}{27}+\frac{4}{27}$$
$$= \frac{12}{27}+\frac{4}{27}+\frac{4}{27} = \frac{20}{27}$$

Q2. $P(\text{team fully fit and win game}) = \frac{7}{10}\times\frac{9}{10} = \frac{63}{100}$

$P(\text{team not fully fit and win}) = \frac{3}{10}\times\frac{4}{10} = \frac{12}{100}$

$$\therefore \; P(\text{team wins next home game}) = \frac{63}{100}+\frac{12}{100}$$
$$= \frac{75}{100} = 0.75$$

Q3. $P(E) = \frac{1}{5}$ $\quad P(F) = \frac{1}{7}$

(i) Since events are independent,

$$\therefore \; P(E\cap F) = P(E)\times P(F)$$
$$= \frac{1}{5}\times\frac{1}{7}$$
$$= \frac{1}{35}$$

(ii) $P(E\cup F) = P(E)+P(F)-P(E\cap F)$

$$= \frac{1}{5}+\frac{1}{7}-\frac{1}{35}$$
$$= \frac{11}{35}$$

Q4. (i) $P(\text{student does not study Biology}) = \dfrac{21}{56}$

$$= \frac{3}{8}$$

(ii) Number of students who study at least 2 subjects = 26

$P(\text{student studying 2 subjects at least does not study Biology}) = \dfrac{4}{26}$

$$= \frac{2}{13}$$

(iii) There are 56 students in the class.

$$P(\text{both students picked randomly study Physics}) = \frac{\binom{28}{2}}{\binom{56}{2}} = \frac{378}{1540} = \frac{27}{110}$$

(iv) 25 students study Chemistry. $C \cap B = 13$ students studying both.

P(one of the two students picked studying Chemistry studies Biology) $= \dfrac{13}{25}$

P(Biology, not biology) **or** P(Not biology, biology)

i.e. $\left(\dfrac{13}{25}\times\dfrac{12}{24}+\dfrac{12}{25}\times\dfrac{13}{24}\right)$

$$= \frac{13}{25}$$

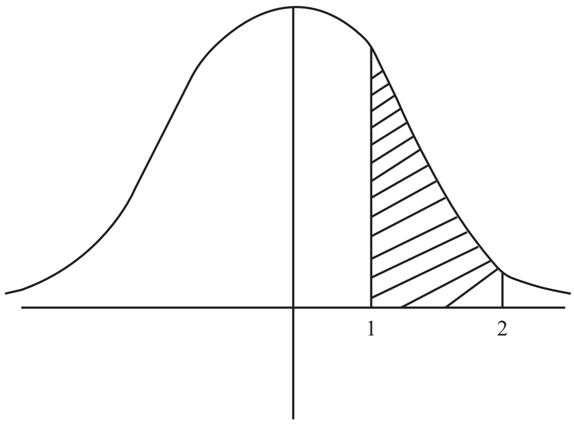
Q5. (i) $P(1 < z < 2)$

$P(z < 2)$ i.e. left side of 2 is 0.9772

$P(z > 1) = 0.8413$

$\therefore P(1 < z < 2) = 0.9772 - 0.8413$

$$= 0.1359$$



Q6. (i) $P(A \text{ qualifies for 5,000 m race}) = \dfrac{3}{5}$

$P(A \text{ qualifies for 10,000 m race}) = \dfrac{1}{4}$

$\therefore P(A \text{ qualifies for both races}) = \dfrac{3}{5}\times\dfrac{1}{4}$

$$= \frac{3}{20} = 0.15$$

(ii) P(exactly one of the athletes qualifies for 5,000 m)

$$= P(A \text{ qualifies and } B \text{ does not}) \text{ or } P(A \text{ does not qualify \& } B \text{ does})$$

$$= \left(\frac{3}{5} \times \frac{1}{3}\right) + \left(\frac{2}{3} \times \frac{2}{5}\right)$$

$$= \frac{3}{15} + \frac{4}{15}$$

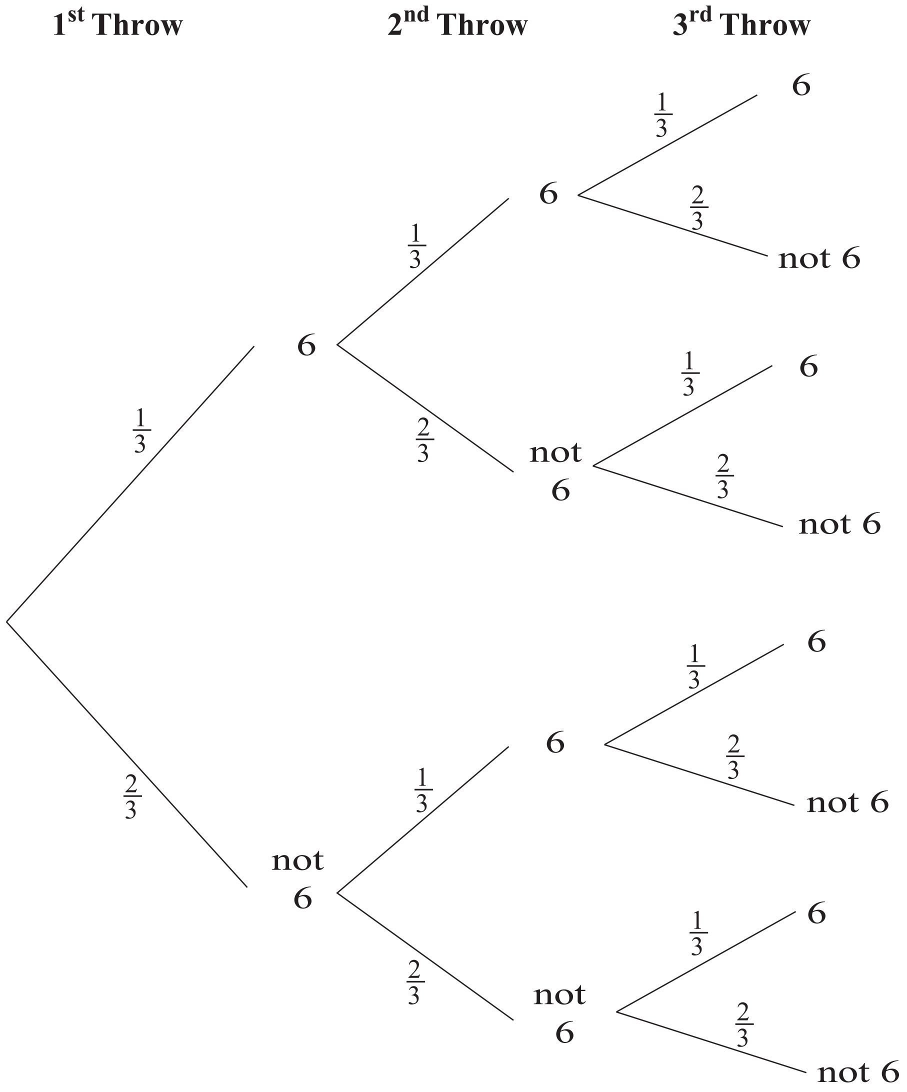
$$= \frac{7}{15}$$

(iii) $P(\text{athlete } A \text{ qualifies for 10,000 m}) = \dfrac{1}{4}$

$P(\text{athlete } B \text{ qualifies for 10,000 m}) = \dfrac{2}{5}$

$P(\text{both athletes qualify for 10,000 m race}) = \dfrac{1}{4} \cdot \dfrac{2}{5} = \dfrac{2}{20} = \dfrac{1}{10}$

Q7. (i)



$$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$$

$$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{27}$$

$$\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{27}$$

$$\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27}$$

$$\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{27}$$

$$\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{27}$$

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27}$$

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{6}{27}$$

At least 1 six in three throws means 1 six, or 2 sixes, or 3 sixes.

$$\therefore P(\text{at least one six in 3 throws}) = \frac{1}{27} + \frac{2}{27} + \frac{2}{27} + \frac{4}{27} + \frac{2}{27} + \frac{4}{27} + \frac{4}{27} = \frac{19}{27}$$

(ii) Given :

$$P(A)=\frac{2}{3} \qquad P(A\cup B)=\frac{3}{4} \qquad P(A\cap B)=\frac{5}{12}$$

To find $P(B)$:

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\frac{3}{4}=\frac{2}{3}+P(B)-\frac{5}{12}$$

$$\frac{3}{4}-\frac{2}{3}+\frac{5}{12}=P(B)$$

$$\frac{6}{12}=P(B)$$

$$\therefore P(B)=\frac{1}{2}$$

Q8. Expected value of payout

Payout (x)	Probability (P)	$x\times P$
€50	$\frac{1}{4}$	$12\frac{1}{2}$
€10	$\frac{1}{4}$	$2\frac{1}{2}$
€5	$\frac{1}{3}$	$1\frac{2}{3}$
€20	$\frac{1}{6}$	$3\frac{1}{3}$

$$\sum x.P(x)=12.5+2.5+1.6666+3.3333$$

$$=€20$$

$\therefore$ Expected value of the payout is €20.

But it costs €25 to spin the spinner, so you expect to lose €5.

This game is not fair since expected payout does not equal zero.

Q9. (i) $n=6 \qquad P(\text{six})=\frac{1}{6} \qquad P(\text{not }6)=\frac{5}{6}$

P(two sixes in first 6 rolls)

$$\therefore \;=\binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$$

$$=15\times\frac{1}{36}\times\frac{625}{1296}$$

$$=\frac{3125}{15{,}552}\;=0.2$$

(ii) P(second 6 on sixth roll) **and**

P(a six in the first 5 rolls)

$$= \binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4$$

$$= 5 \times \frac{1}{6} \times \frac{625}{1296}$$

$$= \frac{3125}{7776}$$

$$= 0.40187$$

$P(\text{a six on } 6^{\text{th}} \text{ roll}) = \frac{1}{6}$

$\therefore P$(a second 6 on the 6^{th} roll)

$$= 0.40187 \times \frac{1}{6}$$

$$= 0.0669$$

$$= 0.067$$

Q10. (i) Given : $P(E) = \frac{2}{3} \quad P(E \mid F) = \frac{2}{3} \quad P(F) = \frac{1}{4}$

To find $P(E \cap F)$:

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

$$\therefore \frac{2}{3} = \frac{P(E \cap F)}{\frac{1}{4}}$$

$$\therefore P(E \cap F) = \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{2}{12} = \frac{1}{6}$$

(ii) $$P(F \mid E) = \frac{P(F \cap E)}{P(E)}$$

$$P(F \mid E) = \frac{\frac{1}{6}}{\frac{2}{3}}$$

$$= \frac{1}{6} \cdot \frac{3}{2} = \frac{3}{12}$$

$$= \frac{1}{4}$$

Yes, E and F are independent events as $P(E \cap F) = P(E) \times P(F)$.

Test Yourself 3

C – Questions

Q1. (i) Possible paths are:

ABEH and ACEH

(ii) Paths from A:

ABDGL	ABDGM
ABDHM	ABDHN
ABEHM	ABEHN
ABEJN	ABEJP
ACEJN	ACEJP
ACFJP	ACFJN
ACEHN	ACFKQ
ACFKP	ACEHM

$P(\text{marble passes through } H \text{ or } J)$

$$= \frac{12}{16} = \frac{3}{4}$$

(iii) $P(\text{marble lands at } N)$

$$= \frac{6}{16} = \frac{3}{8}$$

(iv) $P(\text{two marbles from } A \text{ land at } P) = \dfrac{1}{16}$

Both go separately but there is only 1 way.

Q2. (i) $P(\text{success}) = 0.7, \quad P(\text{failure}) = 0.3$

$P(1^{\text{st}} \text{ goal on } 3^{\text{rd}} \text{ attempt}) =$

$P(\text{not goal}).P(\text{not goal}).P(\text{goal})$

$$= 0.3 \cdot 0.3 \cdot 0.7$$

$$= 0.063$$

(ii) $P(\text{score exactly 3 goals in 5 attempts})$

$$= \binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$$

$$= 1\cancel{0} \cdot \frac{343}{1,000} \cdot \frac{9}{10\cancel{0}} = \frac{3087}{10000}$$

$$= 0.3087$$

$$= 0.309$$

(iii) P(two goals in six attempts)

$$=\binom{6}{2}\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^4$$

$$=15\times\frac{49}{100}\times\frac{81}{10,000}$$

$$=0.059$$

P(a goal on seventh attempt)

$$=\frac{7}{10}$$

$\therefore P$(third goal on seventh attempt)

$$=0.059\times\frac{7}{10}$$

$$=0.0416$$

$$=0.042$$

Q3. (a) Given $P(A)=\frac{13}{25},\quad P(B)=\frac{9}{25},\quad P(A\mid B)=\frac{5}{9}$

(i) To find $P(A \text{ and } B)$, i.e. $P(A\cap B)$;

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}$$

$$\frac{5}{9}=\frac{P(A\cap B)}{\frac{9}{25}}$$

$$\therefore P(A\cap B)=\frac{\cancel{5}^{1}}{9}\times\frac{9}{\cancel{25}_{5}}=\frac{9}{45}$$

$$=\frac{1}{5}$$

(ii) $$P(B\mid A)=\frac{P(B\cap A)}{P(A)}$$

$$=\frac{\frac{1}{5}}{\frac{13}{25}}$$

$$\therefore \frac{1}{\cancel{5}_{1}}\times\frac{\cancel{25}^{5}}{13}=\frac{5}{13}$$

(iii) $$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{13}{25}+\frac{9}{25}-\frac{1}{5}$$

$$=\frac{17}{25}$$

(b) $P(6) = p$

$P(1) = P(2) = P(3) = P(4) = P(5) = \frac{1-p}{5}$

With a fair dice, all throws $1-6$ have a probability of $\frac{1}{6}$.

Number of possible outcomes with 2 dice = 36.

Scores totalling 7 are $(3,4), (4,3), (5,2), (2,5), (6,1), (1,6)$; all independent of p.

$$\therefore P(\text{rolling a total of 7}) = \frac{6}{36} = \frac{1}{6}$$

Q4. (i) $x = 60, \quad \mu = 48, \quad \sigma = 8$

$$z\text{-score} = \frac{60-48}{8} = \frac{12}{8} = 1\frac{1}{2}$$

$$\therefore P(z > 1.5) = 1 - 0.9332 = 0.0668$$

(ii) $$z\text{-score} = \frac{35-48}{8} = \frac{-13}{8} = -1.625$$

$$\therefore P(z < -1.625) = 1 - 0.9484 = 0.0516 = 0.052$$

Q5. (i) Bag has 4 red, 6 green counters.

4 counters drawn at random.

$P(\text{all counters drawn are green})$

$$= \frac{\binom{6}{4}}{\binom{10}{4}} = \frac{15}{210} = \frac{1}{14}$$

(ii) $P(\text{at least one counter of each colour is drawn})$

$\therefore P(1R, 3G)$ or $P(2R, 2G)$ or $P(3R, 1G)$

$$\therefore \frac{\binom{4}{1}\binom{6}{3}}{\binom{10}{4}} + \frac{\binom{4}{2}\binom{6}{2}}{\binom{10}{4}} + \frac{\binom{4}{3}\binom{6}{1}}{\binom{10}{4}}$$

$$\therefore \frac{4 \times 20}{210} + \frac{6 \times 15}{210} + \frac{(4)(6)}{210} = \frac{194}{210} = \frac{97}{105}$$

$$\therefore P(\text{one at least of each colour is drawn}) = \frac{97}{105}$$

(iii) $P(\text{at least 2 green counters drawn})$

$\therefore P(2R, 2G) + P(1R, 3G) + P(\text{all 4 green})$

$$= \frac{\binom{4}{2}\binom{6}{2}}{210} + \frac{\binom{4}{1}\binom{6}{3}}{210} + \frac{\binom{6}{4}}{210}$$

$$= \frac{90}{210} + \frac{80}{210} + \frac{15}{210}$$

$$= \frac{185}{210}$$

$$= \frac{37}{42}$$

(iv) $P(\text{at least } 2G \text{ drawn given that at least one of each colour is drawn})$

Choices are:

$1R, 3G$ or $2R, 2G$

$$P = \frac{\binom{4}{1}\binom{6}{3}}{210} + \frac{\binom{4}{2}\binom{6}{2}}{210}$$

$$= \frac{4.20}{210} + \frac{6.15}{210}$$

$$= \frac{80+90}{210} = \frac{170}{210}$$

$$= \frac{17}{21}$$

The two events are not independent since the answers in (iii) and (iv) are different.

Q6. (i) $P(-k \le z \le k) = 0.8438$

Since this is a normal distribution, and because of symmetry,

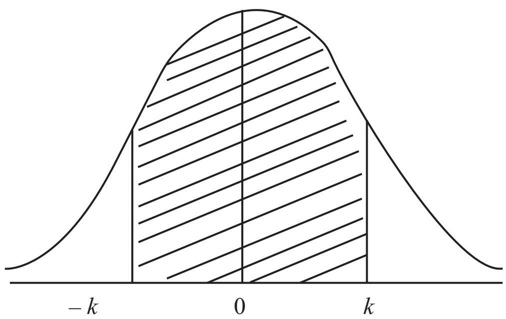
$$P(0 < z \le k) = \frac{1}{2}(0.8438)$$

$$= 0.4219$$

$\therefore P(-k \le z \le k) = 0.5 + 0.4219$

$= 0.9419$ (formulae & tables p 36 & 37)

$\therefore z = 1.42$



(ii) (a) Given $P(X)=\frac{2}{3}, \quad P(X|Y)=\frac{2}{3}, \quad P(Y)=\frac{1}{4}$

$P(X\cap Y)$ is found by using

$$P(X|Y)=\frac{P(X\cap Y)}{P(Y)}$$

$$\therefore \frac{2}{3}=\frac{P(X\cap Y)}{\frac{1}{4}}$$

$$\therefore P(X\cap Y)=\frac{2}{3}\times\frac{1}{4}$$

$$=\frac{2}{12} \quad =\frac{1}{6}$$

(b) $$P(Y|X)=\frac{P(Y\cap X)}{P(X)}$$

$$=\frac{\frac{1}{6}}{\frac{2}{3}} \quad =\frac{1}{6}\times\frac{3}{2}$$

$$=\frac{3}{12} \quad =\frac{1}{4}$$

$$\therefore P(Y|X)=\frac{1}{4}$$

Q7. (i)(a) Since $\sum$ probabilities $=1$

$\therefore \quad 0.1+a+b+0.2+0.1=1$

$\therefore \quad a+b=0.6$

(ii) $\sum x.P(x)=2.9$

$\therefore 0.1+2a+3b+0.8+0.5=2.9$

$\therefore 2a+3b=2.9-1.4$

$\therefore 2a+3b=1.5$

Solve:

$$\begin{array}{ll} a+b=0.6 & \cancel{2a}+3b=1.5 \\ \underline{2a+3b=1.5} & \underline{\cancel{2a}+2b=1.2} \quad \text{(subtract)} \\ & b=0.3 \end{array}$$

$a+b=0.6$

$a+0.3=0.6$

$\therefore \quad a=0.3$

$\therefore \quad a=0.3, \quad b=0.3$

(b) 16 girls 8 boys

12 study french

let girl studying french = x

let boy studying french = y

$\therefore x+y=12$ (i)

$P(\text{girl study } F)=\dfrac{x}{16}$ $P(\text{boy study } F)=\dfrac{y}{8}$

$\therefore \dfrac{x}{16}=\dfrac{3}{2}\left(\dfrac{y}{8}\right)$ (ii)

$x+y=12$, so $\therefore x=12-y$

$\therefore \dfrac{12-y}{16}=\dfrac{3y}{16}$ so $\therefore$ $12-y=3y$

$\therefore 4y=12$ $\therefore y=3$ (boy)

Hence, $x=12-3=9$ (girl)

$\therefore$ 3 boys and 9 girls study french.

Q8. (i) The spinner since scores are added.

(ii) Ann: Dice

Outcome (x)	1	2	3	4	5	6
Probability (P)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$x \times P(x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1

$\therefore \sum x.P(x)=3.5$

Jane: Spinners

Outcome (x)	1	2	3
Probability (P)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$x \times P(x)$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$2[x.P(x)]$	$\frac{2}{3}$	$\frac{4}{3}$	2

$\therefore \sum x.P(x)=4$

Spinners have a better chance of reaching 20 points first as expected outcome is 4, whereas for the dice it is 3.5.

Q9. (i) $P(H)=\frac{1}{2}$ $\qquad P(T)=\frac{1}{2}$

$$P(3H, 2\text{ tails})=\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=10\times\frac{1}{4}\times\frac{1}{8}$$

$$=10\times\frac{1}{32}=\frac{10}{32}=\frac{5}{16}$$

i.e. 16 outcomes with 5 showing 3H's, 2 tails.

Note: You can fully write out the outcomes also.

(ii) If the 5 coins are tossed 8 times:

$\therefore$ probability $(3H, 2T)=\frac{5}{16}$

$\therefore P(\text{not getting } 3H, 2T)=\frac{11}{16}$

$\therefore P(\text{getting } 3H, 2T \text{ exactly 4 times})$

$$=\binom{8}{4}\left(\frac{5}{16}\right)^4\left(\frac{11}{16}\right)^4$$

$$=70\cdot\frac{625\times 14{,}641}{4{,}294{,}967{,}296}$$

$=0.1491$

$=0.149$

Q10. (i) The mean, median and mode of a normal distribution are all the same.

(ii) A normal distribution is smooth and bell-shaped, it is symmetrical, and the empirical rule applies.

(iii) (a) $\mu=12{,}000 \qquad \sigma=300$

$P(\text{bulb will last less than 11,400 hrs})$

$$= z\text{-score of } \frac{11{,}400-12{,}000}{300}$$

$$=\frac{-600}{300}=-2$$

$\therefore P(z<-2)=1-P(z>2)$

$=1-0.9772$

$=0.0228$

(b) $P(\text{bulb last between 11,400 and 12,600 hrs})$

$$z\text{-score} \quad \frac{12,600-12,000}{300}=\frac{600}{300}$$
$$=2$$

$$P(-2<z<2)=0.9772-(1-0.9772)$$
$$=0.9772-0.0228$$
$$=0.9544$$

$P(\text{bulbs lasting longer than 12, 600 hrs})$

$$=P(z>2)=0.0228$$

When 5,000 are tested, then

$$5,000\times 0.0228$$

bulbs last longer than 12,600 hrs

$$=114 \text{ bulbs}$$

Q11. Given $10\,\boxed{R}$, $15\,\boxed{G}$, $8\,\triangle R$, $12\,\triangle G$

E = event $\boxed{}$ is drawn.

F = event that green shape is drawn.

$$\therefore P(E)=\frac{25}{45}$$

$$\therefore P(F)=\frac{27}{45}$$

(i) $P(E\cap F)=P(\text{a square that is green})$

$$=\frac{15}{45}=\frac{1}{3}$$

(ii) $P(E\cup F)=P(\text{square drawn } or \text{ a green shape drawn})$

$$=\frac{10+15+12}{45}=\frac{37}{45}$$

(iii) Yes, events E and F are independent as $P(E\cap F)=P(E)\times P(F)$.

$P(E\cap F)=\frac{1}{3}$ and $P(E)\times P(F)$

$$=\frac{25}{45}\times\frac{27}{45}$$
$$=\frac{675}{2025}=\frac{1}{3}$$

(iv) No, E and F are not mutually exclusive events as

$$P(E\cup F)\neq P(E)+P(F)\ \left(\text{i.e. } \frac{37}{45}\neq\frac{25}{45}+\frac{27}{45}\right)$$

Chapter 4: Statistics 2

Exercise 4.1

Q1. (i) Since y increases as x increases graphs C and E show positive correlation.

(ii) Since y decreases as x increases graphs A and F show negative correlation.

(iii) In graphs B and D, the variables x and y show no linear pattern so we say there is no correlation.

(iv) Graph A shows a strong negative correlation, as the variables are in a straight line.

(v) Graph F can be described as reasonally strong negative correlation.

Q2. (i) Graph B shows the strongest positive correlation with y increasing as x increases.

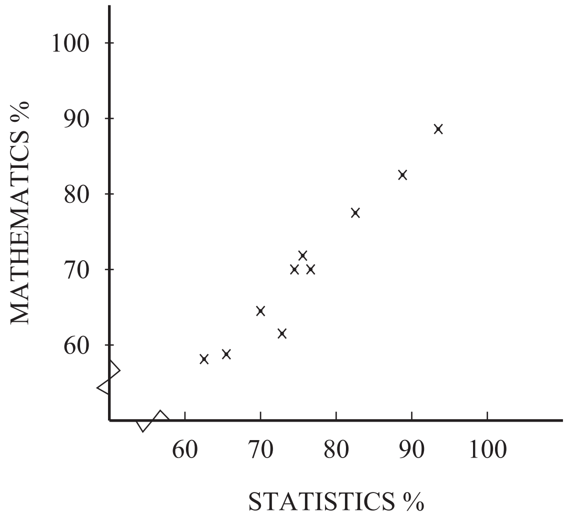
(ii) In graph C the variables x and y have a negative correlation with y decreasing as x increases.

(iii) The weakest correlation is shown in graph D as the points are more widely spread out.

Q3. (i) The correlation can be described as **strongly** positive.

(ii) The better grade a student gets in her mock exams, the better he/she tends to do in the final exam.

Q4. (i)



(ii) A strong positive correlation.

(iii) There is a tendency for those who do better at statistics to also do better at mathematics.

Q5. (i) Negative: The older the boat, it is likely its second-hand selling price decreases.

(ii) Positive: Generally, as children age they grow taller.

(iii) None.

(iv) Negative: The more time spent watching TV means there is less time for studying.

(v) Positive: There is a greater likelihood of accidents when there are higher numbers of vehicles travelling on a route.

Q6. (i) B: As boys get taller they generally require larger shoe sizes as their feet also increase in size.

(ii) C: There is no relationship between mens weight and time taken to complete a crossword puzzle.

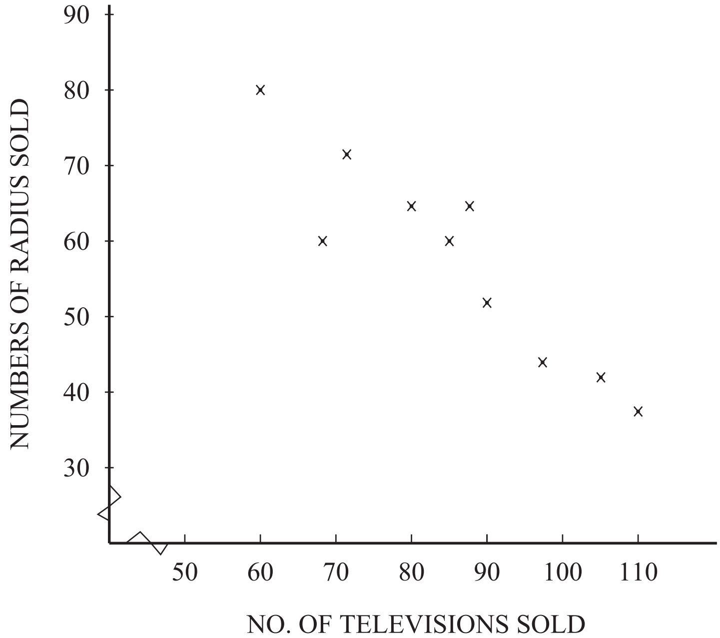
(iii) A: As cars age, the selling price is reduced.

(iv) D: Students generally get similar grades in maths paper 1 and paper 2. There is a positive correlation.

Q7. (i) Reasonably strong negative correlation.

(ii) Yes, as the age of the bike increases, it causes the price to decrease.

Q8. (i)



(ii) A strong negative.

(iii) No, there is not a causal relationship. An increase in sales of one does not cause a decrease in sales of the other.

Exercise 4.2

Q1. $A = 0.6$

$B = -1$

$C = -0.4$

$D = 0.8$

Q2. (i) 0.9 is strong positive correlation.

(ii) – 0.8 is strong negative correlation.

(iii) 0 is no correlation.

(iv) – 1 is perfect negative correlation.

(v) – 0.1 is a very weak negative correlation.

(vi) 0.2 is a very weak positive correlation.

Q3. (i) Line of best fit.

(ii) Approximately an equal number of points lie on either side of the line.

(iii) Draw a line from the height (cm) axis at 150 cm to cut the line of best fit and read the answer on the weight (kg) axis

solution: 55 kg.

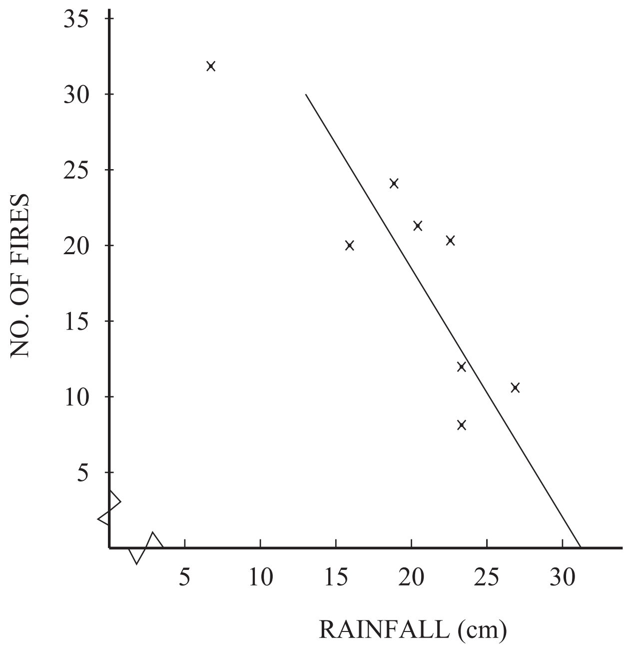
(iv) Strong positive.

Q4. Solution: 0.86

Use your calculator methods (Appendix 1 p.178)

Q5. 0.86

Q6. (i)



(ii) Line of best fit

(iii) $r = -0.9$

(iv) From the graph, points (27.5, 8.0) (24, 12) are 2 points on the line of best fit.

The equation of the line of best fit is of the form

$y = mx + c$ or in this case

$y = a + bx$

slope of the line of best fit

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{12 - 8}{24 - 27.5}$$

$$= \frac{4}{-3.5} = -1.14$$

Equation of the line of best fit.

$$y - y_1 = m(x - x_1)$$

$$y - 12 = -1.14(x - 24)$$

$$y - 12 = -1.14(x) + 27.36$$

$$y = 39.36 - 1.1x$$

$$\therefore y = 39 - 1.1x$$

The equation of the line of best fit can be worked out using a calculator. Using this method, the solution was found to be

$$y = 41 - 1.1x$$

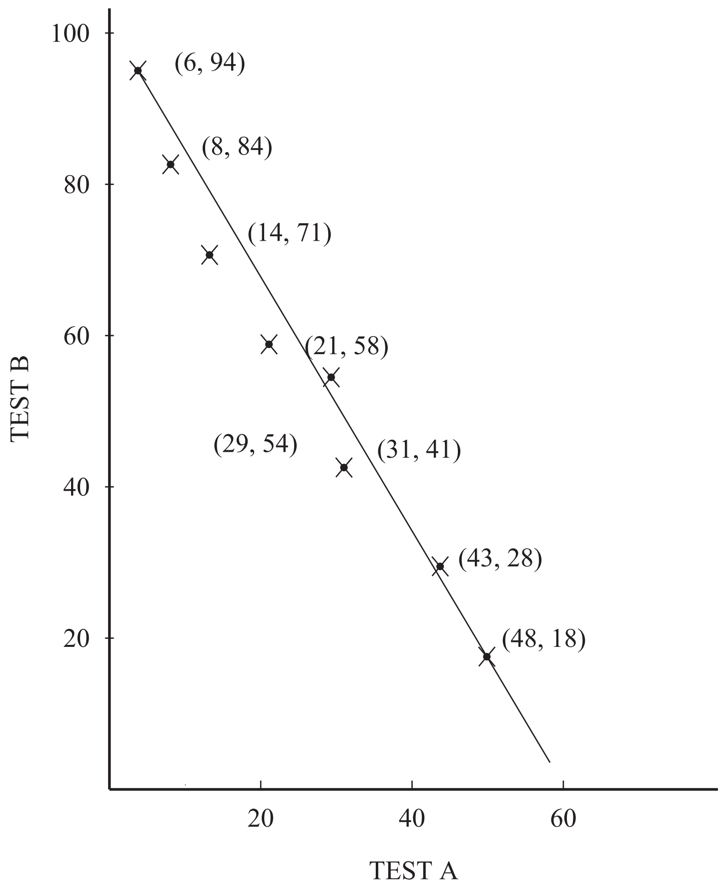
(v) Substitute $y = 25$ in the equation

$$25 = 41 - 1.1x$$

$$\therefore 25 - 41 = -1.1x \quad \Rightarrow \quad x = 14.5$$

$\therefore$ Approximately 15 fires

Q7. (i)



(ii) Strong negative correlation

(iii) Using two points on the line of best fit the slope is found using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point (48, 18) and (6, 94)

$$\therefore m = \frac{94-18}{6-48} = -1.8$$

Equation of line $y - 18 = -1.8(x - 48)$

$$\therefore y = 104 - 1.8x$$

Using a calculator, the exact equation is

$$y = -1.7x + 98$$

(v) Using the graph, draw from score 18 on Test A to the line of best fit on the diagram and read off the solution on Test B axis.

$\therefore$ The student scored approx 68.

Alternatively: substitute $x = 18$

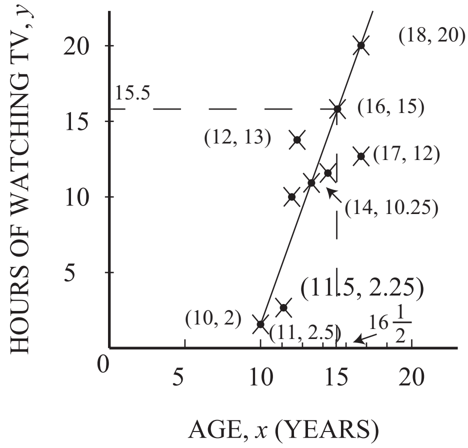
in the linear equation

$$y = -1.7(18) + 98$$

$$\therefore y = 67.4$$

Q8. Calculator value : r = 0.85

Q9.



(iii) Equation of the line of best fit

$$y = 1.9x - 16 \quad \text{(calculator)}$$

(v) Using the graph, draw a line from $16\frac{1}{2}$ on age axis and read where this cuts the line of best fit off the y axis.

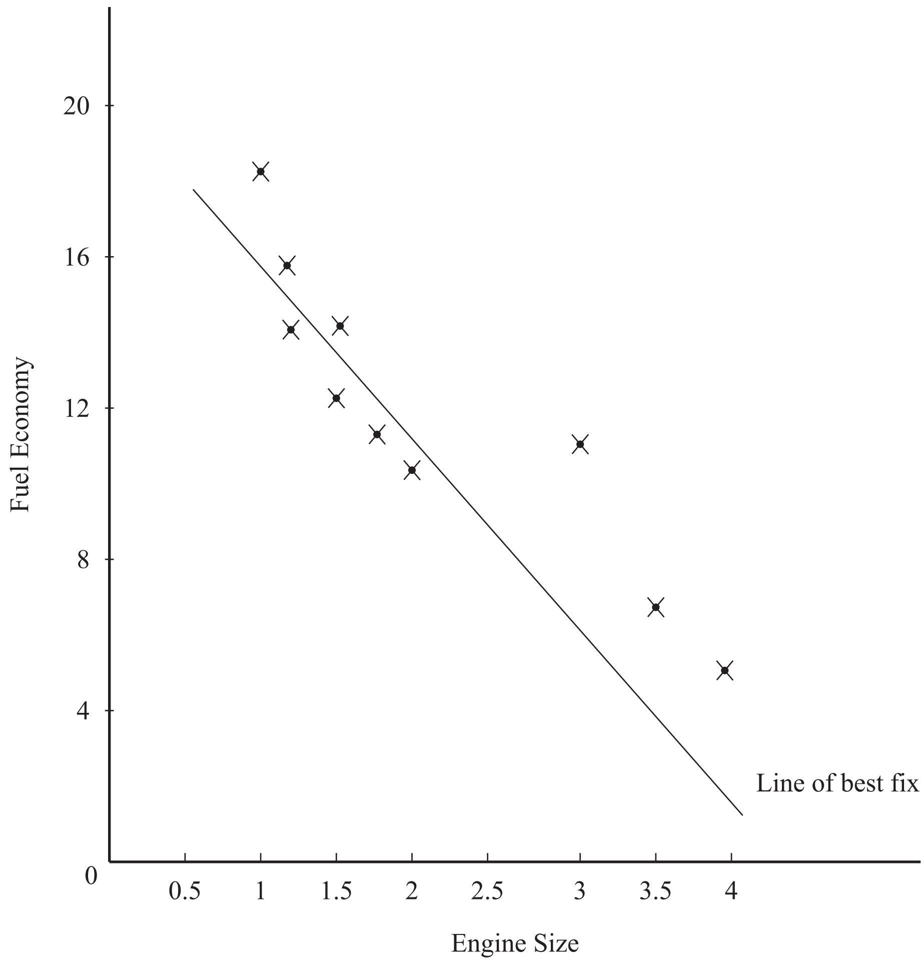
Solution 15.5 approximately.

Alternatively: substituting $x = 16\frac{1}{2}$ in the equation of the line of best fit $y = 1.9x - 16$ gives

$$y = 1.9(16.5) - 16$$
$$= 15.35 \text{ hours}$$

Solution $= 15$ hours (approx)

Q10.



(ii) Strong negative correlation

(iii) $r = -0.9250$ (calculator)

(iv) Line of best fit

$$y = -3x + 18 \quad \text{(calculator)}$$

(Line of best fit is shown in diagram for part (i))

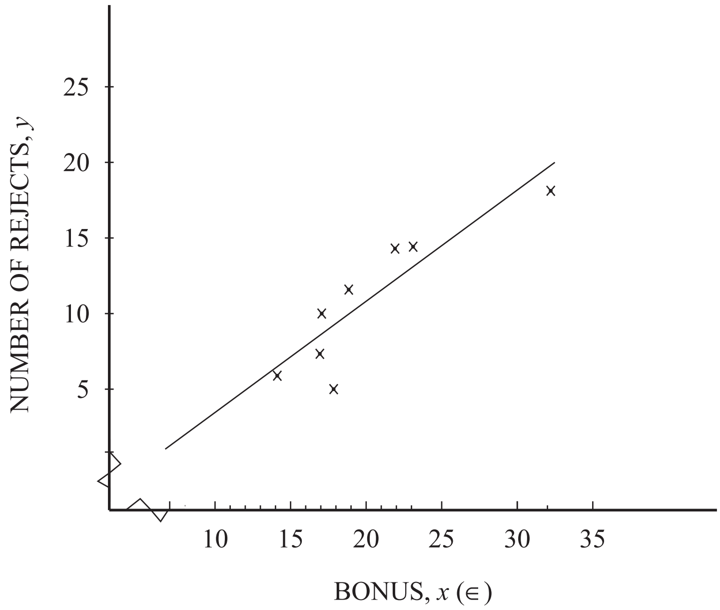
(v) Solution can be read from the graph showing the line of best fit or by substituting into the equation of the line of best fit.

Substitute engine size 5.7 litres

$$\begin{aligned} y &= -3(5.7) + 18 \\ &= -17.1 + 18 \\ &= 0.9 \end{aligned}$$

This result shows the fuel economy of value less than 1, so this may not be reliable.

Q11.



(ii) Fairly strong positive correlation

(iii) $r = 0.8591$ (calculator)

(iv) Using (12, 5) and (30, 16) from the line of best fit the slope $m = \dfrac{16-5}{30-12} = 0.61$

Eq. line is $y - 5 = 0.61(x - 12)$

$\therefore \quad y = 0.61 - 2.3$

Alternatively:

$$y = 0.63x - 2.2 \quad \text{(calculator)}$$

(v) $y = 0.63x - 2.2$

$9 + 2.2 = 0.63x$

$11.2 = 0.63x$

$17.8 = x$

$\therefore$ max bonus should be set at €18 approx.

Exercise 4.3

Q1. (i) The percentage of all the values in the shaded area is 68%, as it is a characteristic of a normal distribution that 68% lie within one standard deviation of the mean.

(ii) Again, according to the Empirical Rule, 95% of values lie within two standard deviations of the mean.

(iii) Values between $-\sigma$ and $0 = \frac{1}{2}(68\%)$

Values between 0 and $2\sigma = \frac{1}{2}(95\%)$

$$\therefore 34\% + 47\frac{1}{2}\% = 81\frac{1}{2}\%$$

$\therefore 81\frac{1}{2}\%$ of values lie between $-\sigma$ and 2σ

(iv) $\mu = 60, \quad \sigma = 4$

$56 = 60 - 4 = \mu - \sigma$

$64 = 60 + 4 = \mu + \sigma$

There are 68% of all values in the range $[\mu - \sigma$ and $\mu + \sigma]$

$\therefore$ 68% of values lie between 56 and 64

Q2. $\mu = 72, \quad \sigma = 6$

$60 = 72 - 12 = \mu - 2\sigma$

$78 = 72 + 6 = \mu + \sigma$

(i) There are $\frac{1}{2}$ (68%) of values in the range 72 to 78

$\therefore$ 34% of teenagers are that height.

(ii) The percentage of teenagers taller than 78 cm is

$$50\% - 34\%$$
$$= 16\%$$

(iii) $\mu = 72, \quad \sigma = 6$

$60 = 72 - 12 = \mu - 2\sigma = \frac{1}{2}(95\%)$

$78 = 72 + 6 = \mu + \sigma = \frac{1}{2}(68\%)$

$\therefore 47\frac{1}{2}\% + 34\%$

$= 81\frac{1}{2}\%$

$\therefore 81\frac{1}{2}\%$ of teenagers are between 60 cm and 78 cm in height

Q3. (i) $\mu = 55$ km/h, $\sigma = 9$ km/h

given z-score $= -1$

$$\frac{x-55}{9} = -1$$

$$x - 55 = -9$$

$$x = 55 - 9$$

$$= 46 \text{ km/h}$$

(ii) Two standard deviations above the mean

$\Rightarrow$ z-score $= 2$

$\therefore \quad \dfrac{x-55}{9} = 2$

$\therefore \quad x - 55 = 18$

$\therefore \quad x = 55 + 18$

$= 73$ km/h

(iii) Three standard deviations above the mean

$\Rightarrow \quad \dfrac{x-55}{9} = 3$

$\therefore \quad x - 55 = 27$

$x = 55 + 27$

$= 82$ km/h

Q4. $\mu = 60$ $\sigma = 5$

Using $z\text{-score} = \dfrac{x-\mu}{\sigma}$

$\pm 1 = \dfrac{x-60}{5}$

$\therefore \quad x - 60 = \pm 5(1)$

$\therefore \quad x - 60 = 5$ or $x - 60 = -5$

$\Rightarrow \quad x = 65$ $\Rightarrow \quad x = 60 - 5$

$= 55$

Hence, the range within which 68% of the distribution lies is

$55 < x < 65$

(ii) 95% will lie between $\pm 2\sigma$ of the mean

$\pm 2 = \dfrac{x-60}{5}$

$\therefore \quad x - 60 = \pm 10$

$\therefore \quad x - 60 = 10$ or $x - 60 = -10$

$\therefore \quad x = 70$ or $\therefore \quad x = 50$

$\therefore$ the range within which 95% of the distribution lies is

$50 < x < 70$

Q5. (i) 68% of the sample will lie between $\pm 1\sigma$ of the mean

$\mu = 170, \qquad \sigma = 8$

$$z\text{-score} = \frac{x-\mu}{\sigma}$$

$$= \frac{x-170}{8} = \pm 1$$

$\therefore \quad x - 170 = \pm 8$

$\therefore \quad x = 170 + 8 \qquad \text{or} \qquad x = 170 - 8$

$\qquad = 178 \qquad \text{or} \qquad x = 162$

$\therefore$ the limits within which 68% of the heights lie are $[162, 178]$ cm.

(ii) 99.7% of the sample will lie between $\pm 3\sigma$ of the mean

Using z-score

$$\frac{x-170}{8} = \pm 3$$

$\therefore \quad x - 170 = \pm 24$

$\therefore \quad x - 170 = 24 \qquad \text{or} \qquad x - 170 = -24$

$\therefore \quad x = 170 + 24 \qquad\qquad x = 170 - 24$

$\therefore \quad x = 194 \qquad\qquad x = 146$

$\therefore$ 99.7% of the heights lie

within the limits $[146, 194]$ cm

Q6. (i) $35 - 23 = 12 \;\Rightarrow\; 2\sigma$ below the mean

$47 - 35 = 12 \;\Rightarrow\; 2\sigma$ above the mean

There are 95% of all values in the

range $[\mu - 2\sigma$ and $\mu + 2\sigma]$

$\therefore$ 95% of all workers take

23 to 47 minutes to get to work.

(ii) Since approximately 47.5% of time values

lie within μ plus two standard deviations of the mean

$\therefore 50\% - 47\frac{1}{2}\%$

$\Rightarrow$ approx 2.5% lie above 47 minutes

$\therefore$ Approx 2.5% of workers take more than 47 minutes

to get to work.

(iii) 95% take 23 to 47 minutes to get to work

With 600 workers:

$\therefore 600 \times 95$

$= 570$ workers

Q7. (i) 68% of bulbs tested lie within $\pm 1\sigma$ of the mean

$\Rightarrow$ 68% of $12{,}000 = 8{,}160$ bulbs

$\therefore$ the lifetime of 8,160 bulbs lie within one standard deviation of the mean.

(ii) $\mu = 620$ hrs $\quad \sigma = 12$ hours

$644 = \mu + 2\sigma$

$\therefore$ $47\frac{1}{2}\%$ of bulbs tested would lie in this range 620 to 644.

$\therefore$ $12{,}000 \times 47\frac{1}{2}\%$

$= 5{,}700$ bulbs

(iii) $50\% - 47\frac{1}{2}\% = 2.5\%$ lie more than two standard deviations above the mean

$2\frac{1}{2}\%$ of $12{,}000 = 300$ bulbs

Q8. $\mu = 134$ cm $\quad \sigma = 3$ cm

Balls with rebound less than 128 cm rejected

The range 128 cm to 134 cm

is $134 - 2\sigma$ i.e. $\mu - 2\sigma$

$\therefore \frac{1}{2}(95\%)$ of balls lie in this range and are accepted

$\therefore 47\frac{1}{2}\%$ are accepted

$\therefore 50\% - 47\frac{1}{2}\% = 2\frac{1}{2}\%$ of balls are rejected

$2\frac{1}{2}\%$ of $1000 = 25$ balls

Q9. (i) The range 140 g to 180 g is

(a) $160 \pm 2\sigma$

$\therefore$ 95% of the portions have weights between 140 g and 180 g

(b) The range 130 g to 190 g is

$160 \pm 3\sigma$

$\therefore$ 99.7% of the weights lie in this range

(ii) The number of portions expected to weigh between 140 g and 190 g is

$160-2\sigma$ to $160+3\sigma$

$\therefore\ 47\frac{1}{2}\%$ + 49.75%

$$=97\frac{1}{4}\%$$

Of a box with 100 portions approx 97 are expected to be of this weight

Q10. (i) $x=84,\quad \mu=80,\quad \sigma=4$

$z\text{-score}=\dfrac{x-\mu}{\sigma}$

$$=\frac{84-80}{4}=1$$

(ii) $x=72,\quad \mu=80,\quad \sigma=4$

$z\text{-score}=\dfrac{72-80}{4}=-2$

(iii) $x=86,\quad \mu=80,\quad \sigma=4$

$z\text{-score}=\dfrac{86-80}{4}=1.5$

(iv) $x=70,\quad \mu=80,\quad \sigma=4$

$z\text{-score}=\dfrac{70-80}{4}=-2.5$

Q11. (i) A z-score of 2 means a value which lies 2 standard deviations above the mean.

(ii) A z-score of -1.5 means a value which lies $1\frac{1}{2}$ standard deviations below the mean.

Q12. (i) Karl's mark is 1.8 standard deviations above the mean which was 70 marks.
Tanya's mark is 0.6 standard deviations below that same mean of 70 marks.

(ii) Karl's z-score $= 1.8$, his mark, x,

$\mu = 70$ marks $\quad \sigma = 15$ marks

Using $z = \dfrac{x-\mu}{\sigma}$

$$1.8 = \frac{x-70}{15}$$

$\therefore x - 70 = 15(1.8)$

$\therefore \quad x = 70 + 27$

$= 97$ marks

Tanya's z-score is -0.6, her mark

is x, $\mu = 70$ marks, $\sigma = 15$ marks

$$\therefore \quad -0.6 = \frac{x-70}{15}$$

$\therefore x - 70 = 15(-0.6)$

$\therefore x = 70 - 9$

$= 61$ marks

Q13. Weight:

$x = 48$ kg, $\quad \mu = 44$ kg, $\quad \sigma = 8$ kg

Use z-score formula $z = \dfrac{x-\mu}{\sigma}$

$$\therefore z = \frac{48-44}{8} = 0.5$$

Height: $x = 160$ cm, $\quad \mu = 175$ cm, $\quad \sigma = 10$ cm

$$z = \frac{160-175}{10} = \frac{-15}{10} = -1.5$$

Q14. Anna's score for **Maths**

Mark$(x) = 80$, $\quad \mu = 75$ mark, $\quad \sigma = 12$ mark

$$z\text{-score} = \frac{80-75}{12} = \frac{5}{12} = 0.417$$

$\therefore$ maths z-score $= 0.417$

Anna's score for **History**

mark $= 70$, $\quad \mu = 78$, $\quad \sigma = 10$

$$z\text{-score} = \frac{70-78}{10} = \frac{-8}{10} = -0.8$$

$\therefore$ Anna's history z-score $= -0.8$

(ii) Anna performed best in maths as she is found to have a higher z-score in the subject.

(iii) Ciara's history z-score $= 0.5$

$$\therefore \quad 0.5 = \frac{x-78}{10}$$

$\therefore \quad x - 78 = 10(0.5)$

$\therefore \quad x = 78 + 5$

$\quad = 83$

$\therefore$ Ciara got 83 marks in history

Q15. (i) A z-score of 1.8 in a maths test means that Sarah-Jane's mark was 1.8 standard deviations above the mean.

(ii) $x = 80, \quad \sigma = 12, \quad$ find μ

Using z-score

$$1.8 = \frac{80-\mu}{12}$$

$\therefore \quad 80 - \mu = 12(1.8)$

$\therefore \quad 80 - \mu = 21.6$

$\therefore \quad -\mu = -80 + 21.6$

$\quad -\mu = -58.4$

$\therefore \quad \mu = 58.4 =$ mean

(iii) Senan scores 50 in the same test

i.e. $x = 50, \quad \mu = 58.4, \quad \sigma = 12$

$$z\text{-score} = \frac{50-58.4}{12} = -0.7$$

$\therefore$ Senan's z-score $= -0.7$

Q16. Paper 1:

(i) Sarah's French mark $= 59$, $\mu = 45$, $\sigma = 8$

$$z\text{-score} = \frac{59-45}{8} = \frac{14}{8} = 1.75$$

(ii) To do equally well on Paper 2,

Sarah would need a z-score of 1.75

Paper 2:

marks $= x, \quad \mu = 56, \quad \sigma = 12$

z-score $= 1.75$

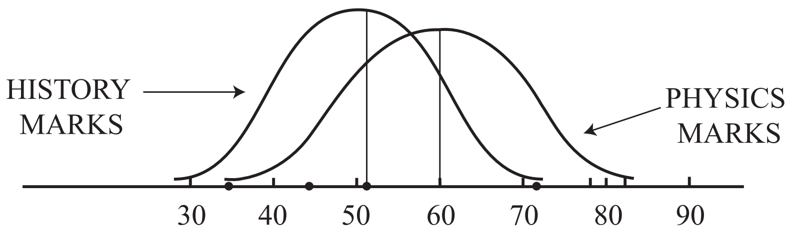
$$\therefore \quad 1.75 = \frac{x-56}{12}$$

$\therefore x - 56 = 12(1.75)$

$\therefore x = 56 + 21$

$\therefore x = 77$ marks

Q17. (i)



HISTORY : 34 – 70
PHYSICS : 36 – 84

(ii) Kelly: History

$x = 64, \qquad \mu = 42, \qquad \sigma = 6$

$z\text{-score} = \dfrac{64-52}{6} = \dfrac{12}{6} = 2$

Kelly: Physics

$x = 72, \qquad \mu = 60, \qquad \sigma = 8$

$z\text{-score} = \dfrac{72-60}{8} = \dfrac{12}{8} = 1.5$

So yes, Kelly did better in history so her claim to be better at history is supported.

Q18. Beach 1: $\mu = 8$ mm, $\sigma = 1.4$ mm

z-score when $x = 10$ mm long

$z = \dfrac{10-8}{1.4} = \dfrac{2}{1.4} = 1.428$

$\therefore z\text{-score} = 1.43$

Beach 2: $\mu = 9$ mm, $\sigma = 0.8$ mm

z-score when $x = 10$ mm long

$z = \dfrac{10-9}{0.8} = \dfrac{1}{0.8} = 1.25$

$\therefore z\text{-score} = 1.25$

$\therefore$ it can be concluded that Alison's claim is correct

Exercise 4.4

Q1. (i) The sample proportion, $\hat{p} = \frac{150}{500} = 0.3$

(ii) Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{500}}$

$= 0.04$

(iii) Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$\therefore \; 0.3 - 0.04 < p < 0.3 + 0.04$

$\therefore \; 0.26 < p < 0.34$

Q2. (i) The sample proportion, $\hat{p} = \frac{136}{\sqrt{400}}$

$= 0.34 = 34\%$

$\therefore$ 34% of computer shops are selling below the list price

(ii) Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{400}} = \frac{1}{20} = 0.05$

Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$= 0.34 - 0.05 < p < 0.34 + 0.05$

$= 0.29 < p < 0.39$

This means that the interval obtained works
for 95% of the time and would give this result.

Q3. The sample proportion, $\hat{p} = \frac{36,000}{10,000} = 0.36$

Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{10,000}} = \frac{1}{100} = 0.01$

95%, confidence interval $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$= 0.36 - 0.01 < p < 0.36 + 0.01$

$= 0.35 < p < 0.37$

Q4. The sample proportion, $\hat{p} = \frac{45}{150} = 0.3$

Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{150}} = 0.082$

Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$= 0.3 - 0.082 < p < 0.3 + 0.082$

$= 0.218 < p < 0.382$

Q5. Sample proportion, $\hat{p} = \frac{57}{80} = 0.713$

$\therefore$ Sample proportion *not* in favour $= 1 - 0.713 = 0.287$

Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{80}} = 0.111$

Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$= 0.287 - 0.111 < p < 0.287 + 0.111$$
$$= 0.176 < p < 0.398$$
$$\text{or } 17.6\% < p < 39.8\%$$

Q6. (i) Margin of error $= \frac{1}{\sqrt{n}}$

$$\frac{1}{\sqrt{n}} = 0.05 \qquad \text{since } 5\% = 0.05$$
$$\left(\frac{1}{\sqrt{n}}\right)^2 = (0.05)^2$$
$$\therefore \frac{1}{n} = (0.05)^2$$
$$\therefore n = \frac{1}{(0.05)^2}$$
$$= 400 = \text{sample size}$$

(ii) Margin of error $= \frac{1}{\sqrt{n}}$ $\quad 3\% = 0.03$

$$\frac{1}{\sqrt{n}} = 0.03$$
$$\left(\frac{1}{\sqrt{n}}\right)^2 = (0.03)^2$$
$$\therefore \frac{1}{n} = (0.03)^2$$
$$\therefore n = \frac{1}{(0.03)^2}$$
$$= 1{,}111 = \text{sample size}$$

(iii) Margin of error $= \frac{1}{\sqrt{n}}$, $\quad 1.5 = 0.015$

$$\frac{1}{\sqrt{n}} = 0.015$$
$$\left(\frac{1}{\sqrt{n}}\right)^2 = (0.015)^2$$
$$\therefore \frac{1}{n} = (0.015)^2$$
$$\therefore n = \frac{1}{(0.015)^2}$$
$$\therefore n = 4{,}444 = \text{sample size}$$

Q7. Sample proportion, $\hat{p} = \dfrac{84}{200} = 0.42$

Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{200}} = 0.07$

Confidence interval (95% level) $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$= 0.42 - 0.07 < p < 0.42 + 0.07$

$= 0.35 < p < 0.49$

Q8. Sample proportion, $\hat{p} = \dfrac{357}{1,000} = 0.357$

Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{1,000}} = 0.0316$

95% confidence interval $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$= 0.357 - 0.0316 < p < 0.357 + 0.0316$

$\therefore \quad 0.325 < p < 0.389$

No. The leader's belief is not justified as 0.4 is outside the above range at the 95% confidence interval

* Step 1: State H_0 and H_1

H_0: The true proportion is 0.4

H_1 : The true proportion is not 0.4

Step 2 : Sample proportion $\hat{p}$ (above)

Step 3 : Margin of error, (above)

Step 4 : Confidence interval (above)

Step 5 : The population proportion 0.4 is not within the confidence interval. So we reject the null hypothesis and accept H_1. We conclude that the leaders belief is not justified at the 95% confidence level.

Q9. 1. H_0 : The college admits equal numbers

H_1 : The college does not admit equal numbers

2. Sample proportion, $\hat{p} = \dfrac{267}{500} = 0.534$

3. Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{500}} = 0.0447$

4. Confidence interval $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$= 0.534 - 0.0447 < p < 0.534 + 0.0447$

$0.489 < p < 0.5787$

$\therefore \quad 0.489 < p < 0.579$

5. There is evidence to suggest that the college is not evenly divided in admitting equal numbers of men and women, since 0.5 is within the confidence range found for men at the 95% level.

Q10. (i) Sample proportion, $\hat{p} = \frac{52}{240}$

$= 0.2166$

(ii) Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{240}} = 0.065$

(iii) Probability of throwing a $6 = 0.1667$

(iv) H_0 : The dice is not biased

H_1 : The dice is biased

From above $\hat{p} = 0.2166$

Margin of error $= 0.065$

$\therefore$ Confidence interval

$= 0.216 - 0.064 < p < 0.216 + 0.064$

$= 0.152 < p < 0.28$

Since 0.1667 is within the 95% confidence interval found we accept H_0 and conclude that the dice is not biased.

Q11. 1. H_0 : The proportion of overdue books had not decreased

H_1 : The proportion of overdue books had decreased

2. Sample proportion, $\hat{p} = \frac{15}{200} = 0.075$

3. Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{200}} = 0.07$

4. Confidence interval $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$= 0.075 - 0.07 < p < 0.075 + 0.07$

$= 0.005 < p < 0.145$

$\therefore$ confidence interval at the 95% level is

$0.5\% < p < 14.5\%$

5. Since 12% lies in this interval the survey is correct and the University's claim that the proportion of overdue books had decreased is not justified.

Q12. 1. H_0 : The company claims 20% will not have red flowers

H_1 : The company claims 20% will have red flowers

2. Sample proportion, $\hat{p} = \frac{11}{82} = 0.134$

3. Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{82}} = 0.11$

4. Confidence interval $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$= 0.134 - 0.11 < p < 0.134 + 0.11$$
$$= 0.024 < p < 0.244$$
$$\therefore \quad 2.4\% < p < 24.4\%$$

5. Since the claim of 20% of plants will have red flowers lies within the 95% confidence interval the company's claim is correct.

Q13. 1. H_0 : at least 60% of its readers do not have third level degrees.
H_1 : at least 60% of its readers do have third level degrees.

2. Sample proportion, $\hat{p} = \frac{208}{312} = 0.6666$

3. Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{312}} = 0.0566$

4. Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$\therefore \quad 0.6666 - 0.0566 < p < 0.6666 + 0.0566$$
$$= 0.61 < p < 0.723$$
$$\therefore \quad 61\% < p < 72.3\%$$

5. Hence the "Daily Mensa's" claim that at least 60% of its readers have third level degrees is justified.

Q14. Sample proportion, $\hat{p} = \frac{45}{300} = 0.15$

Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{300}} = 0.057$

(i) Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$\therefore \quad 0.15 - 0.057 < p < 0.15 + 0.057$$
$$= 0.093 < p < 0.207$$
$$= 0.09 < p < 0.21$$

(ii) If 100 samples were taken we would expect 95 of them to have defective items ranging between 9% and 21% (or between 27 items and 63 items)

(iii) If 200 such tests were performed we would expect 2 × 95 of them to have defective items $\therefore$ 190 defective items.

Test Yourself 4

A – Questions

Q1. On left-hand side of 0,

between -2σ and 0 there is $\frac{1}{2}(95\%) = 47.5\%$

Between 0 and 1σ, there is $\frac{1}{2}(68\%) = 34\%$

$\therefore$ shaded region under curve $= 81.5\%$

Q2. (i) $B -$ positive correlation

(ii) $A -$ negative correlation

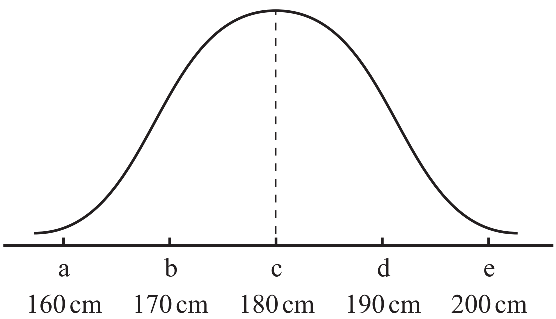
(iii) $C -$ no correlation

(iv) $A -$ negative correlation

(v) $B -$ correlation coefficient of approx 0.7

Q3. $\mu = 180$ cm $\sigma = 10$ cm

(i)



(ii) $z\text{-score} = \dfrac{190-180}{10} = \dfrac{10}{10} = 1$

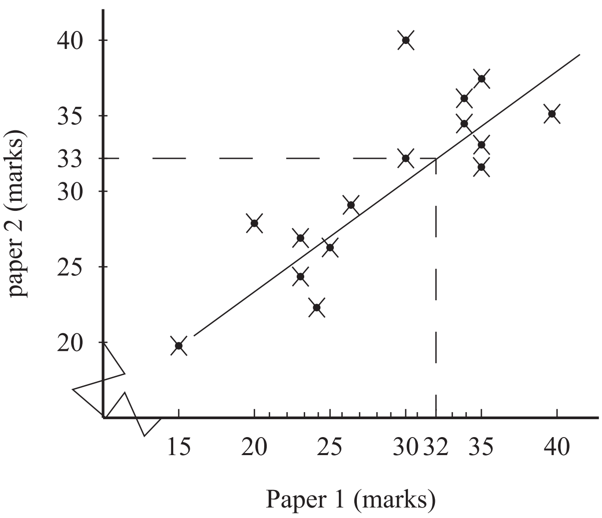
$\therefore z = 1$

(iii) 34% of sample have height between 180 and 190.

$\therefore 50\% - 34\% = 16\%$

$\therefore$ 16% have height greater than 190 cm

Q4. (i)



(ii) Strong positive

(iii) Line on graph

(iv) Taking two points on the line of best fit

$$(25, 27.5) \qquad (40, 37.5)$$

$$\text{slope } m = \frac{37.5-27.5}{40-25} = \frac{10}{15} = 0.666$$

$$\Rightarrow m = 0.7$$

Eq. of line

$$y - 27.5 = 0.7(x-25)$$

$$y - 27.5 = 0.7x - 17.5$$

$$\therefore y = 0.7x + 10$$

Using calculator line of best fit is

$$y = 0.713x + 9.74$$

(v) Drawing in the line from $x = 32$

on the graph gives $y =$ approx 33.

or

Substituting $x = 32$ into the equation of the line of best fit

$$x = 32$$

$$\therefore y = 0.713(32) + 9.74$$

$$= 22.816 + 9.74$$

$$= 32.556$$

$\therefore$ score is 33 marks

Q5. $\mu = 175$ cm

$x = 160 + 15$ $\qquad$ $x = 190 - 15$

$= 175$ $\qquad$ $= 175$

$\therefore \quad 160 = 175 - 1\sigma$

$\therefore \quad 190 = 175 + 1\sigma$

Given 95% of students have heights between 160 and 190

i.e. $\mu \pm 2\sigma$

$$\therefore 2\sigma = 15$$

$$\therefore \sigma = 7.5$$

Q6. (i) Sample proportion, $\hat{p} = \dfrac{170}{250} = 0.68$

Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{250}} = 0.063$

(ii) Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$= 0.68 - 0.063 < p < 0.68 + 0.063$

$\therefore \quad 0.617 < p < 0.743$

$\therefore \quad 0.62 < p < 0.74$

is confidence interval for the proportion of households that own at least one pet.

Q7. (i) Correlation is a measure of the strength of the linear relationship between two sets of variables.

(ii) (a) $r = 0.916$ (calculator)

(b) It is very likely that a student who has done well in test 1 will also have done well in test 2.

Q8. (i) Since 95% of a sample lies between $\pm 2\sigma$ of the mean, then diagram (i) has a 95% probability that a bamboo cane will have length falling in the shaded area.

(ii) Here in diagram (ii), $\frac{1}{2}$ (95%) is shaded so the probability of a bamboo cane having a length falling in the shaded area = 47.5%

Q9. (i) Simon's French test:

$x = 76$ marks, $\mu = 68$ marks, $\sigma = 10$ marks

$z\text{-score} = \frac{78-68}{10} = \frac{8}{10} = 0.8$

(ii) Simon's German test:

$x = 78$ marks, $\mu = 70$ marks, $\sigma = 12$ marks

$z\text{-score} = \frac{78-70}{12} = \frac{8}{12} = 0.66$

(iii) Simon did better in his French test

Q10. There may be a strong positive correlation between house prices and car sales but that does not imply that one increase **causes** the other.

Test Yourself 4

B – Questions

Q1. (i) Sample proportion, $\hat{p} = \dfrac{527}{2,000} = 0.26$

Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{2,000}} = 0.022$

(ii) Confidence interval (95% level) $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$= 0.26 - 0.022 < p < 0.26 + 0.022$

$= 0.241 < p < 0.286$

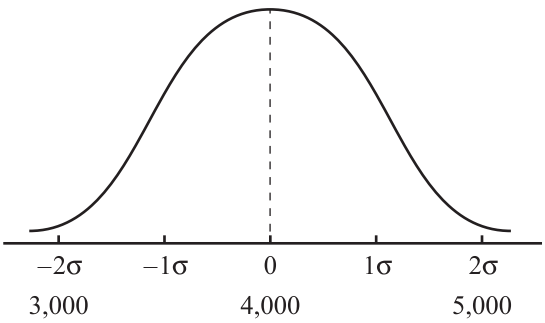
Q2. (i) $x = 3,000$ hours, $\mu = 4,000$ hrs, $\sigma = 500$ hrs

$z\text{-score} = \dfrac{3,000 - 4,000}{500} = \dfrac{-1000}{500} = -2$

$\therefore \dfrac{1}{2}(95\%) = 47.5\%$ of bulbs last between 3,000 and 4,000 hours

$\therefore 50\% - 47.5\%$ last less than 3,000 hours

$\therefore 2.5\%$ last less than 3,000 hrs



(ii) The probability that a tube will last between 3,000 and 5,000 hours

i.e. $\mu \pm 2\sigma = 0.95$

(iii) $2\frac{1}{2}\%$ of the tubes will be expected to be working after 5,000 hours.

In a batch of 10,000 tubes = 250

Q3. (i) $r = 0.959$ (calculator)

(ii) This value shows a very strong positive correlation between the number of employees and the units produced.

Q4. H_0 : the party has 23% support

H_1 : the party does not have 23% support

(i) Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{1111}} = 0.03$ at 95% confidence

(ii) Sample proportion, $\hat{p} = \dfrac{234}{1{,}111} = 0.21$

Confidence interval $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$$= 0.21 - 0.03 < p < 0.21 + 0.03$$
$$= 0.18 < p < 0.24$$
$$\therefore 18\% < p < 24\%$$

The political party has claimed to have 23% support of the electorate.

This is within the confidence interval. Hence, this is not sufficient to reject the party's claim.

Q5. Tree 1:

$x = 7$ cm, $\mu = 5$ cm, $\sigma = 1$ cm

z-score $= \dfrac{7-5}{1} = \dfrac{2}{1} = 2$

Tree 2:

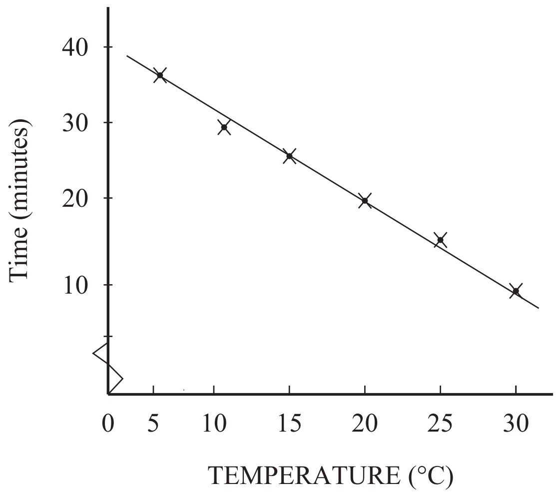
$x = 7$ cm, $\mu = 8$ cm, $\sigma = 1.5$ cm

z-score $= \dfrac{7-8}{1.5} = \dfrac{-1}{1.5} = -0.666$

$$= -0.67$$

Mr. Cross is correct since $z = -0.67$ has a greater chance of happening on the normal curve than $z = 2$.

Q6. (i)



(ii) Strong negative correlation

(iii) Two points on line of best fit are (10, 29) and (30, 8)

$$\text{slope} = \frac{8-29}{30-10} = \frac{-21}{20}$$

$$\therefore m = -1.05$$

Equation of line is $y = mx + c$

$$\therefore 29 = -1.05(10) + c$$

$$29 = -10.5 + c$$

$$\therefore c = 39.5$$

$$\therefore y = -1.05x + 39.5$$

Using calculator

$y = -1.12x + 41.6$ is the equation of the line of best fit.

(iv) When temp is = 0°C

$$0 = -1.12x + 41.6$$

$$\therefore \quad x = \frac{41.6}{1.12}$$

$$= 37.5$$

$$= 38 \text{ minutes}$$

(v) $r = -1$ (calculator)

Q7. 1. H_0 : 20% purchase at least one product

H_1 : 20% do not purchase at least one product

2. Sample proportion, $\hat{p} = \frac{64}{400} = 0.16$

3. Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{400}} = 0.05$

4. Confidence interval (95% level) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$= 0.16 - 0.05 < p < 0.16 + 0.05$$

$$= 0.11 < p < 0.21$$

$$\therefore \quad 11\% < p < 21\%$$

5. (ii) 20% is within this interval. Hence, there is no evidence to reject the company’s claim that 20% of the visitors purchase at least one of its products

Q8. (i) $\mu = 135$ cm, $x = 120$ cm, $\sigma = 10$ cm

$$z\text{-score} = \frac{120-135}{10} = \frac{-15}{10} = -1\frac{1}{2}$$

$\therefore$ David's height is -1.5σ below the mean

(ii) $\mu = 180$ cm, $\sigma = 18$ cm, z-score $= -1.5$

$$z\text{-score} = \frac{x-\mu}{\sigma}$$

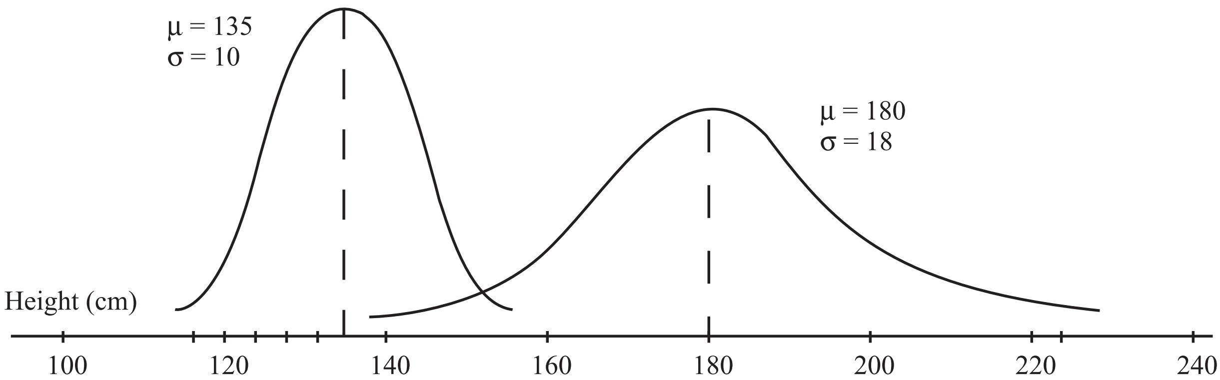
$$\therefore -1.5 = \frac{x-180}{18}$$

$$\therefore x - 180 = -1.5(18)$$

$$\therefore x = 180 - 27$$

$$= 153 \text{ cm tall}$$

(iii)



Q9. 1. H_0 : 70% are claimed to be in favour of change
H_1 : 70% are claimed to not be in favour of change

2. Sample proportion, $\hat{p} = \dfrac{134}{180} = 0.744$

3. Margin of error $= \dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{180}} = 0.0745$

4. Confidence interval (95% level) $= \hat{p} - \dfrac{1}{\sqrt{n}} < p < \hat{p} + \dfrac{1}{\sqrt{n}}$

$$= 0.744 - 0.0745 < p < 0.744 + 0.0745$$

$$= 0.669 < p < 0.8185$$

$$\therefore \quad 66.9\% < p < 81.85\%$$

$$\therefore \quad 66.9\% < p < 81.9\%$$

5. Since 70% is within this range at the 95% confidence level the NCCB's beliefs are borne out and the claim that 70% are in favour of syllabus change accepted.

Q10. 1. H_0 : Claim is that 10% of apples attacked

H_1 : Claim is that 10% of apples have not been attacked

2. Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{2,500}} = 0.02$

3. Sample proportion, $\hat{p} = \frac{274}{2,500} = 0.1096$

4. Confidence interval (at 95%) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$= 0.02 - 0.1096 < p < 0.02 + 0.1096$$
$$0.0896 < p < 0.1296$$
$$\therefore 8.96\% < p < 12.96\%$$
$$9\% < p < 13\%$$

5. Yes, the owner's claim is justified at the 95% confidence level as 10% is within the above range

Test Yourself 4

C – Questions

Q1. (i)(a) $\mu = 20$ mm, $\sigma = 3$ mm

$17 \text{ mm} = \mu - 1\sigma$

$23 \text{ mm} = \mu + 1\sigma$

$\therefore 17 - 23 \text{ mm} = 20 \pm 1\sigma$

68% of a normal distribution lies within this area (Empirical rule)

(b) $14 \text{ mm} = \mu - 2\sigma$

$23 \text{ mm} = \mu + \sigma$

$\therefore 14 \text{ mm} = 2\sigma$ below mean $= 47.5\%$

$\therefore 23 \text{ mm} = 1\sigma$ above mean $= 34\%$

$\therefore$ the percentage of nails measured 14 mm $-$ 23 mm is

$47.5\% + 34\%$

$= 81.5\%$

(ii) $17 = 1\sigma$ below mean $= 34\%$

$26 = 2\sigma$ above mean $= 47.5\%$

$\therefore 81.5\%$ are of $17 - 26$ mm nails.

When 10,000 are measured

$\therefore \dfrac{81.5}{100} \times 10,000 = 8150$ nails

(iii) $23 \text{ mm} = 1\sigma$ above μ

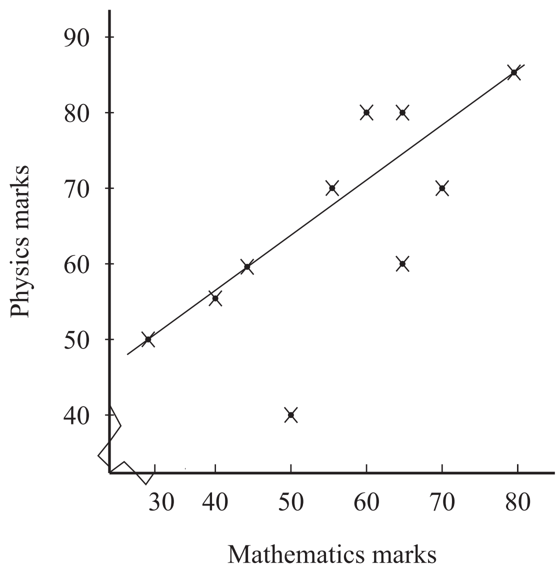
$= 34\%$

50% of all nails are > 20 (mean length)

$\therefore 50\% - 34\%$ of nails are more than 23 mm long

$= 16\%$

Q2.



(ii) Equation of line of best fit

$y = 0.7x + 25$ (calculator)

Using two points:

$(30, 50)$ $(80, 85)$

$$\text{slope} = \frac{85-50}{80-30} = \frac{35}{50} = 0.7$$

$$y - 50 = 0.7(x-30)$$

$$y - 50 = 0.7x - 21$$

$$y = 0.7x - 21 + 50$$

$$y = 0.7x + 29$$

(iii) $r = 0.737$ (calculator)

(iv) There is a fairly strong positive correlation between the mathematics and physics results of the students

Q3. $\hat{p} = \frac{352}{400} = 0.88$ i.e. 88%

(i) Margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{400}} = 0.05$

(ii) Confidence interval (95%) $= \hat{p} - \frac{1}{\sqrt{n}} < p < \hat{p} + \frac{1}{\sqrt{n}}$

$$= 0.88 - 0.05 < p < 0.88 + 0.05$$

$$0.83 < p < 0.93$$

$$83\% < p < 93\%$$

H_0 : There is no difference in opinion between Cork and Dublin

H_1 : The is a difference in opinion between Cork and Dublin

Sample proportion, $\hat{p} = \frac{810}{1000} = 0.81$

$= 81\%$

Confidence interval is

$83\% < p < 93\%$.

The company's claim in *not* justified at the 95% confidence level as 81% (the Dublin population proportion) is not within the confidence limits, so we reject the null Hypothesis and accept their claim is not justified, and there is a difference in opinion between Cork and Dublin samples.

Q4. $\mu = 60$ yrs, $\sigma = 8$ yrs

(i) (a) Abdul z-score:

$$z = \frac{x-\mu}{\sigma} = \frac{70-60}{8} = 1.25$$

(b) Marie z-score:

$$\frac{52-60}{8} = \frac{-8}{8} = -1$$

(c) George z-score:

$$\frac{60-60}{8}=0$$

(d) Elsie z-score:

$$z=\frac{92-60}{8}=4$$

(ii) $76=60+16=\mu+2\sigma$

$$=47.5\%$$

Hence, the percentage of people more than 76 years is

$50\%-47.5\%=2.5\%$

(iii) Ezra

$$2.5=\frac{x-60}{8}$$

$$\therefore x-60=8(2.5)$$

$$\therefore x-60=20$$

$$\therefore \quad x=60+20$$

$$=80 \text{ years}$$

(iv) $x=40$ yrs

$$z=\frac{40-60}{8}=\frac{-20}{8}=-2.5$$

Since the z-score $=-2.5$ it is very unlikely as the probability will be less than 1%

Q5. (i) $r=-0.85$ approx

(ii) Outlier: age $=37$, bpm $=139$

(iii) Read from x (age value) = 44 to cut the line of best fit and read y (bpm value)
Solution (44, 180 bpm)

(iv) Possible points: (20, 200) (80, 150)

$$\text{slope}=\frac{150-200}{80-20}=\frac{-50}{60}=-0.833$$

$$\therefore m=-0.8$$

(v) Equation of the line of best fit

$$y-y_1=m(x-x_1)$$

$$y-200=-0.833(x-20)$$

$$y-200=-0.833x+20(0.833)$$

$$y=-0.8x+16$$

$$=200+16-0.8(\text{age})$$

Replacing y with MHR

MHR $=216-0.8$ (age)

(vi)

Age	Old rule	New rule
20	200	200
50	170	176
70	150	160

For a younger person (20 years) the MHRs are roughly the same. For an older person (50 years or 70 years) the new rule gives a higher MHR reading.

(vii) At 65 years, the old rule gives MHR = 155 and the new rule gives MHR = 164. To get more benefit from exercise, he should increase his activity to 75% of 164 instead of 75% of 155.